Prolog Lecture 2

- Rules
- Lists
- Arithmetic
- Last-call optimisation
- Backtracking
- Generate and Test

Rules have a head which is true if the body is true

- Our Prolog databases have contained only facts
  - e.g. `lecturer(prolog,dave).`
- Most programs require more complex rules (p8)
  - Not just “this is true”, but “this is true if that is true”

$$\underbrace{\texttt{rule(X,Y)}}_{\text{head}}\ \texttt{:-}\ \underbrace{\texttt{part1(X), part2(X,Y).}}_{\text{body}}$$

- You can read this as: “rule(X,Y) is true if part1(X) is true and part2(X,Y) is true”
  - Note: X and Y also need to be unified appropriately

Variables can be internal to a rule

The variable Z is not present in the clause head:

```
rule2(X) :- thing(X,Z), thang(Z).
```

Read this as “rule2(X) is true if there is a Z such that thing(X,Z) is true and thang(Z) is true”

Prolog and first order logic

The :- symbol is an ASCII-art arrow pointing left

- The “neck” (it's between the clause head and body!)

The arrow represents logical implication

- Mathematically we'd usually write `clause→head`
- It's not as clean as a graphical arrow ...
- In practice Prolog is not as clean as logic either!

Note that quantifiers (∀ and ∃) are not explicitly expressed in Prolog

Rules can be recursive

```
rule3(ground).
rule3(In) :- anotherRule(In,Out),
             rule3(Out).
```

In a recursive reading `rule3(ground)` is a base case, and the other clause is the recursive case.

In a declarative reading both clauses simply represent a situation in which the rule is true.

Prolog identifies clauses by name and arity

We refer to a rule using its clause's head term

The clause

- rule.

is referred to as rule/0 and is different to:

- rule(A).

which is referred to as rule/1 (i.e. it has arity 1)

- rule(_,Y).

would be referred to as rule/2, etc.

Prolog has built-in support for lists

- Items are put within square brackets, separated by commas, e.g.[1,2,3,4] (p61)
  - The empty list is denoted []
- A single list may contain terms of any kind:
  - [1,2,an_atom,5,Variable,compound(a,b,c)]
- Use a pipe symbol to refer to the tail of a list
  - Examples: [Head|Tail] or [1|T] or [1,2,3|T]
  - Try unifying [H|T] and [H1,H2|T] with [1,2,3,4]
    - i.e. ?- [H|T] = [1,2,3,4].

We can write rules to find the first and last element of a list

Like functional languages, Prolog uses linked lists

```
first([H|_],H).
```

```
last([H],H).
last([_|T],H) :- last(T,H).
```

Make sure that you (eventually) understand what this shows you about Prolog's list representation:

write_canonical([1,2,3]).

Question

```
last([H],H).
last([_|T],H):-
    last(T,H).
```

What happens if we ask: last([],X). ?

a) pattern-match exception

b) Prolog says no

c) Prolog says yes, X = []

d) Prolog says yes, X = ???

You should include tests for your clauses in your source code

Example last.pl:

```
last([H],H).
last([_|T],H) :- last(T,H).

% this is a test assertion
:- last([1,2,3],A), A=3.
```

What happens if the test assertion fails?

What happens if we ask:

?- last(List,3).

Prolog provides a way to trace through the execution path

Query trace/0, evaluation then goes step by step

- Press enter to “creep” through the trace
- Pressing s will “skip” over a call

```
?- [last].
% last compiled 0.01 sec, 604 bytes

Yes
?- trace,last([1,2],A).
   Call: (8) last([1, 2], _G187) ? creep
   Call: (9) last([2], _G187) ? creep
   Exit: (9) last([2], 2) ? creep
   Exit: (8) last([1, 2], 2) ? creep

A = 2
Yes
```

Arithmetic Expressions

(AKA "Why Prolog is a bit special/different/surprising")

What happens if you ask Prolog:

```
?- A = 1+2.
```

Arithmetic equality is not the same as Unification

```
?- A = 1+2.
A = 1+2
Yes

?- 1+2 = 3.
No
```

This should raise anyone's procedural eyebrows...

Arithmetical operators get no special treatment!

Unification, unification, unification

In Prolog “=” is not assignment!

“=” does not evaluate expressions!

“=” means “try to unify two terms”

Arithmetic equality is not the same as Unification

```
?- A = money+power.
A = money+power
Yes

?- money+power = A,
     A = +(money,power).
A = money+power
Yes
```

Plus (+) is just forming compound terms

We discussed this in lecture 1

Use the “is” operator to evaluate arithmetic

The “is” operator tells Prolog: (p81)

(1) evaluate the right-hand expression numerically

(2) then unify the expression result with the left

```
?- A is 1+2.
A = 3
Yes

?- A is money+power.
ERROR: is/2: Arithmetic: `money/0' is not a function
```

Ensure that you can explain what will happen here:

?- 3 is 1+2   ?- 1+2 is 3

The right hand side must be a ground term (no variables)

```
?- A is B+2.
ERROR: is: Arguments are not sufficiently
 instantiated

?- 3 is B+2.
ERROR: is: Arguments are not sufficiently
 instantiated
```

It seems that “is” is some sort of magic predicate

- Our predicates do not force instantiation of variables!

