A Novel Parallel Approach for 3D Seismological Problems

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MATHMATICAl DESCRIPTION OF THE GEOLOGICAL SITE
Initial Boundary Value Problem

\[
\begin{aligned}
\rho(x) \frac{\partial^2 u}{\partial t^2} &= \nabla \cdot \sigma + F(x, t), \quad \forall (x, t) \in \Omega \times (0, +\infty), \\
\sigma &= (\lambda(x) + \mu(x)) \nabla u, \\
\mathbf{u}(x, 0) &= 0, \quad \forall x \in \Omega, \\
\frac{\partial}{\partial t} \mathbf{u}(x, 0) &= 0, \quad \forall x \in \Omega, \\
\frac{\partial}{\partial n} \sigma(x, t) &= 0, \quad \forall (x, t) \in \partial\Omega \times (0, +\infty),
\end{aligned}
\]
Numerical Approach

Non structured mesh on the integration domain

- Numerical reconstruction of the energy shearing processes;
- Full agreement between interfaces and elements;

Numerical Discretization

- **FEM**: Finite Element Method;
- Newmark Method

\[ A \ddot{U}^{n+1} = b^{n+1} \] (1)
**Block Matrix Properties**

- **Sparse**
  \(\Rightarrow\) *optimized C.S.R. Format*;

- **Symmetric Pattern**;

- **Diagonally Dominant a.e.**
  \(\Rightarrow\) *Gauss-Seidel Method*.
The Reason of Parallel Approach

Modelling Seismological IBVP more and more realistic

- Wider Domains
- High Frequencies
- Dissipation

- More Elements
- Higher Computational Cost
PARALLEL APPROACH

1. Distributed Memory ⇒ **M.P.I.** on CASPUR Cluster MATRIX (320 nodes);
2. Domain Decomposition:

**MeTiS**

http://glaros.dtc.umn.edu/gkhome/node/105
Let us consider an homogeneous cube with side $L = 700$ m, $v_{\text{min}} = 1600$ Km/s and $f_{\text{max}} = 15$ Hz, discretized by 3375 nodes connected by 16464 tetraedra, distributed on 10 subdomains.

**Figure:** MeTiS distribution
Speed up and Efficiency

Figure: Local Mass Matrices by 4 processors
**Figure:** Speed up: \( S_p = \frac{T_1}{T_p} \)
\textbf{Efficiency}

\textbf{Figure:} Efficiency: \( E_p = \frac{S_p}{np} \)
The parallel algorithm can be split in 3 phases.

I. data distribution and node reordering;
II. assemblage of matrices using the optimized CSR format;
III. time integration and linear system solver.

**Table:** Parallel performance on 453,655 elements.

<table>
<thead>
<tr>
<th>PART</th>
<th>1 PROC</th>
<th>4 PROC</th>
<th>8 PROC</th>
<th>16 PROC</th>
<th>24 PROC</th>
<th>32 PROC</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>177.7665</td>
<td>23.9192</td>
<td>15.01584</td>
<td>13.03564</td>
<td>12.77283</td>
<td>13.12143</td>
</tr>
<tr>
<td>II</td>
<td>51.2128</td>
<td>5.82255</td>
<td>2.51800</td>
<td>1.03282</td>
<td>0.62755</td>
<td>0.47853</td>
</tr>
<tr>
<td>III</td>
<td>2140.091</td>
<td>627.961</td>
<td>389.342</td>
<td>205.041</td>
<td>152.33111</td>
<td>114.364</td>
</tr>
</tbody>
</table>

The table reports the time (in seconds) spent by a processor to execute a part of the algorithm.

**Remarks.**
Most of the time is spent on node reordering and reconstruction of the global solution.
Future Work

• Assemblage of global solution in the overlapping nodes
• Scalability
• Simulations with real topography