Security Protocols

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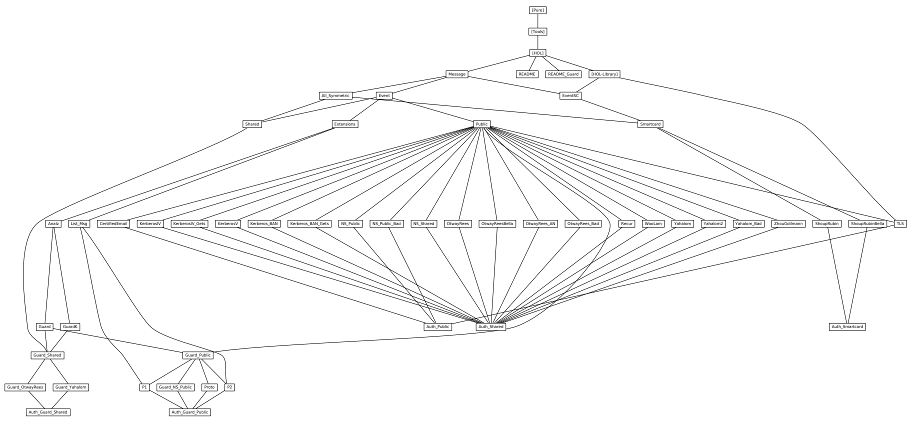

1 Theory of Agents and Messages for Security Protocols

```
theory Message
imports Main
begin


lemma [simp] : "A ∪ (B ∪ A) = B ∪ A"
  ⟨proof⟩

type_synonym
  key = nat

consts
  all_symmetric :: bool          — true if all keys are symmetric
  invKey        :: "key⇒key"    — inverse of a symmetric key

specification (invKey)
  invKey [simp]: "invKey (invKey K) = K"
  invKey_symmetric: "all_symmetric ⟶ invKey = id"
    ⟨proof⟩
```

The inverse of a symmetric key is itself; that of a public key is the private key and vice versa

```
definition symKeys :: "key set" where
  "symKeys == {K. invKey K = K}"

datatype  — We allow any number of friendly agents
  agent = Server | Friend nat | Spy

datatype
     msg = Agent  agent      — Agent names
         | Number nat        — Ordinary integers, timestamps, ...
         | Nonce  nat        — Unguessable nonces
         | Key    key        — Crypto keys
         | Hash   msg        — Hashing
         | MPair  msg msg    — Compound messages
         | Crypt  key msg    — Encryption, public- or shared-key
```

Concrete syntax: messages appear as ⦃A,B,NA⦄, etc...

```
syntax
  "_MTuple" :: "['a, args] ⇒ 'a * 'b"  (‹(‹indent=2 notation=‹mixfix message
tuple››⦃_,/ _⦄)›)
syntax_consts
  "_MTuple" ⇌ MPair
translations
  "⦃x, y, z⦄" ⇌ "⦃x, ⦃y, z⦄⦄"
  "⦃x, y⦄" ⇌ "CONST MPair x y"


definition HPair :: "[msg,msg] ⇒ msg" (‹(4Hash[_] /_)› [0, 1000]) where
    — Message Y paired with a MAC computed with the help of X
```

```
  "Hash[X] Y == {|Hash{|X,Y|}, Y|}"
```

definition *keysFor :: "msg set ⇒ key set"* **where**
— Keys useful to decrypt elements of a message set
"keysFor H == invKey ‘ {K. ∃X. Crypt K X ∈ H}"

1.1 Inductive Definition of All Parts of a Message

inductive_set
parts :: "msg set ⇒ msg set"
for *H :: "msg set"*
where

```
    Inj [intro]: "X ∈ H ⟹ X ∈ parts H"
  | Fst:         "{|X,Y|} ∈ parts H ⟹ X ∈ parts H"
  | Snd:         "{|X,Y|} ∈ parts H ⟹ Y ∈ parts H"
  | Body:        "Crypt K X ∈ parts H ⟹ X ∈ parts H"
```

Monotonicity

lemma *parts_mono_aux: "⟦G ⊆ H; X ∈ parts G⟧ ⟹ X ∈ parts H"*
⟨*proof*⟩

lemma *parts_mono: "G ⊆ H ⟹ parts(G) ⊆ parts(H)"*
⟨*proof*⟩

Equations hold because constructors are injective.