In fact it can be implemented in logic

- See the supervision worksheet

We can now write a rule about the length of a list

List length:

```
len([],0).
len([_|T],N) :- len(T,M), N is M+1.
```

This uses O(N) stack space for a list of length N

List length using O(N) stack space

- Evaluate len([1,2],A).
- Apply len([1| [2]],A_0) :- len([2],M_0), A_0 is M_0+1
  - Evaluate len([2],M_0)
  - Apply len([2 | []],M_0) :- len([],M_1), M_0 is M_1+1
    - Evaluate len([],M_1)
    - Apply len([],0) so M_1 = 0
    - Evaluate M_0 is M_1+1 so M_0 = 1

    Stack Frame 2
  - Evaluate A_0 is M_0+1 so A_0 = 2

  Stack Frame 1
- Result len([1,2],2)
- This takes O(N) space because of the variables in each frame

List length using O(1) stack space

List length using an accumulator:

```
len2([],Acc,Acc).
len2([_|Tail],Acc,Result) :-
     AccNext is Acc + 1,
     len2(Tail,AccNext,Result).

len2(List,Result) :-
     len2(List,0,Result).
```

We are passing variables to the recursive len2 call that we do not need to use in future evaluations

- Make sure that you understand an example trace

List length using O(1) stack space

- Evaluate len2([1,2],0,R)
- Apply len2([1| [2]],0,R) :- AccNext is 0+1,
  len2([2],AccNext,R).
  - Evaluate AccNext is 0+1 so AccNext = 1
  - Evaluate len2([2],1,R)

Stack Frame 1

- Apply len2([2| []],1,R) :- AccNext is 1+1,
  len2([],AccNext,R).
  - Evaluate AccNext is 1+1 so AccNext = 2
  - Evaluate len2([],2,R).

Stack Frame 2

- Apply len2([],2,2) so R = 2
- I didn't need to use any subscripts on variable instances!

Last Call Optimisation turns recursion into iteration

Any decent Prolog implementation will apply "Last Call Optimisation" to tail recursion (p186)

- The last query in a clause body can re-use the stack frame of its caller
- This "tail" recursion can be implemented as iteration, drastically reducing the stack space required

Can only apply LCO to rules that are determinate

- The rule must have exhausted all of its options for change: no further computation or backtracking

We can demonstrate that Prolog is applying last call optimisation

Trace will not help

- The debugger will likely interfere with LCO!

How about a “test to destruction”?

```
biglist(0,[]).
biglist(N,[N|T]) :-
    M is N-1,
    biglist(M,T),
    M=M.
```

Prolog uses depth-first search to find answers

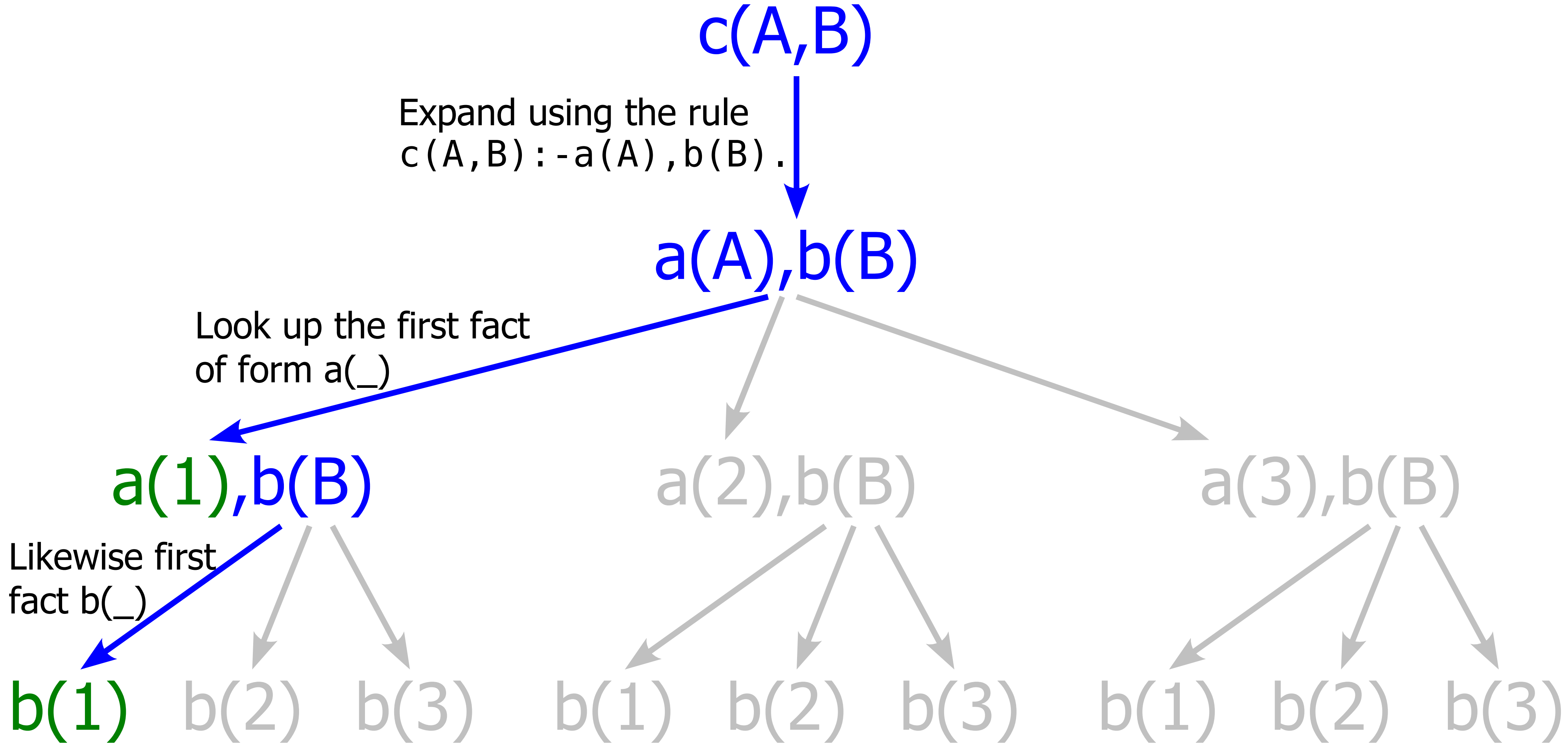
Here is a (boring) program:

```
a(1).
a(2).
a(3).
b(1).
b(2).
b(3).
c(A,B) :- a(A), b(B).
```

What does Prolog do when given this query?

c(A,B).

