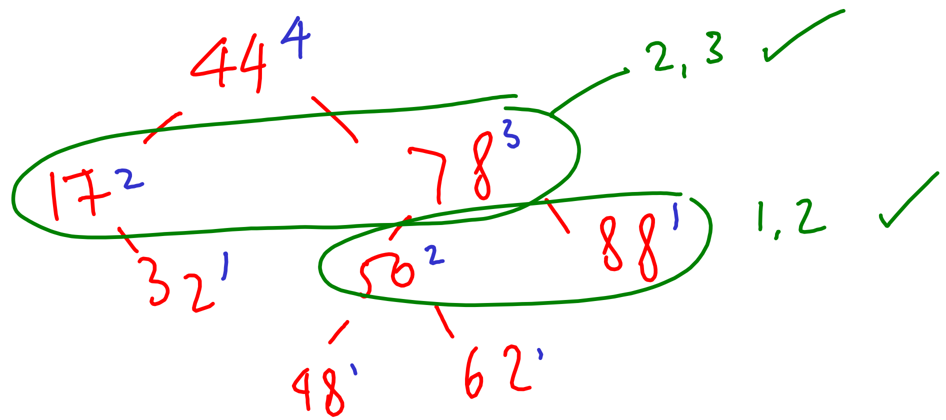
AVL Trees

AVL Trees

- Another, more simple way to auto-balance a BST
  - (Named after inventors)
  - It is essentially our manual approach of applying rotations, but using a nice metric to decide when and what to rotate

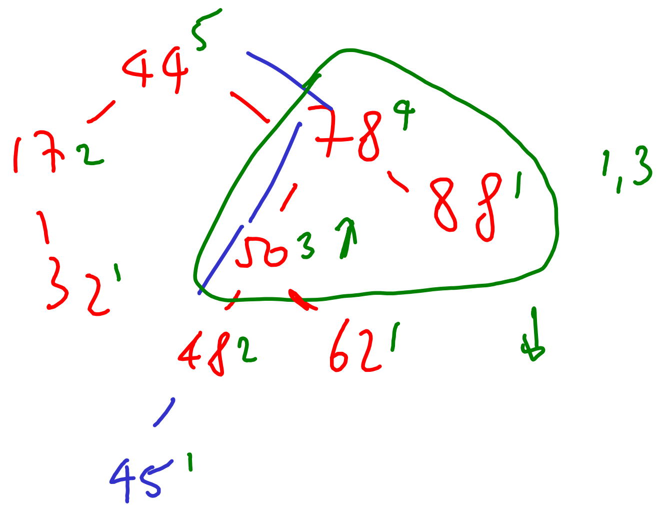
The rule:

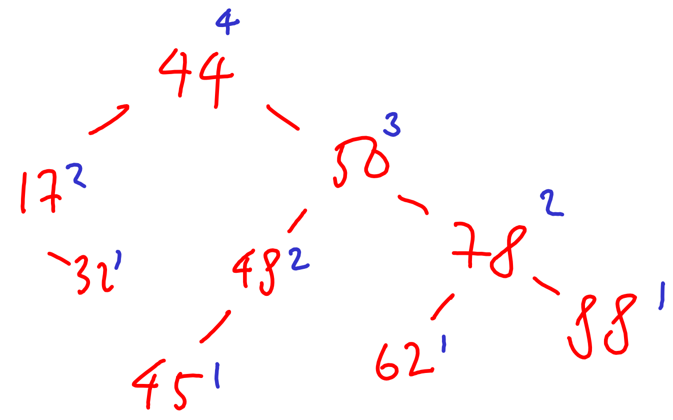
The heights of sibling nodes must differ by no more than one



