Can you ...

- show sorting is $\Omega(n \log n)$?
- define the class P?
- define the class NP?
- show 3SAT is NP-complete? (at least at a high level)
- show that TAUTOLOGY is in Co-NP?
- define a one-way function?
- understand every relationship in

$$L \subseteq NL \subseteq P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXP?$$
Can you do these reductions?

- $3SAT \leq_P IND$
- $IND \leq_P CLIQUE$
- $3SAT \leq_P 3$-Colourability
- $3SAT \leq_P HAM$
- $HAM \leq_P TSP$
- $3SAT \leq_P 3DM$
- $3DM \leq_P XSC$ (Exact Set Cover)
- $XSC \leq_P SC$ (Set Cover)
- $XSC \leq_P KNAPSACK$
(Undirected) Hamiltonian Path problem (HAM-PATH)

HAM-PATH

Given a graph $G = (V, E)$, does it contain a path that visits every node exactly once?

HAM-PATH is NP-complete.

Proof (Papadimitriou, pages 193 to 198): The problem is in NP since we can guess a path and check it in polynomial time. To show that it is NP-complete we do a reduction from 3SAT. First, we need a gadget to represent each variable $x$:

![Diagram]

We construct a chain of these gadgets, one for each variable.
The exclusive or (XOR) gadget
Representing clauses

For each clause of three literals \((l_1 \lor l_2 \lor l_3)\), we construct a (virtual) triangle where each “edge” is associated with a literal, and connected with an XOR gadget to the virtual link that of the literal’s variable that would make the literal true.
Example

The initial end-points are connected to a new node with label 1, the terminal end-points are connected to a dotted node (All dotted nodes will be connected in one large clique.) Finally, we add one arc with a dotted node and a node labeled 3. Thus, any Hamiltonian path must link nodes 1 and 3.

\[(x_1 \lor x_2 \lor x_3) \land (\neg x_1 \lor \neg x_2 \lor x_3)\]
Eulerian Path Problem is in P

Eulerian Path Problem

Given a graph \( G = (V, E) \), does it contain a path that visits every edge exactly once?

1. Pick any vertex to start.
2. From the current vertex, pick any edge, but never cross a *bridge* in the *reduced graph* (the graph with marked edges deleted), unless there is no other choice. A bridge is an edge whose deletion would increase the number of connected components.
3. Mark the edge (so will not use it again).
4. *Traverse* the edge, picking the node at the other end.
5. Repeat steps 2 through 4, until back at the starting point (or failure).

Exercise

Prove that this algorithm is correct.
Directed Hamiltonian Path problem (DHAM-PATH)

HAM-PATH

Given a directed graph $G = (V, E)$, does it contain a path that visits every node exactly once?

DHAM-PATH is NP-complete.

The problem is in NP since we can guess a path and check it in polynomial time. To show that it is NP-complete we do a reduction from 3SAT.
We construct a graph that looks like this ...
Each variable diamond has a chain ...
... chains are connected to clause nodes

\[ x_i \text{ appears on } C_j \]

\[ \neg x_i \text{ appears on } C_j \]