Depth-first solution of query c(A,B)



c(A,B)

Expand using the rule
`c(A,B):-a(A),b(B).`

a(A),b(B)

Look up the first fact
of form a(_)

a(1),b(B) a(2),b(B) a(3),b(B)

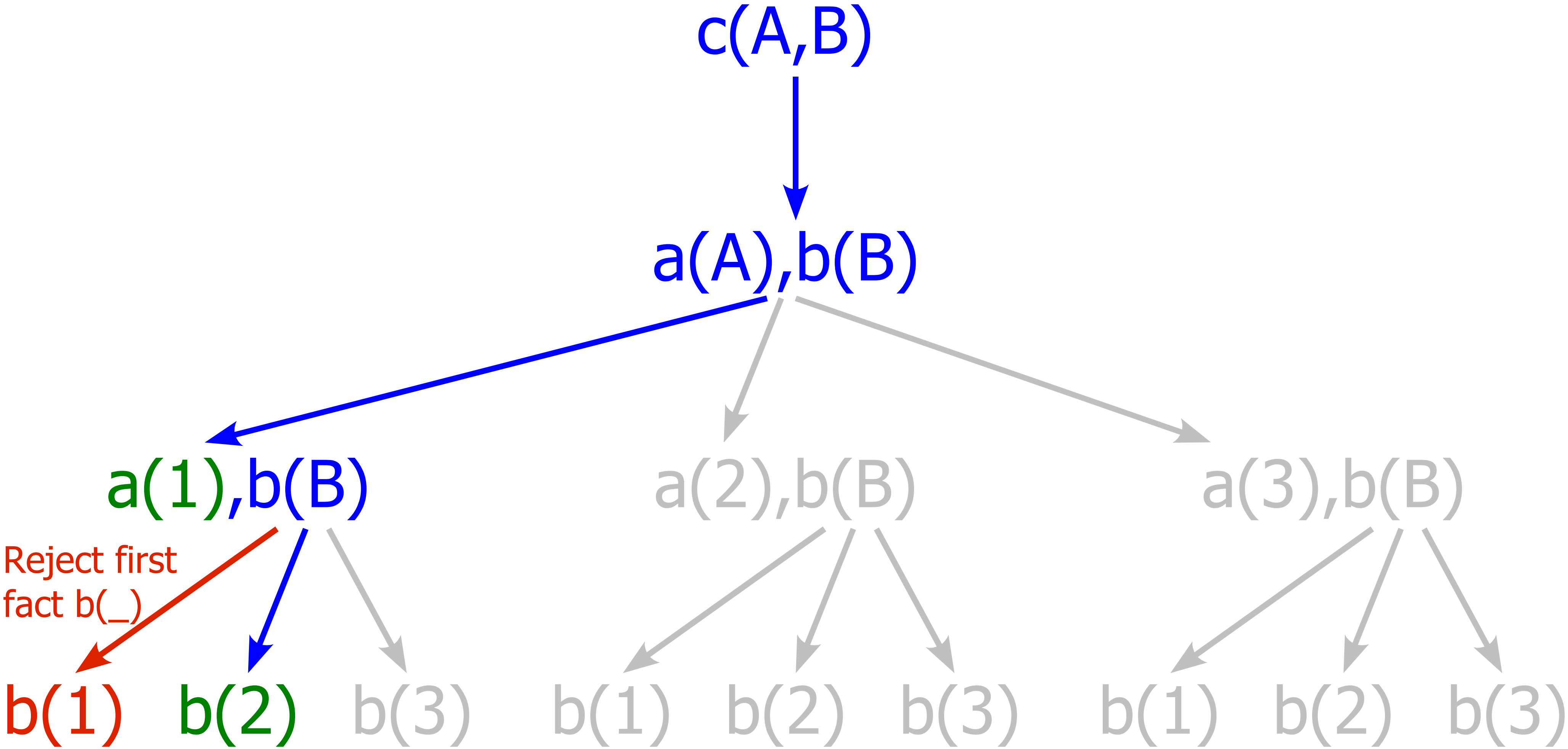
Likewise first
fact b(_)

b(1) b(2) b(3) b(1) b(2) b(3) b(1) b(2) b(3)

We've found
a solution!

Variable bindings: A=1, B=1

Backtrack to find the next solution



c(A,B)

a(A),b(B)

a(1),b(B)

a(2),b(B)

a(3),b(B)

