Category Theory Exercises: Week 4

November 2009

These exercises are not compulsory, and they will not contribute to your final grade. Please send your solutions or questions by e-mail to bk291@cam.ac.uk, or leave them in Bartek Klin’s pigeonhole next to Reception.

**Exercise 1.** Prove that if a functor preserves limits then it preserves monos.  
*Hint:* Use Exercise 1 for week 3.

**Exercise 2.** Given any monoid \((M, \cdot, 1)\), one can define a relation \(\leq\) on the set \(M\) by:

\[ m \leq n \iff \exists k \in M. m \cdot k = n. \]

It is easy to check that \(\leq\) is a preorder (i.e., it is reflexive and transitive).

Show how the above construction extends to a functor from the category \(\text{Mon}\) of monoids and monoid homomorphisms to the category \(\text{Preord}\) of preorders and monotonic functions.