lemma *Friend_image_eq [simp]: "(Friend x ∈ Friend‘A) = (x ∈A)"*
⟨*proof*⟩

lemma *Key_image_eq [simp]: "(Key x ∈ Key‘A) = (x ∈A)"*
⟨*proof*⟩

lemma *Nonce_Key_image_eq [simp]: "(Nonce x ∉ Key‘A)"*
⟨*proof*⟩

1.2 Inverse of keys

lemma *invKey_eq [simp]: "(invKey K = invKey K’) = (K=K’)"*
⟨*proof*⟩

1.3 The *keysFor* operator

lemma *keysFor_empty [simp]: "keysFor {} = {}"*
⟨*proof*⟩

lemma *keysFor_Un [simp]: "keysFor (H ∪ H’) = keysFor H ∪ keysFor H’"*
⟨*proof*⟩

lemma *keysFor_UN [simp]: "keysFor (⋃i ∈A. H i) = (⋃i ∈A. keysFor (H i))"*
⟨*proof*⟩

Monotonicity

lemma *keysFor_mono: "G ⊆ H ⟹ keysFor(G) ⊆ keysFor(H)"*
⟨*proof*⟩

lemma *keysFor_insert_Agent [simp]: "keysFor (insert (Agent A) H) = keysFor H"*
⟨*proof*⟩

lemma *keysFor_insert_Nonce [simp]: "keysFor (insert (Nonce N) H) = keysFor H"*
⟨*proof*⟩

lemma *keysFor_insert_Number [simp]: "keysFor (insert (Number N) H) = keysFor H"*
⟨*proof*⟩

lemma *keysFor_insert_Key [simp]: "keysFor (insert (Key K) H) = keysFor H"*
⟨*proof*⟩

lemma *keysFor_insert_Hash [simp]: "keysFor (insert (Hash X) H) = keysFor H"*
⟨*proof*⟩

lemma *keysFor_insert_MPair [simp]: "keysFor (insert ⦃X,Y⦄ H) = keysFor H"*
⟨*proof*⟩

lemma *keysFor_insert_Crypt [simp]:*
"keysFor (insert (Crypt K X) H) = insert (invKey K) (keysFor H)"
⟨*proof*⟩

lemma *keysFor_image_Key [simp]: "keysFor (Key‘E) = {}"*
⟨*proof*⟩

lemma *Crypt_imp_invKey_keysFor: "Crypt K X ∈ H ⟹ invKey K ∈ keysFor H"*
⟨*proof*⟩

1.4 Inductive relation "parts"

lemma *MPair_parts:*
"⟦⦃X,Y⦄ ∈ parts H;
⟦X ∈ parts H; Y ∈ parts H⟧ ⟹ P⟧ ⟹ P"
⟨*proof*⟩

declare *MPair_parts [elim!] parts.Body [dest!]*

NB These two rules are UNSAFE in the formal sense, as they discard the compound message. They work well on THIS FILE. *MPair_parts* is left as SAFE because it speeds up proofs. The Crypt rule is normally kept UNSAFE to avoid breaking up certificates.

lemma *parts_increasing: "H ⊆ parts(H)"*
⟨*proof*⟩

lemmas *parts_insertI = subset_insertI [THEN parts_mono, THEN subsetD]*

lemma *parts_empty_aux: "X ∈ parts{} ⟹ False"*
⟨*proof*⟩

lemma *parts_empty [simp]: "parts{} = {}"*
⟨*proof*⟩

lemma *parts_emptyE [elim!]: "X ∈ parts{} ⟹ P"*
⟨*proof*⟩

WARNING: loops if H = Y, therefore must not be repeated!

lemma *parts_singleton: "X ∈ parts H ⟹ ∃ Y ∈H. X ∈ parts {Y}"*
⟨*proof*⟩

1.4.1 Unions

lemma *parts_Un [simp]: "parts(G ∪ H) = parts(G) ∪ parts(H)"*
⟨*proof*⟩

lemma *parts_insert: "parts (insert X H) = parts {X} ∪ parts H"*
⟨*proof*⟩

TWO inserts to avoid looping. This rewrite is better than nothing. But its behaviour can be strange.

lemma *parts_insert2:*
"parts (insert X (insert Y H)) = parts {X} ∪ parts {Y} ∪ parts H"
⟨*proof*⟩

lemma *parts_image [simp]:*
"parts (f ‘ A) = (⋃x ∈A. parts {f x})"
⟨*proof*⟩

Added to simplify arguments to parts, analz and synth.