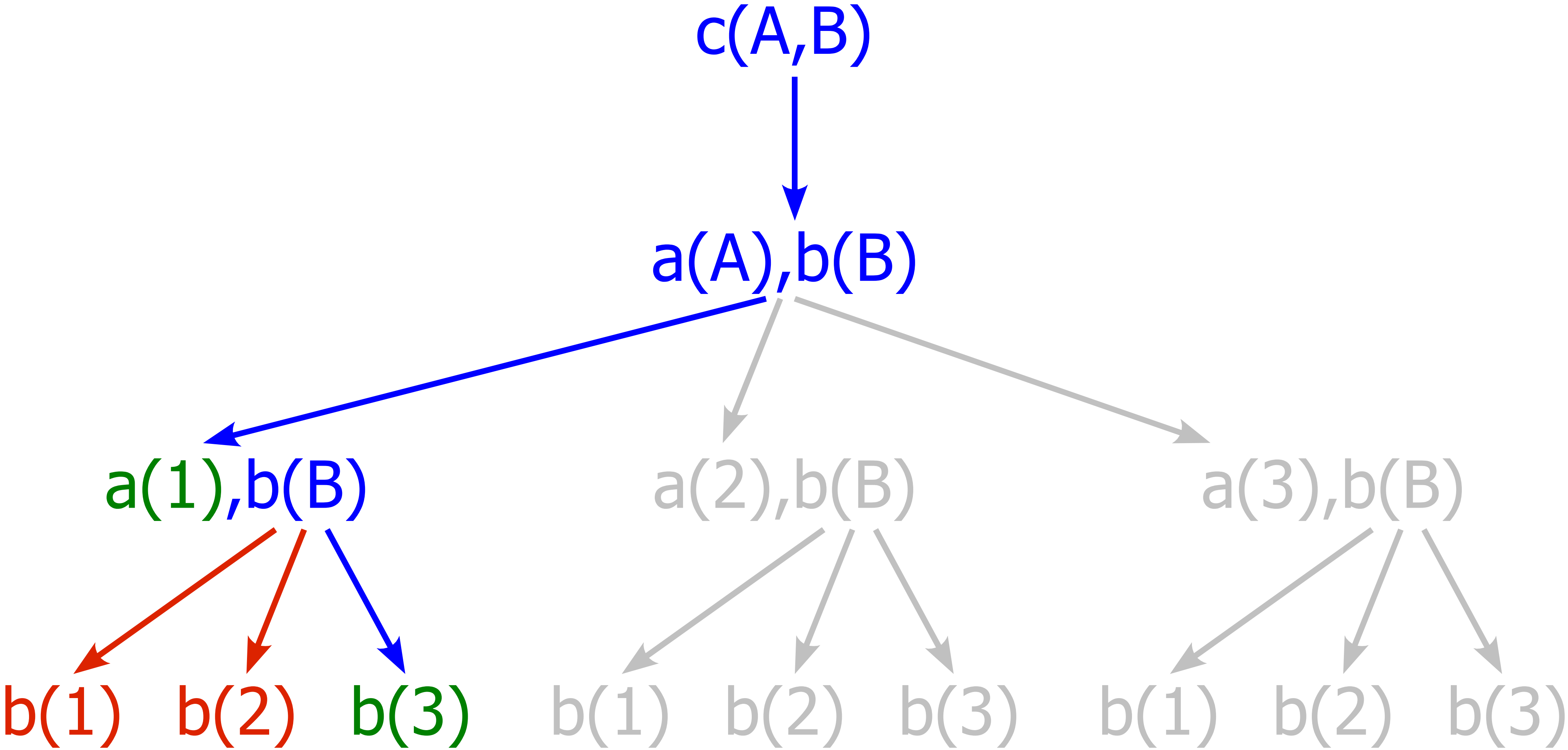
Reject first
fact b(_)

b(1) b(2) b(3) b(1) b(2) b(3) b(1) b(2) b(3)

We've found the
next solution

Variable bindings: A=1, B=2

Backtrack to find another solution



c(A,B)

a(A),b(B)

a(1),b(B)

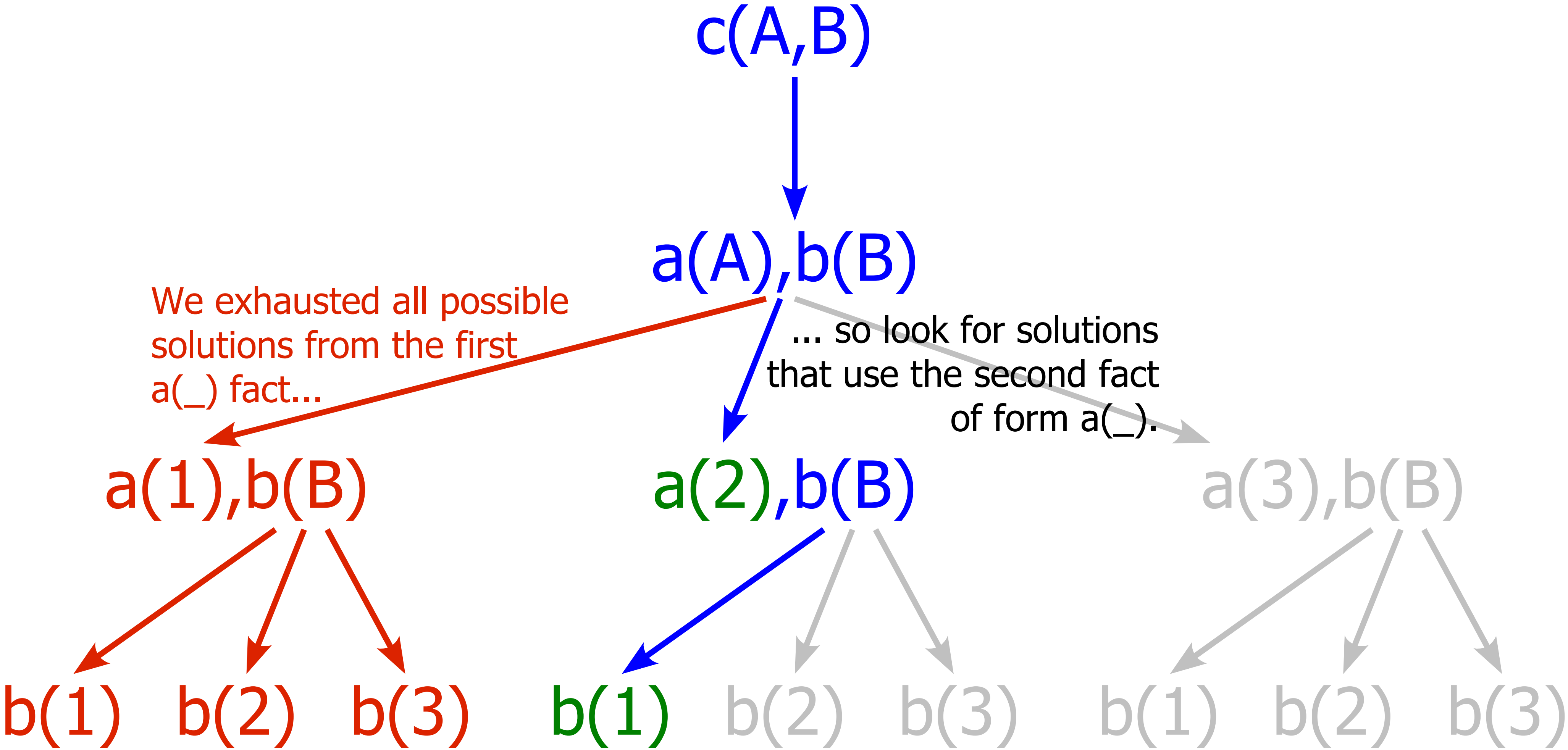
a(2),b(B)