Insertion

- Insert as usual, then look for violations





Rotate to fix

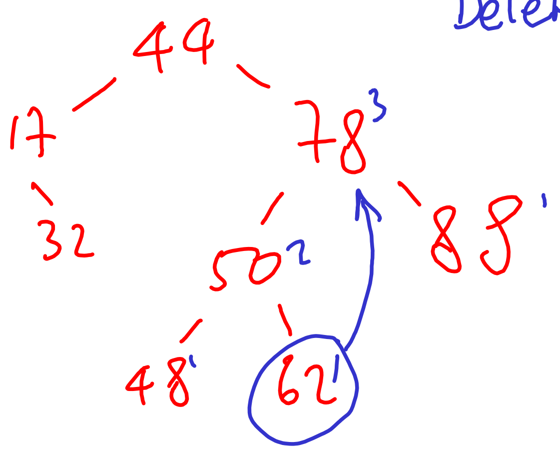
"Rebalance the tree"

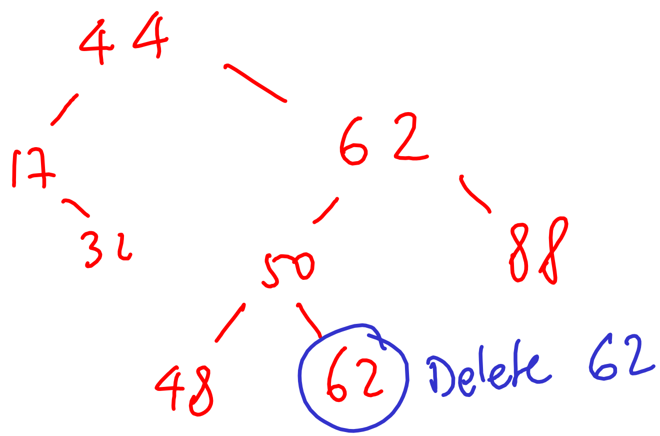
Deletion

Leaf node ⇒ Delete, rebalance

Non-leaf node ⇒ Decide left/right based on height
⇒ Find predecessor or successor

Delete 78





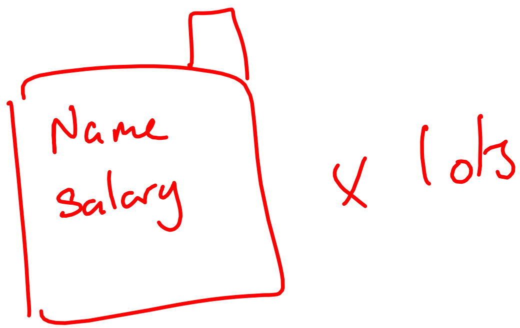
AVL vs Red-Black

- AVL trees are more rigidly balanced than RB trees
  - i.e. on average they are shorter than their RB equivalents
  - Marginally faster to search
- But the extra work needed to get that shorter tree means insertions and deletions are slower
- In most cases, not much to choose between them

B-Trees

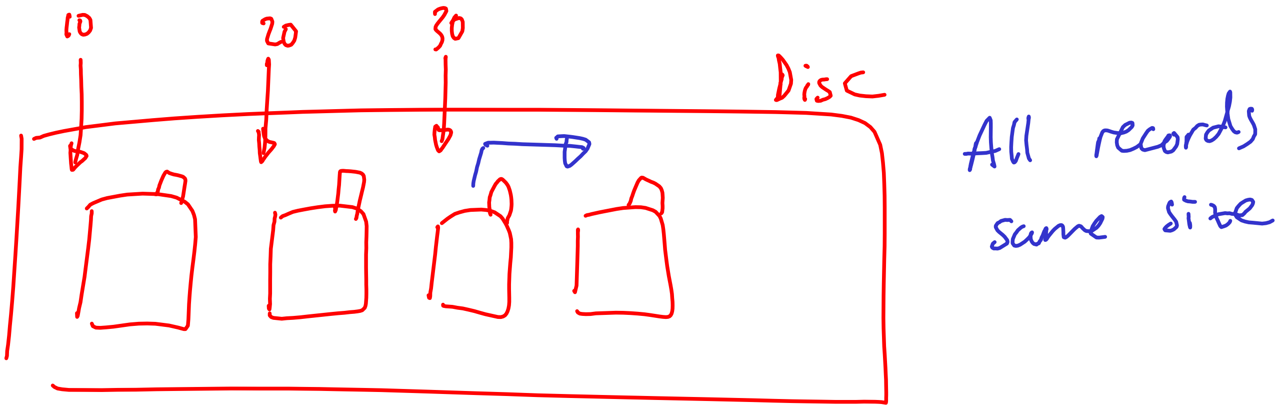
Databases

- A database is just a collection of records
  - Each record has a series of data fields
  - We want to be able to find a given record quickly
  - But there are typically *lots* of records