This allows *blast* to simplify occurrences of *parts (G ∪ H)* in the assumption.

lemmas *in_parts_UnE = parts_Un [THEN equalityD1, THEN subsetD, THEN UnE]*

declare *in_parts_UnE [elim!]*

lemma *parts_insert_subset: "insert X (parts H) ⊆ parts(insert X H)"*
⟨*proof*⟩

1.4.2 Idempotence and transitivity

lemma *parts_partsD [dest!]: "X ∈ parts (parts H) ⟹ X ∈ parts H"*
⟨*proof*⟩

lemma *parts_idem [simp]: "parts (parts H) = parts H"*
⟨*proof*⟩

lemma *parts_subset_iff [simp]: "(parts G ⊆ parts H) = (G ⊆ parts H)"*
⟨*proof*⟩

lemma *parts_trans: "⟦X ∈ parts G; G ⊆ parts H⟧ ⟹ X ∈ parts H"*
⟨*proof*⟩

Cut

lemma *parts_cut:*
```
"⟦Y ∈ parts (insert X G);  X ∈ parts H⟧ ⟹ Y ∈ parts (G ∪ H)"
```
⟨*proof*⟩

lemma *parts_cut_eq [simp]: "X ∈ parts H ⟹ parts (insert X H) = parts H"*
⟨*proof*⟩

1.4.3 Rewrite rules for pulling out atomic messages

lemmas *parts_insert_eq_I = equalityI [OF subsetI parts_insert_subset]*

lemma *parts_insert_Agent [simp]:*
```
"parts (insert (Agent agt) H) = insert (Agent agt) (parts H)"
```
⟨*proof*⟩

lemma *parts_insert_Nonce [simp]:*
```
"parts (insert (Nonce N) H) = insert (Nonce N) (parts H)"
```
⟨*proof*⟩

lemma *parts_insert_Number [simp]:*
```
"parts (insert (Number N) H) = insert (Number N) (parts H)"
```
⟨*proof*⟩

lemma *parts_insert_Key [simp]:*
```
"parts (insert (Key K) H) = insert (Key K) (parts H)"
```
⟨*proof*⟩

lemma *parts_insert_Hash [simp]:*
```
"parts (insert (Hash X) H) = insert (Hash X) (parts H)"
```
⟨*proof*⟩

lemma *parts_insert_Crypt [simp]:*
```
"parts (insert (Crypt K X) H) = insert (Crypt K X) (parts (insert X H))"
```
⟨*proof*⟩

lemma *parts_insert_MPair [simp]:*
```
"parts (insert ⦃X,Y⦄ H) = insert ⦃X,Y⦄ (parts (insert X (insert Y H)))"
```
⟨*proof*⟩

lemma *parts_image_Key [simp]: "parts (Key‘N) = Key‘N"*
⟨*proof*⟩

In any message, there is an upper bound N on its greatest nonce.

lemma *msg_Nonce_supply: "∃N. ∀n. N≤n ⟶ Nonce n ∉ parts {msg}"*
⟨*proof*⟩

1.5 Inductive relation "analz"

Inductive definition of "analz" – what can be broken down from a set of messages, including keys. A form of downward closure. Pairs can be taken apart; messages decrypted with known keys.

inductive_set

```
analz :: "msg set ⇒ msg set"
for H :: "msg set"
where
  Inj [intro,simp]: "X ∈ H ⟹ X ∈ analz H"
| Fst:     "⦃X,Y⦄ ∈ analz H ⟹ X ∈ analz H"
| Snd:     "⦃X,Y⦄ ∈ analz H ⟹ Y ∈ analz H"
| Decrypt [dest]:
  "⟦Crypt K X ∈ analz H; Key(invKey K) ∈ analz H⟧ ⟹ X ∈ analz H"
```