a(3),b(B)

b(1) b(2) b(3) b(1) b(2) b(3) b(1) b(2) b(3)

Variable bindings: A=1, B=3

Backtrack to find another solution



We exhausted all possible solutions from the first a(_) fact…

… so look for solutions that use the second fact of form a(_).

Variable bindings: A=2, B=1

Take from a list

Here is a program that takes an element from a list:

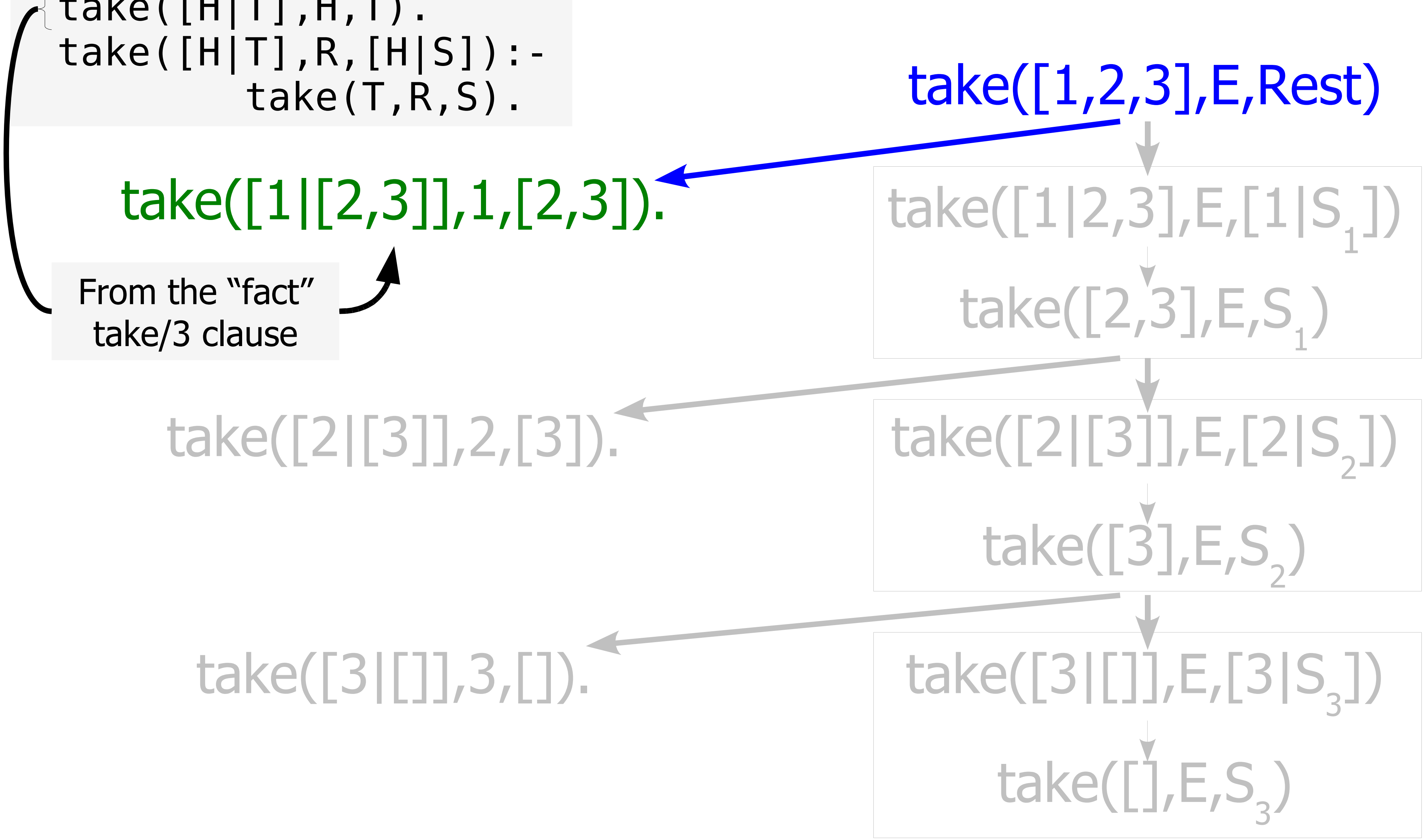
```
take([H|T],H,T).
take([H|T],R,[H|S]) :- take(T,R,S).
```

What does Prolog do when given the query:

take([1,2,3],E,Rest).

All solutions for take([1,2,3],E,Rest)

```
take([H|T],H,T).
take([H|T],R,[H|S]):-
        take(T,R,S).
```



take([1,2,3],E,Rest)

take([1|[2,3]],1,[2,3]).