Databases

- We end up putting all the data on to hard drives due to size problems
  - We choose a field to sort by and write the records to the disc in sorted order

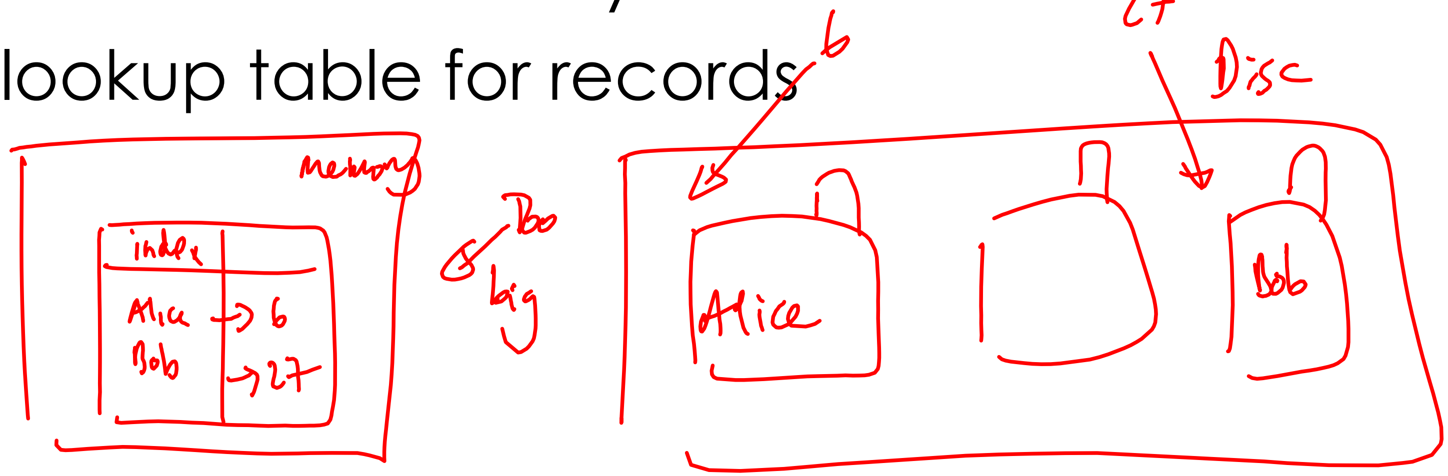


  - A binary search finds a given record in log n comparisons

But…

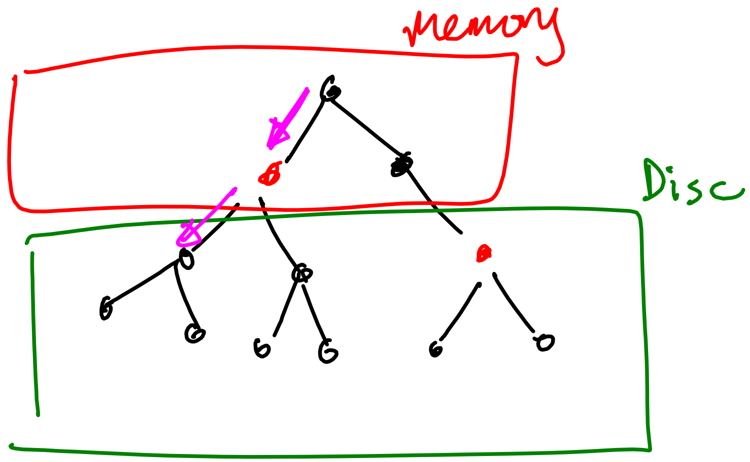
- We have a new problem
  - Every time we follow a branch on the tree it has to load in a record from disc in order to perform a comparison
  - And disc reads are slooooow

- Easy solution: pick out the index fields and keep them in memory
  - A lookup table for records



Apply Red-Black Trees?

