Desirable features in CAD

- **Need to handle *any* surface**
- **Need guaranteed continuity**
  - Continuity of slope (C1)
    - Smooth surfaces
  - Continuity of curvature (C2)
    - Smoothly reflecting surfaces
    - Required for some aerodynamics
- **Need to allow for discontinuities**
  - Edges, creases and holes
- **Needs to be easy to use**

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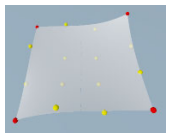
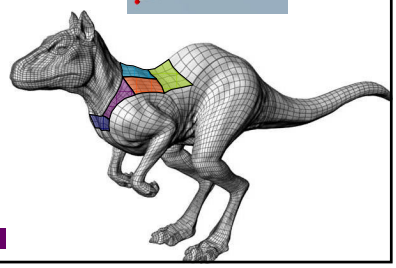
Traditional tools

- **Bezier patches**
- **B-spline patches**
- **NURBS patches**





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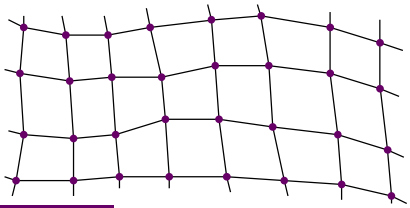
Bezier & B-spline patches

- **A rectangular array of control points**
- **A mathematical function determines where the surface goes based on those points**
- **Move a control point to change the surface**



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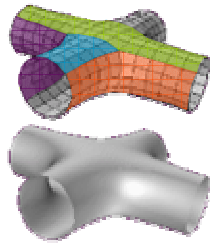
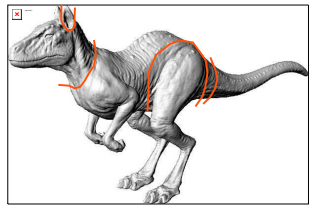
The first problem

- **Very few objects are made up of a single rectangular patch, so we need to join patches together**





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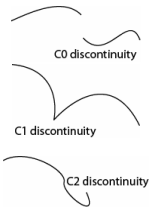
The mathematics of joins

- **We want to preserve certain types of mathematical continuity across joins**
  - C0: continuity of position
    - Prevents holes at the join
  - C1: continuity of slope
    - Prevents a sharp edge at the join
  - C2: continuity of curvature
    - Strongly related to aesthetics
    - Most often visible in reflections
    - Prevents sharp edges in reflected lines
  - These are continuity of the zeroth, first and second derivatives



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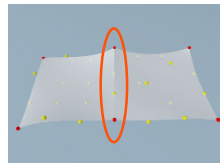
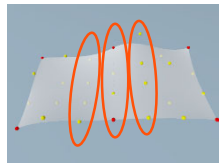
Joining two Bezier patches

- **C0 but not C1**
  - Four edge points are the same



- **C0 and C1**
  - Four edge points are the same
  - Next four points out in either direction are constrained



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An example: the car's roof

- The car

- Curvature plot of its roof

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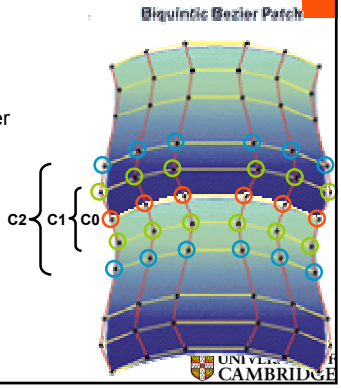
Definition of the car's roof

- 5×2 grid of biquintic Bezier patches
  - 36 control points per patch
  - 286 control points overall
  - Moving one point also moves several others to maintain C2 continuity



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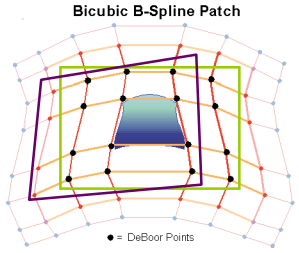
B-spline patches

- A rectangular array of points define a rectangular array of *automatically joined* patches
- Example
  - The black points control the central patch
  - All points together define a surface of many joined patches



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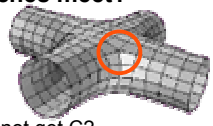
The second problem

- What do we do at special points where other than four patches meet?



  - Either we cannot get C2
    - Which means that curvature is not continuous
  - Or we get C2 be forcing curvature to be zero
    - Which produces a flat spot
  - Or we get C2 using very high degree patches
    - Which are very hard for a designer to control

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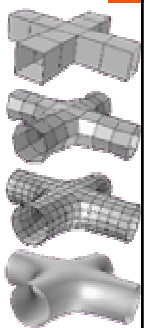
Subdivision surfaces

- Developed in the 1970s, adopted in computer animation in 1990s
- Replace the patch-based representation of B-splines and Beziers
- Base a curve or surface solely on its control points and their connectivity
- A simple mechanism produces a larger, more refined set of control points from the current set
- Iterate refinement until the appropriate level of detail is achieved



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Subdivision

- **Advantages**
  - Reproduces everything which can be done by B-splines
  - Handles extraordinary points much more easily
- **Disadvantages**
  - Cannot get C2 unless you produce a flat spot
  - Generates other visual artefacts, not seen in B-spline surfaces
- **Commercial position**
  - Subdivision is replacing B-splines in computer animation
  - Subdivision is **not** replacing B-splines in CAD

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