From the “fact” take/3 clause

take([1|2,3],E,[1|S_1])

take([2,3],E,S_1)

take([2|[3]],2,[3]).

take([2|[3]],E,[2|S_2])

take([3],E,S_2)

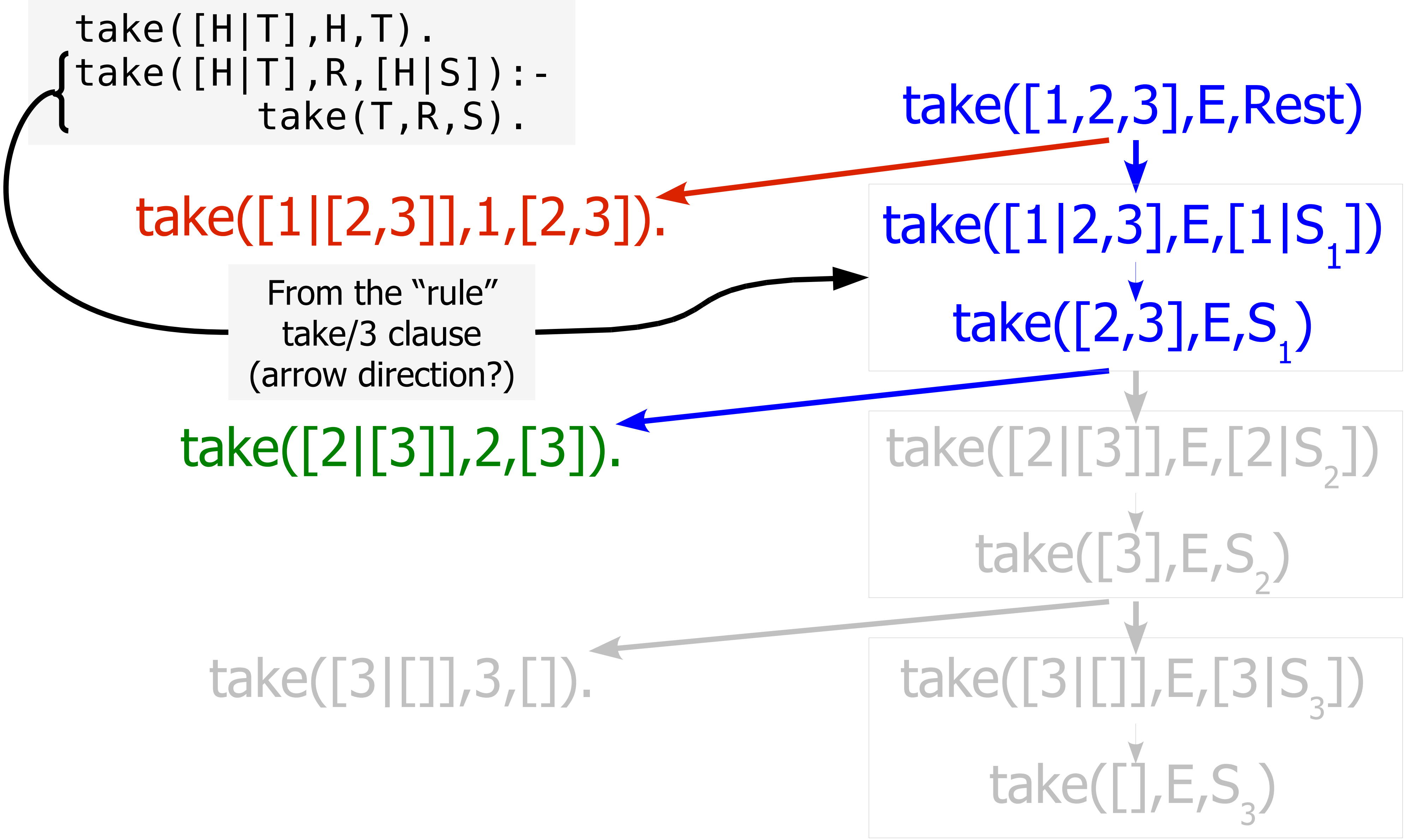
take([3|[]],3,[]).

take([3|[]],E,[3|S_3])

take([],E,S_3)

Variable bindings: E=1, Rest=[2,3]

Backtrack for next solution



```
take([H|T],H,T).
take([H|T],R,[H|S]):-
        take(T,R,S).
```

take([1,2,3],E,Rest)

take([1|[2,3]],1,[2,3]).

take([1|2,3],E,[1|S_1])

take([2,3],E,S_1)

From the “rule”
take/3 clause
(arrow direction?)

take([2|[3]],2,[3]).

take([2|[3]],E,[2|S_2])

take([3],E,S_2)

take([3|[]],3,[]).

