

## Exercises: sheet 1

1. Insert the full amount of parentheses in the following abbreviated terms:
  - (a)  $ux(yz)(\lambda v.vy)$ .
  - (b)  $(\lambda xyz.xz(yz))uvw$ .
  - (c)  $w(\lambda xyz.xz(yz))uv$ .
2. (a) Evaluate (up to  $\alpha$ -conversion) the substitutions
  - i.  $(\lambda y.x(\lambda x.x))[(\lambda y.xy)/x]$ ;
  - ii.  $(y(\lambda v.xv))[(\lambda y.vy)/x]$ .(b) Under what conditions does the equality  $M[N/x][L/y] \equiv_{\alpha} M[L/y][N/x]$  hold?
3. Reduce the following  $\lambda$ -terms to normal form, or indicate that a normal form does not exist:
  - (a)  $P \equiv (\lambda x.x(xy))I$  where  $I \equiv \lambda u.u$ .
  - (b)  $Y \equiv \lambda f.QQ$  where  $Q \equiv (\lambda x.f(xx))$ .
  - (c)  $L \equiv (\lambda x.xxy)(\lambda x.xxy)$ .
  - (d)  $(\lambda x.xL)M$  where  $M \equiv \lambda x.y$  and  $L$  is defined in 3c.
4. Find all possible reduction paths to normal form for the following  $\lambda$ -terms, where  $I \equiv \lambda u.u$ :
  - (a)  $(\lambda x.xx)((\lambda x.xy)I)$ .
  - (b)  $I((\lambda y.Ix)z)$ .
  - (c)  $(\lambda u.v)L$  where  $L \equiv (\lambda x.xxy)(\lambda x.xxy)$ .State and comment on which reduction paths are shortest.
5. Consider the  $\lambda$ -terms given in part 3. Show that the following pairs of  $\lambda$ -terms are equal or prove otherwise:
  - (a)  $P$  and  $(\lambda x.xL)M$ .
  - (b)  $L$  and  $I$ ;
  - (c)  $Yf$  and  $f(Yf)$ .
6. For the  $\lambda$ -terms in part 4, identify which reduction paths satisfy the call-by-name and call-by-value reduction strategies.