Monotonicity; Lemma 1 of Lowe's paper

lemma *analz_mono_aux: "⟦G ⊆ H; X ∈ analz G⟧ ⟹ X ∈ analz H"*
⟨*proof*⟩

lemma *analz_mono: "G⊆H ⟹ analz(G) ⊆ analz(H)"*
⟨*proof*⟩

Making it safe speeds up proofs

lemma *MPair_analz [elim!]:*
"⟦⦃X,Y⦄ ∈ analz H;
⟦X ∈ analz H; Y ∈ analz H⟧ ⟹ P⟧ ⟹ P"
⟨*proof*⟩

lemma *analz_increasing: "H ⊆ analz(H)"*
⟨*proof*⟩

lemma *analz_into_parts: "X ∈ analz H ⟹ X ∈ parts H"*
⟨*proof*⟩

lemma *analz_subset_parts: "analz H ⊆ parts H"*
⟨*proof*⟩

lemma *analz_parts [simp]: "analz (parts H) = parts H"*
⟨*proof*⟩

lemmas *not_parts_not_analz = analz_subset_parts [THEN contra_subsetD]*

lemma *parts_analz [simp]: "parts (analz H) = parts H"*
⟨*proof*⟩

lemmas *analz_insertI = subset_insertI [THEN analz_mono, THEN [2] rev_subsetD]*

1.5.1 General equational properties

lemma *analz_empty [simp]: "analz{} = {}"*
⟨*proof*⟩

Converse fails: we can analz more from the union than from the separate parts, as a key in one might decrypt a message in the other

lemma *analz_Un: "analz(G) ∪ analz(H) ⊆ analz(G ∪ H)"*
⟨*proof*⟩

lemma *analz_insert: "insert X (analz H) ⊆ analz(insert X H)"*
⟨*proof*⟩

1.5.2 Rewrite rules for pulling out atomic messages

lemmas *analz_insert_eq_I = equalityI [OF subsetI analz_insert]*

lemma *analz_insert_Agent [simp]:*
  "analz (insert (Agent agt) H) = insert (Agent agt) (analz H)"
  ⟨*proof*⟩

lemma *analz_insert_Nonce [simp]:*
  "analz (insert (Nonce N) H) = insert (Nonce N) (analz H)"
  ⟨*proof*⟩

lemma *analz_insert_Number [simp]:*
  "analz (insert (Number N) H) = insert (Number N) (analz H)"
  ⟨*proof*⟩

lemma *analz_insert_Hash [simp]:*
  "analz (insert (Hash X) H) = insert (Hash X) (analz H)"
  ⟨*proof*⟩

Can only pull out Keys if they are not needed to decrypt the rest

lemma *analz_insert_Key [simp]:*
  "K ∉ keysFor (analz H) ⟹
  analz (insert (Key K) H) = insert (Key K) (analz H)"
  ⟨*proof*⟩

lemma *analz_insert_MPair [simp]:*
  "analz (insert ⦃X,Y⦄ H) = insert ⦃X,Y⦄ (analz (insert X (insert Y H)))"
⟨*proof*⟩

Can pull out encrypted message if the Key is not known

lemma *analz_insert_Crypt:*
  "Key (invKey K) ∉ analz H
  ⟹ analz (insert (Crypt K X) H) = insert (Crypt K X) (analz H)"
  ⟨*proof*⟩

lemma *analz_insert_Decrypt:*
  assumes *"Key (invKey K) ∈ analz H"*
  shows *"analz (insert (Crypt K X) H) = insert (Crypt K X) (analz (insert X H))"*
⟨*proof*⟩

Case analysis: either the message is secure, or it is not! Effective, but can cause subgoals to blow up! Use with *if_split*; apparently *split_tac* does not cope with patterns such as *analz (insert (Crypt K X) H)*

lemma *analz_Crypt_if [simp]:*
  "analz (insert (Crypt K X) H) =
  (if (Key (invKey K) ∈ analz H)
  then insert (Crypt K X) (analz (insert X H))
  else insert (Crypt K X) (analz H))"
  ⟨*proof*⟩

This rule supposes "for the sake of argument" that we have the key.