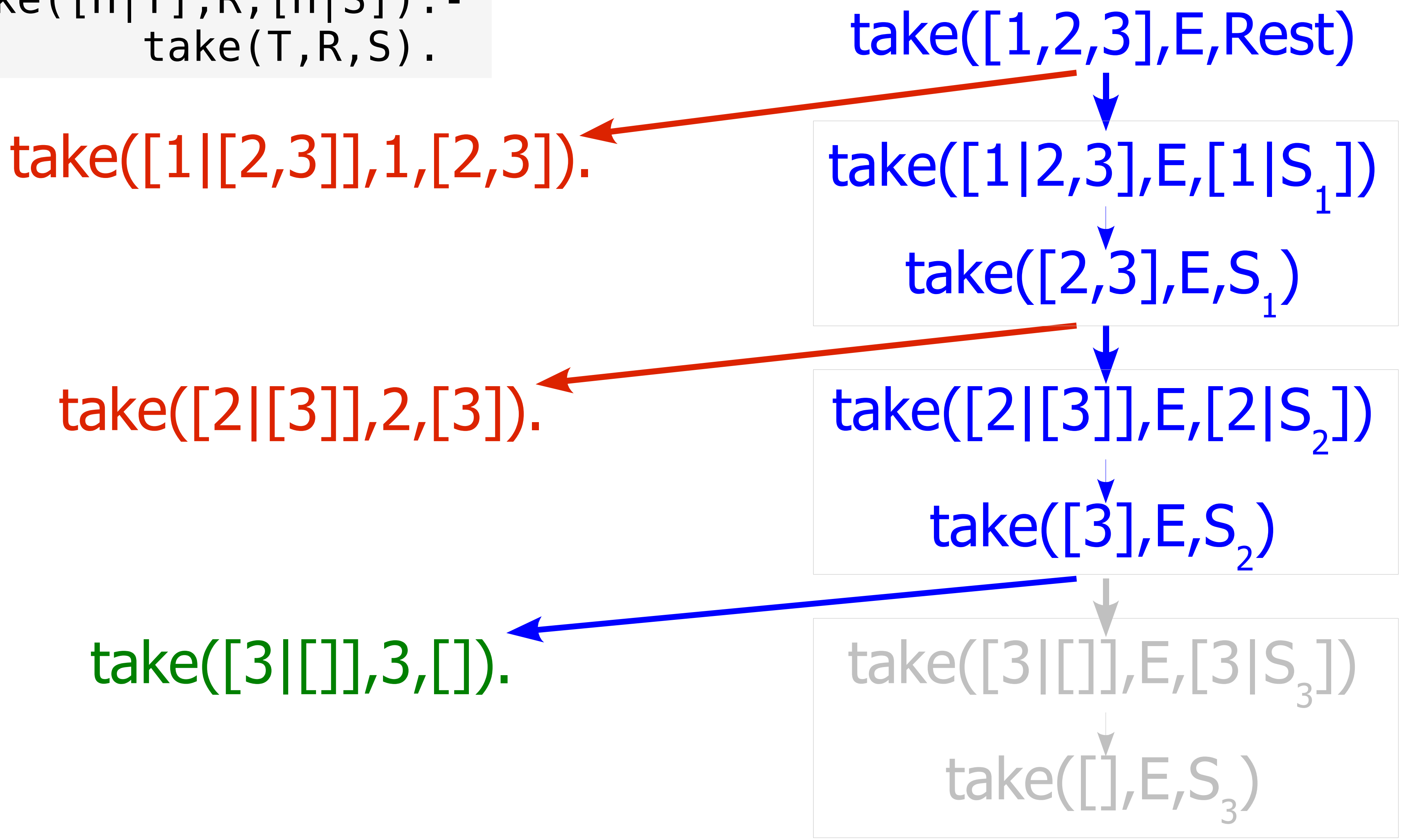
take([3|[]],E,[3|S_3])

take([],E,S_3)

Variable bindings: E=2, Rest=[1,3], S_1=[3]

Backtrack for another solution

```
take([H|T],H,T).
take([H|T],R,[H|S]):-
        take(T,R,S).
```



take([1,2,3],E,Rest)

take([1|2,3],E,[1|S_1])

take([2,3],E,S_1)

take([2|[3]],E,[2|S_2])

take([3],E,S_2)

take([3|[]],E,[3|S_3])

take([],E,S_3)

take([1|[2,3]],1,[2,3]).

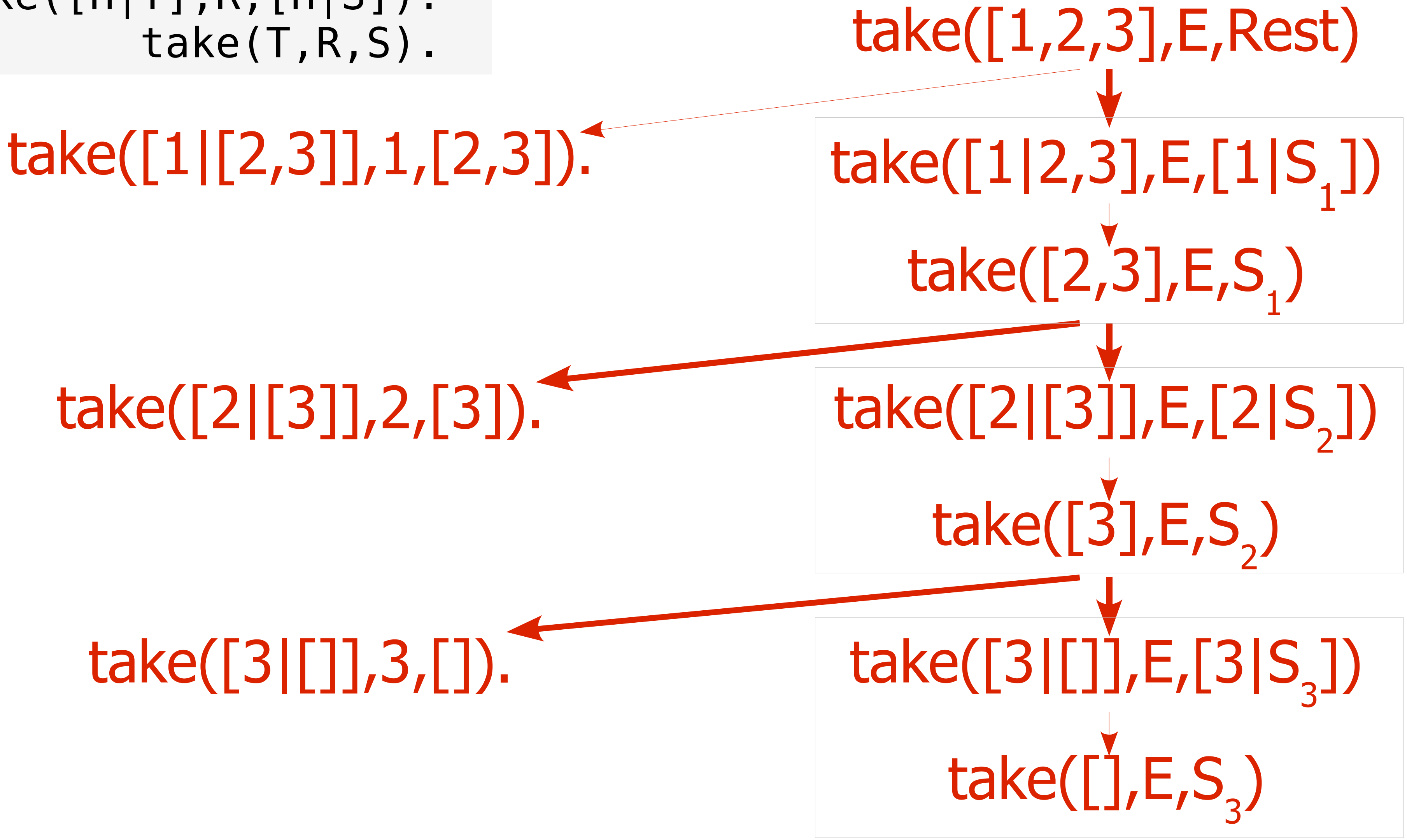
take([2|[3]],2,[3]).

