## Summary of the rules of structured proof.

**Introduction rules**

<table>
<thead>
<tr>
<th>△</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>l.</td>
<td>(P) from ...</td>
<td></td>
</tr>
<tr>
<td>m.</td>
<td>(Q) from ...</td>
<td></td>
</tr>
<tr>
<td>n.</td>
<td>(P \land Q) from (l) and (m) by (\land)-introduction</td>
<td></td>
</tr>
</tbody>
</table>

(\(\Delta\) doesn't matter in what order \(l\) and \(m\) are in)

<table>
<thead>
<tr>
<th>(\lor)</th>
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<tr>
<td>m.</td>
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<tr>
<td>n.</td>
<td>(Q) from ...</td>
<td></td>
</tr>
<tr>
<td>n + 1. (P \lor Q) from (m \land n) by (\lor)-introduction</td>
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<table>
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<tr>
<th>(\neg)</th>
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<tr>
<td>l.</td>
<td>(P) from ...</td>
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<td>m.</td>
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<tr>
<th>(\forall)</th>
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<tr>
<td>m.</td>
<td>Assume (P) from ...</td>
<td></td>
</tr>
<tr>
<td>n.</td>
<td>(Q) from ...</td>
<td></td>
</tr>
<tr>
<td>n + 1. (\forall x. P(x)) from (n \land x) by (\forall)-introduction</td>
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<th>(\exists)</th>
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<tr>
<td>m.</td>
<td>Assume (\neg P) from ...</td>
<td></td>
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<tr>
<td>n.</td>
<td>(P) from ...</td>
<td></td>
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<tr>
<td>n + 1. (\neg P) from (m \land n) by contradiction</td>
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**Elimination rules**

<table>
<thead>
<tr>
<th>△</th>
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<tbody>
<tr>
<td>l.</td>
<td>(P \land Q) from ... by ...</td>
<td></td>
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<tr>
<td>m.</td>
<td>(P) from ...</td>
<td></td>
</tr>
<tr>
<td>n.</td>
<td>(P \land Q) from (m) by (\land)-elimination</td>
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<th>(\lor)</th>
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<td>l.</td>
<td>(P \lor Q) from ... by ...</td>
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<td>m.</td>
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<tbody>
<tr>
<td>l.</td>
<td>(P) by ...</td>
<td></td>
</tr>
<tr>
<td>m.</td>
<td>(\neg P) by ...</td>
<td></td>
</tr>
<tr>
<td>n.</td>
<td>(P) from (l) and (m) by (\neg)-elimination</td>
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<tr>
<td>m.</td>
<td>(\neg (\forall x. P(x))) from ...</td>
<td></td>
</tr>
<tr>
<td>n.</td>
<td>(P) from (m) by (\forall)-elimination</td>
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<tbody>
<tr>
<td>m.</td>
<td>Assume (\exists x. P(x)) from ...</td>
<td></td>
</tr>
<tr>
<td>n.</td>
<td>(P) from ...</td>
<td></td>
</tr>
<tr>
<td>n + 1. (P) from (m \land n), by contradiction</td>
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**Proof by contradiction**

### Notes

- **\(\Delta\)**: A disjunction is derived from both \(P\) and \(Q\).
- **\(\lor\)**: A disjunction is derived from \(P\) or \(Q\).
- **\(\neg\)**: The negation of \(P\) is derived from \(P\).
- **\(\forall\)**: A universal quantification is derived from \(P\) for any \(x\).
- **\(\exists\)**: An existential quantification is derived from \(P\) with witness \(v\).
- **\(\land\)**: A conjunction is derived from both \(P\) and \(Q\).
- **\(\lor\)**: A conjunction is derived from \(P\) and \(Q\).
- **\(\neg\)**: The negation of \(P\) is derived from \(P\).
- **\(\forall\)**: A universal quantification is derived from \(P\) for any \(x\).
- **\(\exists\)**: An existential quantification is derived from \(P\) with witness \(v\).

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*Sam Staton, Jan 2011.*