lemma *analz_insert_Crypt_subset:*

```
"analz (insert (Crypt K X) H) ⊆
        insert (Crypt K X) (analz (insert X H))"
```
⟨proof⟩

lemma *analz_image_Key [simp]: "analz (Key‘N) = Key‘N"*
⟨proof⟩

1.5.3 Idempotence and transitivity

lemma *analz_analzD [dest!]: "X ∈ analz (analz H) ⟹ X ∈ analz H"*
⟨proof⟩

lemma *analz_idem [simp]: "analz (analz H) = analz H"*
⟨proof⟩

lemma *analz_subset_iff [simp]: "(analz G ⊆ analz H) = (G ⊆ analz H)"*
⟨proof⟩

lemma *analz_trans: "⟦X ∈ analz G; G ⊆ analz H⟧ ⟹ X ∈ analz H"*
⟨proof⟩

Cut; Lemma 2 of Lowe

lemma *analz_cut: "⟦Y ∈ analz (insert X H); X ∈ analz H⟧ ⟹ Y ∈ analz H"*
⟨proof⟩

This rewrite rule helps in the simplification of messages that involve the forwarding of unknown components (X). Without it, removing occurrences of X can be very complicated.

lemma *analz_insert_eq: "X ∈ analz H ⟹ analz (insert X H) = analz H"*
⟨proof⟩

A congruence rule for "analz"

lemma *analz_subset_cong:*
```
"⟦analz G ⊆ analz G'; analz H ⊆ analz H'⟧
    ⟹ analz (G ∪ H) ⊆ analz (G' ∪ H')"
```
⟨proof⟩

lemma *analz_cong:*
```
"⟦analz G = analz G'; analz H = analz H'⟧
    ⟹ analz (G ∪ H) = analz (G' ∪ H')"
```
⟨proof⟩

lemma *analz_insert_cong:*
```
"analz H = analz H' ⟹ analz(insert X H) = analz(insert X H')"
```
⟨proof⟩

If there are no pairs or encryptions then analz does nothing

lemma *analz_trivial:*
```
"⟦∀X Y. ⦃X,Y⦄ ∉ H;  ∀X K. Crypt K X ∉ H⟧ ⟹ analz H = H"
```
⟨proof⟩

1.6 Inductive relation "synth"

Inductive definition of "synth" – what can be built up from a set of messages. A form of upward closure. Pairs can be built, messages encrypted with known keys. Agent names are public domain. Numbers can be guessed, but Nonces cannot be.

inductive_set
  synth :: "msg set => msg set"
  for *H :: "msg set"*
  where
```
    Inj    [intro]:   "X ∈ H ⟹ X ∈ synth H"
  | Agent  [intro]:   "Agent agt ∈ synth H"
  | Number [intro]:   "Number n  ∈ synth H"
  | Hash   [intro]:   "X ∈ synth H ⟹ Hash X ∈ synth H"
  | MPair  [intro]:   "⟦X ∈ synth H;  Y ∈ synth H⟧ ⟹ ⦃X,Y⦄ ∈ synth H"
  | Crypt  [intro]:   "⟦X ∈ synth H;  Key(K) ∈ H⟧ ⟹ Crypt K X ∈ synth H"
```

Monotonicity

lemma *synth_mono: "G⊆H ⟹ synth(G) ⊆ synth(H)"*
  ⟨*proof*⟩

NO *Agent_synth*, as any Agent name can be synthesized. The same holds for *Number*

inductive_simps *synth_simps [iff]:*
```
  "Nonce n ∈ synth H"
  "Key K ∈ synth H"
  "Hash X ∈ synth H"
  "⦃X,Y⦄ ∈ synth H"
  "Crypt K X ∈ synth H"
```

lemma *synth_increasing: "H ⊆ synth(H)"*
  ⟨*proof*⟩

1.6.1 Unions

Converse fails: we can synth more from the union than from the separate parts, building a compound message using elements of each.

lemma *synth_Un: "synth(G) ∪ synth(H) ⊆ synth(G ∪ H)"*
  ⟨*proof*⟩

lemma *synth_insert: "insert X (synth H) ⊆ synth(insert X H)"*
  ⟨*proof*⟩

1.6.2 Idempotence and transitivity

lemma *synth_synthD [dest!]: "X ∈ synth (synth H) ⟹ X ∈ synth H"*
  ⟨*proof*⟩

lemma *synth_idem: "synth (synth H) = synth H"*
  ⟨*proof*⟩

lemma *synth_subset_iff [simp]: "(synth G ⊆ synth H) = (G ⊆ synth H)"*

⟨proof⟩

lemma *synth_trans: "⟦X ∈ synth G; G ⊆ synth H⟧ ⟹ X ∈ synth H"*
⟨proof⟩

Cut; Lemma 2 of Lowe

lemma *synth_cut: "⟦Y ∈ synth (insert X H); X ∈ synth H⟧ ⟹ Y ∈ synth H"*
⟨proof⟩

lemma *Crypt_synth_eq [simp]:*
"Key K ∉ H ⟹ (Crypt K X ∈ synth H) = (Crypt K X ∈ H)"
⟨proof⟩

lemma *keysFor_synth [simp]:*
"keysFor (synth H) = keysFor H ∪ invKey`{K. Key K ∈ H}"
⟨proof⟩