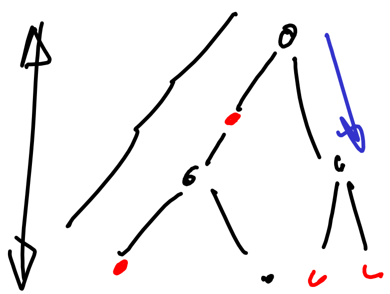
- Sadly, the tree gets so big (lots of keys) we need to split the tree between memory and disc too!

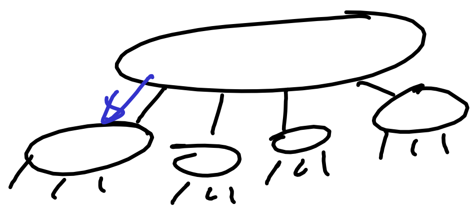


- So we have to go to the disk almost every time we follow a branch
  - Discs are too slow for this

2-3-4 Trees Again

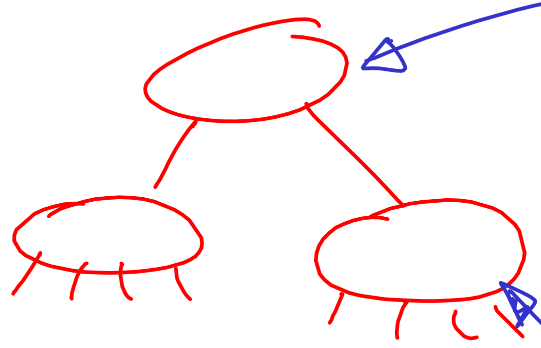
- We disliked 2-3-4 trees because the nodes had a variable size and we'd be wasting lots of memory if not all nodes were 4-nodes.
- But now we're trying to minimise the number of levels and they certainly did that compared to BSTs...





- Generalise to much wider trees: B-trees

B-Tree Definition and Terminology



Root has

$2 \leq n_{children} \leq m$

$1 \leq n_{keys} \leq m-1$

Internal node

$\frac{m}{2} \leq n_{children} \leq m$

1. "B-tree of order x" $\Rightarrow$ $[m = x]$ No more than x children
2. "B-tree of min. degree x" $\Rightarrow$ $[\frac{m}{2} = x]$ At least x children

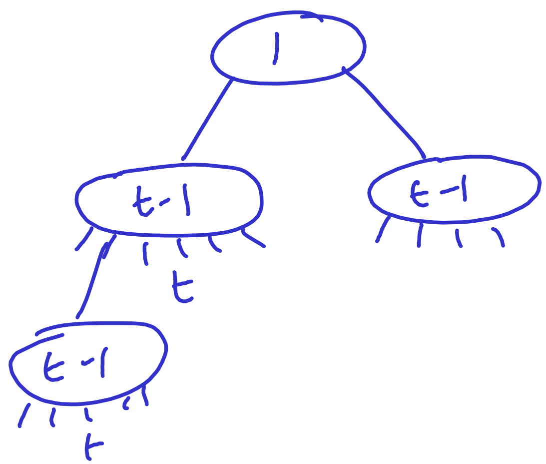
Confused

B-Tree Height

Most operations ⇒ $O(h)$

⇒ what is h??