take([3|[]],3,[]).

Variable bindings: E=3, Rest=[1,2], S_1=[2], S_2=[]

Prolog says “no”

```
take([H|T],H,T).
take([H|T],R,[H|S]):-
        take(T,R,S).
```



take([1,2,3],E,Rest)

take([1|[2,3]],1,[2,3]).

take([1|2,3],E,[1|S_1])

take([2,3],E,S_1)

take([2|[3]],2,[3]).

take([2|[3]],E,[2|S_2])

take([3],E,S_2)

take([3|[]],3,[]).

take([3|[]],E,[3|S_3])

take([],E,S_3)

Variable bindings: none – the predicate is false

“Find list permutation” predicate is very elegant

```
perm([],[]).
perm(List,[H|T]) :- take(List,H,R), perm(R,T).
```

What is the declarative reading of this predicate?

Dutch national flag

The problem was used by Dijkstra as an exercise in program design and proof.

Take a list and re-order such that red precedes white precedes blue

[red,white,blue,white,red]

↓

[red,red,white,white,blue]

“Generate and Test” is a technique for solving problems like this

(1) Generate a solution

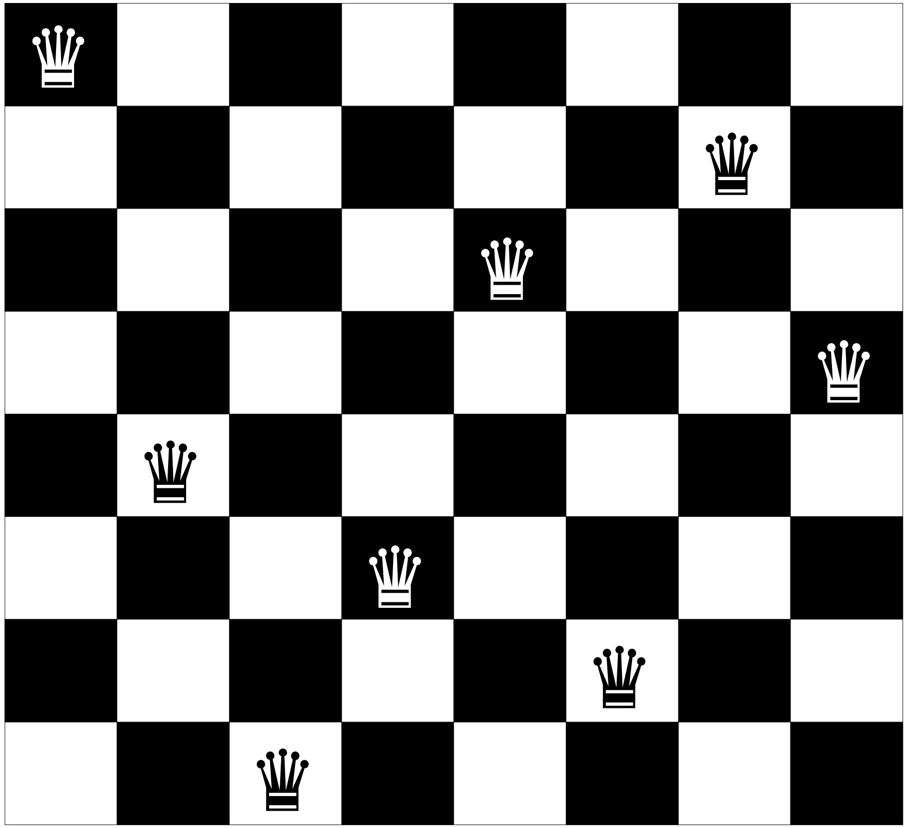
(1) Test if it is valid

(2) If not valid then backtrack to the next generated solution

```
flag(In,Out) :- perm(In,Out),
                checkColours(Out).
```

How can we implement checkColours/1?

Place 8 queens so that none can take any other



[1 , 5 , 8 , 6 , 3 , 7 , 2 , 4]

Generate and Test works for 8 Queens too

```
8queens(R) :- perm([1,2,3,4,5,6,7,8],R),
              checkDiagonals(R).
```

Why do I only need to check the diagonals?

Anagrams

Load the dictionary into the Prolog database e.g.:

- `word([a,a,r,d,v,a,r,k]).`

Generate permutations of the input word and **test** if they are words from the dictionary

or

Generate words from the dictionary and **test** if they are a permutation!

http://www.cl.cam.ac.uk/~dme26/pl/anagram.pl

End

Next lecture:
controlling backtracking with cut,
and negation