

# ACS Introduction to NLP

## Lecture 3: Language Modelling and Smoothing



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- A **language model** is a probability distribution over strings
  - A core component of many language processing systems
    - speech recognition, hand writing recognition, machine translation, ...
  - Many of the estimation techniques carry over to other NLP problems
    - POS tagging, parsing, ...

$$\begin{aligned} P(w_1, \dots, w_n) &= P(w_1)P(w_2|w_1)P(w_3|w_1, w_2) \dots P(w_n|w_1, \dots, w_{n-1}) \\ &\approx P(w_1)P(w_2|w_1)P(w_3|w_1, w_2) \dots P(w_n|w_{n-2}, w_{n-1}) \end{aligned}$$

- Chain rule followed by independence assumptions
  - here the independence assumptions give a **trigram** language model
- Notice chain rule is *exact*
- Note also that we don't have to apply the chain rule using this ordering of the words

- I am happy with the idea of the following probabilities:
  - a fair coin landing heads =  $1/2$  (frequentist)
  - Red Rum winning the Grand National =  $1/5$  (subjectivist)
  - rain tomorrow =  $5/6$  (subjectivist)
  - Radium atom decaying in the next 30s =  $1/2$  (objectivist)
- But what about  $P(\textit{pig})$ ?
  - perhaps conditional probabilities more intuitive:  $P(*|\textit{the, fast})$ ?
- Probabilities of words reflect underlying statistical distribution of the text

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*It must be recognised that the notion of a ‘probability of a sentence’ is an entirely useless one, under any interpretation of this term (Chomsky, 1969)*

[taken from Chapter 1 of Young and Bloothoof, eds, *Corpus-Based Methods in Language and Speech Processing*]

[Colorless green ideas sleep furiously example]

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*Statistics is the science of learning from observations and experience*  
(Ney, 1997)

*By chance we mean something like a guess. Why do we make guesses? We make guesses when we wish to make a judgement but have incomplete information or uncertain knowledge. We want to make a guess as to what things are, or what things are likely to happen. Often we wish to make a guess because we have to make a decision ... Sometimes we make guesses because we wish, with our limited knowledge, to say as much as we can about some situation. Really, any generalization is in the nature of a guess. Any physical theory is a kind of guesswork. There are good guesses and there are bad guesses. The theory of probability is a system for making better guesses. The language of probability allows us to speak quantitatively about some situation which may be highly variable, but which does have some consistent average behavior. (Guess who?)*

[taken from Chapter 1 of Young and Bloothoof, eds, *Corpus-Based Methods in Language and Speech Processing*]

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- Maximum likelihood estimation (relative frequency estimation in our cases) can suffer from *overfitting*
  - Because we are choosing parameters to make the data as probable as possible, the resulting model can look “too much” like the data
  - Classic example of this occurs in language modelling with a word we haven’t seen before

$$\hat{P}(w_3|w_2, w_1) = \frac{f(w_1, w_2, w_3)}{f(w_1, w_2)}$$

- If  $w_1$  followed by  $w_2$  is unseen then estimate is undefined
- If  $w_1$  followed by  $w_2$  followed by  $w_3$  is unseen then estimate is zero
- Zeros propagate through the product to give a zero probability for the whole string
- *Smoothing* techniques are designed to solve this problem (called *smoothing* because the resulting distributions tend to be more uniform)

$$\begin{aligned}\hat{P}(w_i|w_{i-1}) &= \frac{1 + f(w_{i-1}, w_i)}{\sum_{w_i} 1 + f(w_{i-1}, w_i)} \\ &= \frac{1 + f(w_{i-1}, w_i)}{|V| + \sum_{w_i} f(w_{i-1}, w_i)}\end{aligned}$$

- Simple technique not widely used anymore

- Consider *burnish the* and *burnish thou* where:

$$f(\textit{burnish}, \textit{the}) = f(\textit{burnish}, \textit{thou}) = 0$$

- add-1 smoothing would assign the same probability to  $P(\textit{the}|\textit{burnish})$  and  $P(\textit{thou}|\textit{burnish})$
- But *burnish the* intuitively more likely (because *the* more likely than *thou*)

$$\tilde{P}(w_i|w_{i-1}) = \lambda \hat{P}(w_i|w_{i-1}) + (1 - \lambda) \hat{P}(w_i)$$

- $\tilde{P}$  is the interpolated model and  $\hat{P}$  is the maximum likelihood (relative frequency) estimate

$$\begin{aligned} \tilde{P}(w_i|w_{i-1}, w_{i-2}) &= \lambda_1 \hat{P}(w_i|w_{i-1}, w_{i-2}) \\ &+ (1 - \lambda_1)(\lambda_2 \hat{P}(w_i|w_{i-1}) + (1 - \lambda_2) \hat{P}(w_i)) \end{aligned}$$

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$$\begin{aligned}\tilde{P}(w_i|w_{i-1}, w_{i-2}) &= \hat{P}(w_i|w_{i-1}, w_{i-2}) && \text{if } f(w_{i-2}, w_{i-1}, w_i) > 0 \\ &= \alpha_1 \hat{P}(w_i|w_{i-1}) && \text{if } f(w_{i-2}, w_{i-1}, w_i) = 0 \\ & && \text{and } f(w_{i-1}, w_i) > 0 \\ &= \alpha_2 \hat{P}(w_i) && \text{otherwise}\end{aligned}$$

- where the  $\alpha$ s are required to ensure a proper distribution
- although see *Large Language Models in Machine Translation*, Brants et al., based on **2 trillion tokens** which uses “stupid backoff” and ignores the  $\alpha$ s

- Tag sequence probabilities can be smoothed (or backed off):

$$\begin{aligned}\tilde{P}(t_i|t_{i-1}, t_{i-2}) &= \lambda_1 \hat{P}(t_i|t_{i-1}, t_{i-2}) \\ &+ (1 - \lambda_1)(\lambda_2 \hat{P}(t_i|t_{i-1}) + (1 - \lambda_2) \hat{P}(t_i))\end{aligned}$$

- A simple solution for unknown words is to replace them with UNK:

$$P(w_i|t_i) = P(\text{UNK}|t_i)$$

where any word in the training data occurring less than, say, 5 times is replaced with UNK

- Lots of clues as to what the tag of an unknown word might be:
  - proper nouns (NNP) likely to be unknown
  - if the word ends in *ing*, likely to be VBG
  - ...

$$P(w|t) = \frac{1}{Z} P(\text{unknown word}|t) P(\text{capitalized}|t) P(\text{endings}|t)$$

- but now we're starting to see the weaknesses of generative models for taggers
- *Conditional* models can deal with these features directly
  - see Ratnaparkhi's tagger as an example

- Jurafsky and Martin, *Speech and Language Processing*, Chapter on N-grams
- Manning and Schütze, *Foundations of Statistical Natural Language Processing*, Chapter on Statistical Inference: n-gram models over sparse data
- An Empirical Study of Smoothing Techniques for Language Modeling, Chen and Goodman, TR-10-98, 1998 (*lots of technical and empirical details*)
- Chapter 1 of *Corpus-Based Methods in Language and Speech Processing*, ed Steve Young and Gerrit Bloothoof, Kluwer 1997 (*if available*)