Consider smallest tree of height h, min degree t



Nodes	keys
1	1
2	$2(t-1)$
$2t$	$2t(t-1)$

$$n_{keys} = 1 + 2(t-1) + 2t(t-1) + \dots 2t^{h-2}(t-1)$$

$$= 1 + 2(t-1)\sum_{i=0}^{h-2} t^i$$

geometric progression

$$= 1 + 2(t-1)\frac{(t^{h-1}-1)}{t-1}$$

$$= 1 + 2(t^{h-1}-1)$$

$$n \geq 1 + 2(t^{h-1}-1)$$

$$\frac{n+1}{2} \geq t^{h-1}$$

$$\log_t\left(\frac{n+1}{2}\right) \geq h-1$$

$$h \leq \log_t\left(\frac{n+1}{2}\right)$$

Factor $\lg t$ better than RB-tree

Insertion

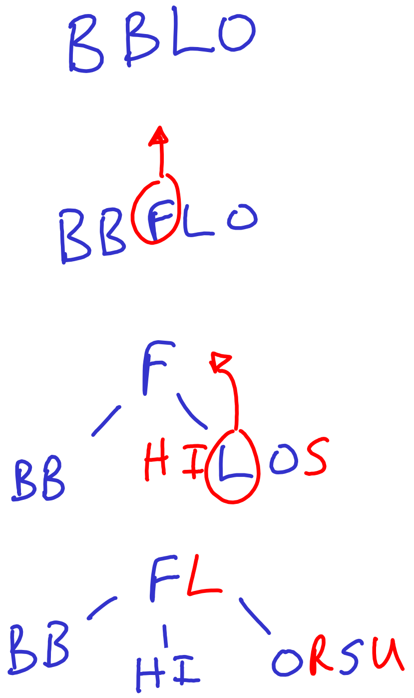
- Same idea as 2-3-4 except we split nodes into two based on the median being pushed up

