Category Theory Exercises: Week 2

October 2009

These exercises are not compulsory, and they will not contribute to your final grade. Please send your solutions or questions by e-mail to `bk291@cam.ac.uk`, or leave them in Bartek Klin's pigeonhole next to Reception.

Exercise 1. Let $\mathbf{D}$ be a subcategory of $\mathbf{C}$, and let $f : X \to Y$ be an arrow in $\mathbf{D}$. Prove that:

- if f is a mono in $\mathbf{C}$ then it is a mono in $\mathbf{D}$,
- if f is a section in $\mathbf{D}$ then it is a section in $\mathbf{C}$.

Exercise 2. A *partial function* from a set A to a set B, denoted $f : A \rightharpoonup B$, is:

- a subset $C \subseteq A$ with
- a function $f : C \to B$.

Note that C is fully determined by f and is therefore omitted in the notation $f : A \rightharpoonup B$. C is called the *domain of definition* of f. Composition of partial functions is defined in the obvious way and is clearly associative; also, identity functions are clearly units for composition. Thus one gets the category of sets and partial functions $\mathbf{Par}$.

Characterize binary products in $\mathbf{Par}$.

Hint: First realize why the Cartesian product of sets does not satisfy the definition of categorical product in $\mathbf{Par}$.