

# Summary of the rules of structured proof.

Discrete Maths I: slides 87–104

	<i>Introduction rules</i>	<i>Elimination rules</i>
$\wedge$	<p>...</p> <p><math>l.</math> <math>P</math> from ...</p> <p>...</p> <p><math>m.</math> <math>Q</math> from ...</p> <p>...</p> <p><math>n.</math> <math>P \wedge Q</math> from <math>l</math> and <math>m</math> by <math>\wedge</math>-introduction</p> <p>(it doesn't matter in what order <math>l</math> and <math>m</math> are in)</p>	<p>...</p> <p><math>m.</math> <math>P \wedge Q</math> from ...</p> <p>...</p> <p><math>n.</math> <math>P</math> from <math>m</math> by <math>\wedge</math>-elimination</p> <p>or</p> <p>...</p> <p><math>m.</math> <math>P \wedge Q</math> from ...</p> <p>...</p> <p><math>n.</math> <math>Q</math> from <math>m</math> by <math>\wedge</math>-elimination</p>
$\vee$	<p>...</p> <p><math>m.</math> <math>P</math> from ...</p> <p>...</p> <p><math>n.</math> <math>P \vee Q</math> from <math>m</math> by <math>\vee</math>-introduction</p> <p>or</p> <p>...</p> <p><math>m.</math> <math>Q</math> from ...</p> <p>...</p> <p><math>n.</math> <math>P \vee Q</math> from <math>m</math> by <math>\vee</math>-introduction</p>	<p><math>l.</math> <math>P \vee Q</math> from ... by ...</p> <div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <p>...</p> <p><math>m_1.</math> Assume <math>P</math></p> <p>...</p> <p><math>m_2.</math> <math>R</math></p> <p>...</p> </div> <div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <p>...</p> <p><math>n_1.</math> Assume <math>Q</math></p> <p>...</p> <p><math>n_2.</math> <math>R</math></p> <p>...</p> </div> <p><math>o.</math> <math>R</math> from <math>l, m_1-m_2, n_1-n_2</math> by <math>\vee</math>-elimination</p> <p>(it doesn't matter what order <math>l, m_1-m_2,</math> and <math>n_1-n_2</math> are in)</p>
$\Rightarrow$	<div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <p>...</p> <p><math>m.</math> Assume <math>P</math></p> <p>...</p> <p><math>n.</math> <math>Q</math> from ... by ...</p> <p>...</p> </div> <p><math>n + 1.</math> <math>P \Rightarrow Q</math> from <math>m-n,</math> by <math>\Rightarrow</math>-introduction</p>	<p>...</p> <p><math>l.</math> <math>P \Rightarrow Q</math> by ...</p> <p>...</p> <p><math>m.</math> <math>P</math> by ...</p> <p>...</p> <p><math>n.</math> <math>Q</math> from <math>l</math> and <math>m</math> by <math>\Rightarrow</math>-elimination</p>
$\neg$	<div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <p>...</p> <p><math>m.</math> Assume <math>P</math></p> <p>...</p> <p><math>n.</math> <math>F</math> from ... by ...</p> <p>...</p> </div> <p><math>n + 1.</math> <math>\neg P</math> from <math>m-n,</math> by <math>\neg</math>-introduction</p>	<p>...</p> <p><math>l.</math> <math>P</math> by ...</p> <p>...</p> <p><math>m.</math> <math>\neg P</math> by ...</p> <p>...</p> <p><math>n.</math> <math>F</math> from <math>l</math> and <math>m</math> by <math>\neg</math>-elimination</p>
$T$	<p>...</p> <p><math>n.</math> <math>T</math></p>	No elimination rule for True.
$F$	No introduction rule for False.	<p>...</p> <p><math>m.</math> <math>F</math> from ... by ...</p> <p>...</p> <p><math>n.</math> <math>P</math> from <math>m,</math> by contradiction</p>
$\forall$	<div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <p>...</p> <p><math>m.</math> Consider an arbitrary <math>x</math> (from domain ...)</p> <p>...</p> <p><math>n.</math> <math>P(x)</math> by ...</p> <p>...</p> </div> <p><math>n + 1.</math> <math>\forall x.P(x)</math> from <math>m-n</math> by <math>\forall</math>-introduction</p>	<p>...</p> <p><math>m.</math> <math>\forall x.P(x)</math> from ...</p> <p>...</p> <p><math>n.</math> <math>P(v)</math> from <math>m</math> by <math>\forall</math>-elimination</p>
$\exists$	<p>...</p> <p><math>m.</math> <math>P(v)</math></p> <p>...</p> <p><math>n.</math> <math>\exists x.P(x)</math> from <math>m</math> by <math>\exists</math>-introduction with witness <math>x = v</math></p>	<p><math>l.</math> <math>\exists x.P(x)</math></p> <p>...</p> <div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <p><math>m.</math> For some actual <math>x_1, P(x_1)</math></p> <p>...</p> <p><math>n.</math> <math>Q</math> (where <math>x_1</math> not free in <math>Q</math>)</p> <p>...</p> </div> <p><math>o.</math> <math>Q</math> from <math>l, m-n,</math> by <math>\exists</math>-elimination</p>
	<div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <p>...</p> <p><math>m.</math> Assume <math>\neg P</math></p> <p>...</p> <p><math>n.</math> <math>F</math> from ... by ...</p> <p>...</p> </div> <p><math>n + 1.</math> <math>P</math> from <math>m-n,</math> by contradiction</p> <p style="text-align: right;">(Proof by contradiction)</p>	