Category Theory Exercises: Week 2

October 2009

These exercises are not compulsory, and they will not contribute to your final grade. Please send your solutions or questions by e-mail to bk291@cam.ac.uk, or leave them in Bartek Klin’s pigeonhole next to Reception.

Exercise 1. Let D be a subcategory of C, and let \( f : X \to Y \) be an arrow in D. Prove that:

- if \( f \) is a mono in \( C \) then it is a mono in \( D \),
- if \( f \) is a section in \( D \) then it is a section in \( C \).

Exercise 2. A partial function from a set \( A \) to a set \( B \), denoted \( f : A \twoheadrightarrow B \), is:

- a subset \( C \subseteq A \) with
- a function \( f : C \to B \).

Note that \( C \) is fully determined by \( f \) and is therefore omitted in the notation \( f : A \twoheadrightarrow B \). \( C \) is called the domain of definition of \( f \). Composition of partial functions is defined in the obvious way and is clearly associative; also, identity functions are clearly units for composition. Thus one gets the category of sets and partial functions \( \text{Par} \).

Characterize binary products in \( \text{Par} \).

Hint: First realize why the Cartesian product of sets does not satisfy the definition of categorical product in \( \text{Par} \).