1. Given a graph $G = (V, E)$, a set $U \subseteq V$ of vertices is called a vertex cover of $G$ if, for each edge $(u, v) \in E$, either $u \in U$ or $v \in U$. That is, each edge has at least one end point in $U$. The decision problem $V$-COVER is defined as:

given a graph $G = (V, E)$, and an integer $K$, does $G$ contain a vertex cover with $K$ or fewer elements?

(a) Show a polynomial time reduction from IND to $V$-COVER.
(b) Use (a) to argue that $V$-COVER is NP-complete.

2. The problem of four dimensional matching, 4DM, is defined analogously with 3DM:

Given four sets, $W, X, Y$ and $Z$, each with $n$ elements, and a set of quadruples $M \subseteq W \times X \times Y \times Z$, is there a subset $M' \subseteq M$, such that each element of $W, X, Y$ and $Z$ appears in exactly one triple in $M'$.

Show that 4DM is NP-complete.

3. Given a graph $G = (V, E)$, a source vertex $s \in V$ and a target vertex $t \in V$, a Hamiltonian Path from $s$ to $t$ in $G$ is a path that begins at $s$, ends at $t$ and visits every vertex in $V$ exactly once. We define the decision problem $\text{HamPath}$ as:

given a graph $G = (V, E)$ and vertices $s, t \in V$, does $G$ contain a Hamiltonian path from $s$ to $t$?

(a) Give a polynomial time reduction from the Hamiltonian cycle problem to $\text{HamPath}$.
(b) Give a polynomial time reduction from $\text{HamPath}$ to the problem of determining whether a graph has a Hamiltonian cycle.

Hint: consider adding a vertex to the graph.