1.6.3 Combinations of parts, analz and synth

lemma *parts_synth [simp]: "parts (synth H) = parts H ∪ synth H"*
⟨proof⟩

lemma *analz_analz_Un [simp]: "analz (analz G ∪ H) = analz (G ∪ H)"*
⟨proof⟩

lemma *analz_synth_Un [simp]: "analz (synth G ∪ H) = analz (G ∪ H) ∪ synth G"*
⟨proof⟩

lemma *analz_synth [simp]: "analz (synth H) = analz H ∪ synth H"*
⟨proof⟩

1.6.4 For reasoning about the Fake rule in traces

lemma *parts_insert_subset_Un: "X ∈ G ⟹ parts(insert X H) ⊆ parts G ∪ parts H"*
⟨proof⟩

More specifically for Fake. See also *Fake_parts_sing* below

lemma *Fake_parts_insert:*
"X ∈ synth (analz H) ⟹
parts (insert X H) ⊆ synth (analz H) ∪ parts H"
⟨proof⟩

lemma *Fake_parts_insert_in_Un:*
"⟦Z ∈ parts (insert X H); X ∈ synth (analz H)⟧
⟹ Z ∈ synth (analz H) ∪ parts H"
⟨proof⟩

H is sometimes *Key ` KK ∪ spies evs*, so can't put *G = H*.

lemma *Fake_analz_insert:*
"X ∈ synth (analz G) ⟹

```
    analz (insert X H) ⊆ synth (analz G) ∪ analz (G ∪ H)"
  ⟨proof⟩

lemma analz_conj_parts [simp]:
  "(X ∈ analz H ∧ X ∈ parts H) = (X ∈ analz H)"
  ⟨proof⟩

lemma analz_disj_parts [simp]:
  "(X ∈ analz H | X ∈ parts H) = (X ∈ parts H)"
  ⟨proof⟩
```

Without this equation, other rules for synth and analz would yield redundant cases

```
lemma MPair_synth_analz [iff]:
  "{|X,Y|} ∈ synth (analz H) ⟷ X ∈ synth (analz H) ∧ Y ∈ synth (analz H)"
  ⟨proof⟩

lemma Crypt_synth_analz:
  "⟦Key K ∈ analz H;  Key (invKey K) ∈ analz H⟧
     ⟹ (Crypt K X ∈ synth (analz H)) = (X ∈ synth (analz H))"
  ⟨proof⟩

lemma Hash_synth_analz [simp]:
  "X ∉ synth (analz H)
    ⟹ (Hash{|X,Y|} ∈ synth (analz H)) = (Hash{|X,Y|} ∈ analz H)"
  ⟨proof⟩
```

1.7 HPair: a combination of Hash and MPair

1.7.1 Freeness

```
lemma Agent_neq_HPair: "Agent A ≠ Hash[X] Y"
  ⟨proof⟩

lemma Nonce_neq_HPair: "Nonce N ≠ Hash[X] Y"
  ⟨proof⟩

lemma Number_neq_HPair: "Number N ≠ Hash[X] Y"
  ⟨proof⟩

lemma Key_neq_HPair: "Key K ≠ Hash[X] Y"
  ⟨proof⟩

lemma Hash_neq_HPair: "Hash Z ≠ Hash[X] Y"
  ⟨proof⟩

lemma Crypt_neq_HPair: "Crypt K X' ≠ Hash[X] Y"
  ⟨proof⟩

lemmas HPair_neqs = Agent_neq_HPair Nonce_neq_HPair Number_neq_HPair
  Key_neq_HPair Hash_neq_HPair Crypt_neq_HPair

declare HPair_neqs [iff]
declare HPair_neqs [symmetric, iff]
```

```
lemma HPair_eq [iff]: "(Hash[X'] Y' = Hash[X] Y) = (X' = X ∧ Y'=Y)"
  ⟨proof⟩

lemma MPair_eq_HPair [iff]:
  "(⦃X',Y'⦄ = Hash[X] Y) = (X' = Hash⦃X,Y⦄ ∧ Y'=Y)"
  ⟨proof⟩

lemma HPair_eq_MPair [iff]:
  "(Hash[X] Y = ⦃X',Y'⦄) = (X' = Hash⦃X,Y⦄ ∧ Y'=Y)"
  ⟨proof⟩
```