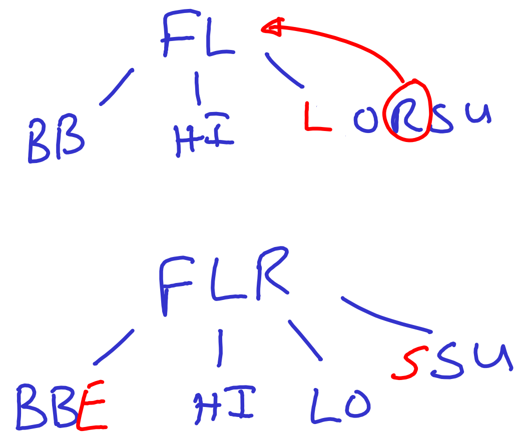
BLOBFISHRULES

n=13

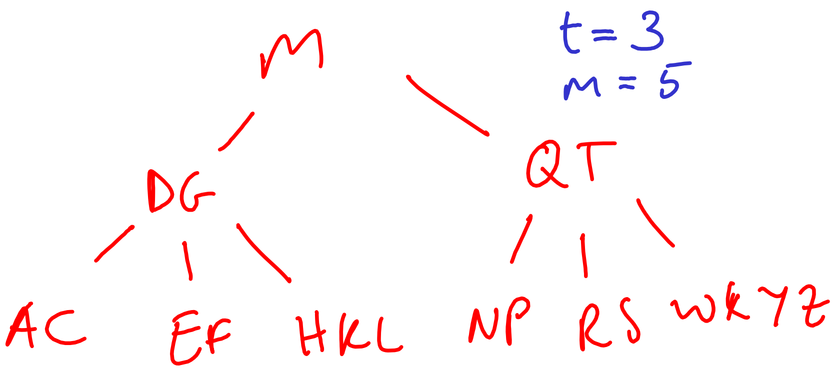
Order 5

min degree t=3



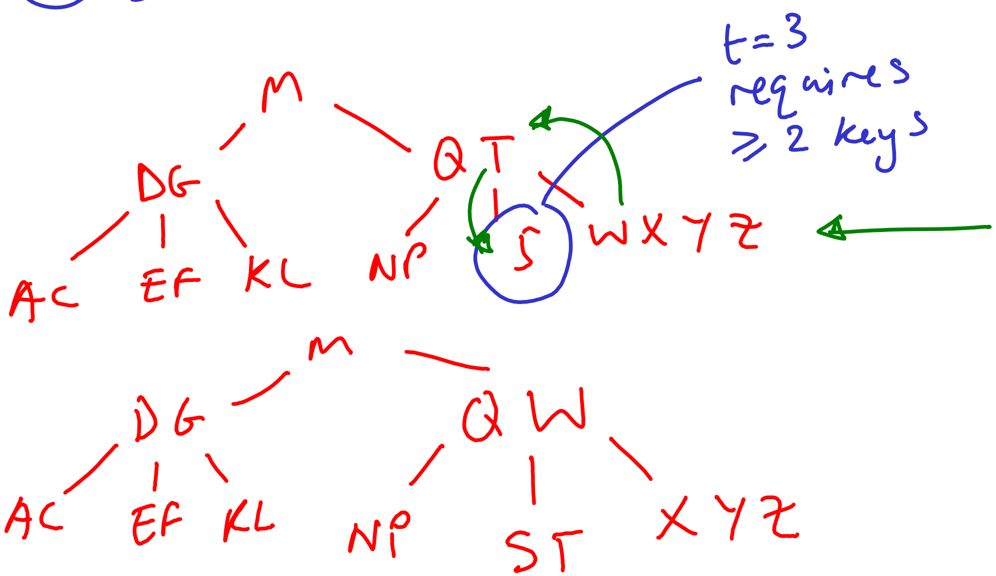


Deletion



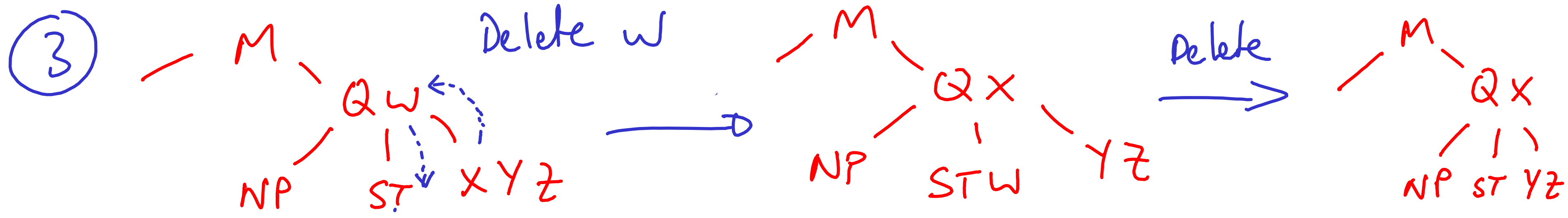
① H ⇒ leaf node
⇒ Delete

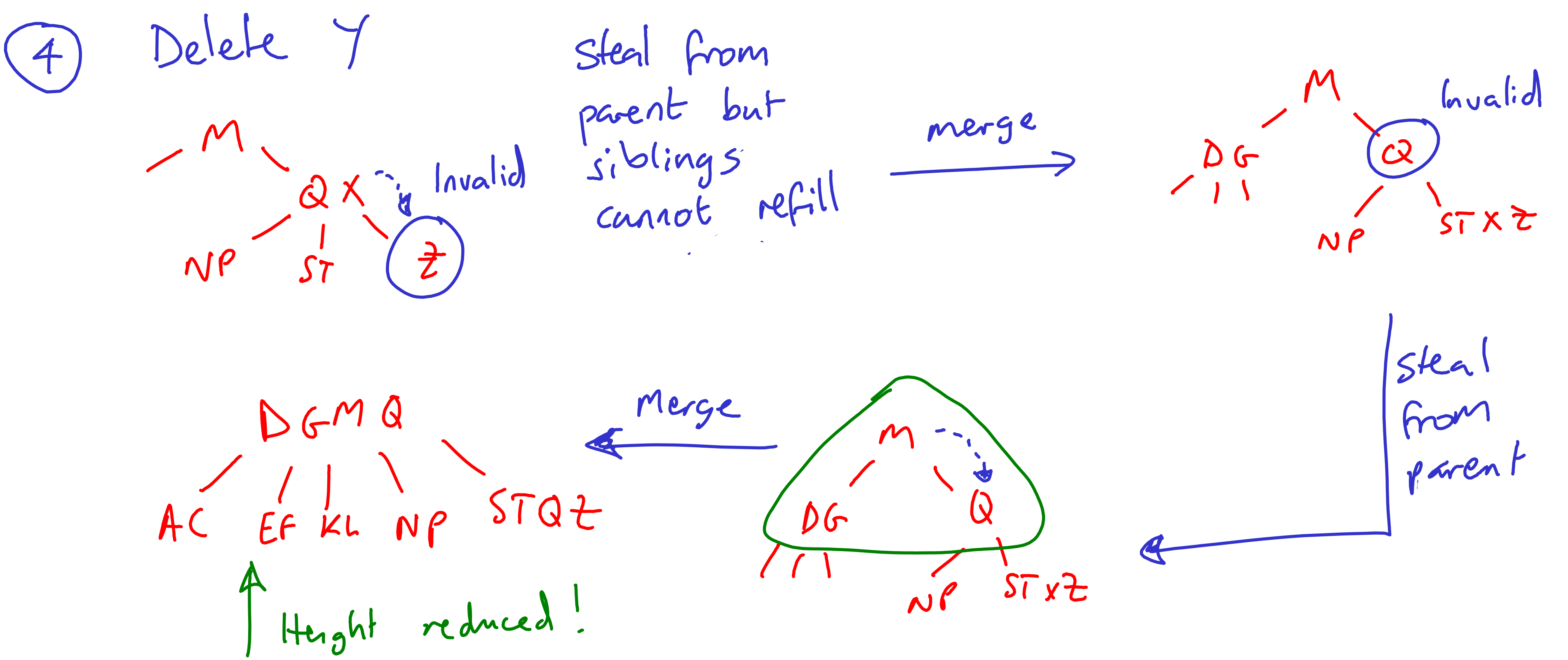
② Delete R



more than half full sibling that can refill the parent if we steal from it

Deletion


3
M
Q W
NP
ST
X Y Z
Delete W
M
Q X
NP
STW
YZ
Delete
M
Q X
NP ST YZ


4
Delete Y
M
Q X
NP
ST
Z
Invalid
Steal from parent but siblings cannot refill
merge
M
DG
Q
Invalid
NP
ST X Z
Steal from parent
M
DG
Q
NP
ST X Z
Merge
DGMQ
AC
EF
KL
NP
STQZ
Height reduced!

Search Costs

