Exercises: sheet 3

1. Use the SECD machine to evaluate the λ-terms:

   (a) $(\lambda x.(\lambda y.yx)I)I$;

   (b) $(\lambda x.xx)((\lambda x.\lambda y.xy)I)$;

   (c) $YI \equiv (\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))I$.

   In part 1c, the SECD evaluation will not terminate. Do enough steps to convince yourself of this.

2. Give de Bruijn terms corresponding to the λ-terms given in part 1.

3. (optional) Given de Bruijn terms s and t corresponding to λ-terms M and N respectively, find an algorithm which directly computes the de Bruijn term for M[N/x] from the de Bruijn terms s and t.

4. Give translations of the λ-terms given in part 1 to combinators, using the operators λ^* and λ^T. For the translations using λ^T, give the graph of the resulting combinators and do some of the reduction steps.

5. Define $\mathbf{B} \equiv \mathbf{S}(\mathbf{KS})\mathbf{K}$ and $\mathbf{C} \equiv \mathbf{S}(\mathbf{BBS})(\mathbf{KK})$. Show that $\mathbf{B}PQR \to_w P(QR)$ and $\mathbf{C}PQR \to_w PQR$. Define a combinator $\mathbf{W}$ in terms of $\mathbf{S}$ and $\mathbf{K}$ such that $\mathbf{W}PQ \to_w PQQ$.

6. Prove that $(\lambda^T x.P)Q \twoheadrightarrow_w P[Q/x]$.