Category Theory Exercises: Week 3

October 2009

These exercises are not compulsory, and they will not contribute to your final grade. Please send your solutions or questions by e-mail to `bk291@cam.ac.uk`, or leave them in Bartek Klin's pigeonhole next to Reception.

Exercise 1. Prove that $f : A \to B$ is a mono if and only if

$$
\begin{array}{ccc}
A & \xrightarrow{\;1_A\;} & A \\
{\scriptstyle 1_A}\downarrow & & \downarrow{\scriptstyle f} \\
A & \xrightarrow[\;f\;]{} & B
\end{array}
$$

is a pullback diagram.

Exercise 2. Prove that if $\mathbf{C}$ is finitely complete then for any object A in $\mathbf{C}$, the slice category $\mathbf{C}/A$ is finitely complete.

Hint: It is enough to show that $\mathbf{C}/A$ has a final object, products and equalizers. A final object is easy to find. To construct products, use pullbacks in $\mathbf{C}$. To construct equalizers, use equalizers in $\mathbf{C}$.