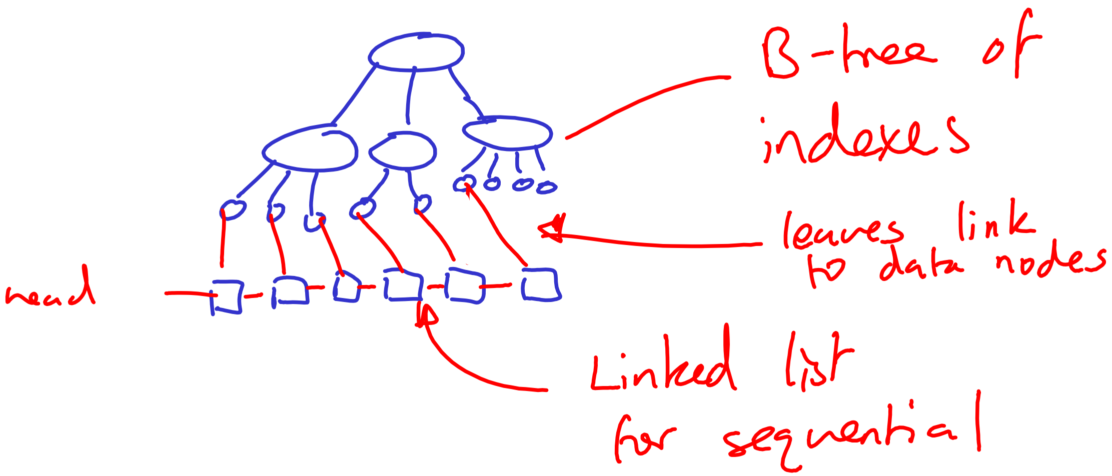
Limitations

- B-trees only make sense when we want to minimise the tree height i.e. when traversing branches is costly
- B-trees mess up your lectures.
- Consider traversing the records in order
  - Just like with the BST, we often need to explore branches to find successors
  - Which means we have to load in lots of records
  - Slow
- Unfortunately, we often want both random and sequential access to records...

$\Rightarrow B^+$ Tree

Idea

- Store *all* the records at the leaves of the B-tree
  - Replace 'keys' with 'indexes'



  - Now we always traverse (h-1) links on search
  - BUT we can sequentially link our records
  - Called a "B+ tree"