1.7.2 Specialized laws, proved in terms of those for Hash and MPair

```
lemma keysFor_insert_HPair [simp]: "keysFor (insert (Hash[X] Y) H) = keysFor
H"
  ⟨proof⟩

lemma parts_insert_HPair [simp]:
  "parts (insert (Hash[X] Y) H) =
     insert (Hash[X] Y) (insert (Hash⦃X,Y⦄) (parts (insert Y H)))"
  ⟨proof⟩

lemma analz_insert_HPair [simp]:
  "analz (insert (Hash[X] Y) H) =
     insert (Hash[X] Y) (insert (Hash⦃X,Y⦄) (analz (insert Y H)))"
  ⟨proof⟩

lemma HPair_synth_analz [simp]:
  "X ∉ synth (analz H)
    ⟹ (Hash[X] Y ∈ synth (analz H)) =
        (Hash ⦃X, Y⦄ ∈ analz H ∧ Y ∈ synth (analz H))"
  ⟨proof⟩
```

We do NOT want Crypt... messages broken up in protocols!!

```
declare parts.Body [rule del]
```

Rewrites to push in Key and Crypt messages, so that other messages can be pulled out using the `analz_insert` rules

```
lemmas pushKeys =
  insert_commute [of "Key K" "Agent C"]
  insert_commute [of "Key K" "Nonce N"]
  insert_commute [of "Key K" "Number N"]
  insert_commute [of "Key K" "Hash X"]
  insert_commute [of "Key K" "MPair X Y"]
  insert_commute [of "Key K" "Crypt X K'"]
  for K C N X Y K'

lemmas pushCrypts =
  insert_commute [of "Crypt X K" "Agent C"]
  insert_commute [of "Crypt X K" "Agent C"]
  insert_commute [of "Crypt X K" "Nonce N"]
  insert_commute [of "Crypt X K" "Number N"]
  insert_commute [of "Crypt X K" "Hash X'"]
```

```
insert_commute [of "Crypt X K" "MPair X' Y"]
for X K C N X' Y
```

Cannot be added with `[simp]` – messages should not always be re-ordered.

lemmas *pushes = pushKeys pushCrypts*

1.8 The set of key-free messages

```
inductive_set
  keyfree :: "msg set"
  where
    Agent:  "Agent A ∈ keyfree"
  | Number: "Number N ∈ keyfree"
  | Nonce:  "Nonce N ∈ keyfree"
  | Hash:   "Hash X ∈ keyfree"
  | MPair:  "⟦X ∈ keyfree;  Y ∈ keyfree⟧ ⟹ ⦃X,Y⦄ ∈ keyfree"
  | Crypt:  "⟦X ∈ keyfree⟧ ⟹ Crypt K X ∈ keyfree"
```

```
declare keyfree.intros [intro]
```

```
inductive_cases keyfree_KeyE: "Key K ∈ keyfree"
inductive_cases keyfree_MPairE: "⦃X,Y⦄ ∈ keyfree"
inductive_cases keyfree_CryptE: "Crypt K X ∈ keyfree"
```

```
lemma parts_keyfree: "parts (keyfree) ⊆ keyfree"
  ⟨proof⟩
```

```
lemma analz_keyfree_into_Un: "⟦X ∈ analz (G ∪ H); G ⊆ keyfree⟧ ⟹ X ∈
parts G ∪ analz H"
⟨proof⟩
```

1.9 Tactics useful for many protocol proofs

⟨ML⟩

By default only `o_apply` is built-in. But in the presence of eta-expansion this means that some terms displayed as `f ∘ g` will be rewritten, and others will not!

```
declare o_def [simp]
```

```
lemma Crypt_notin_image_Key [simp]: "Crypt K X ∉ Key ` A"
  ⟨proof⟩
```

```
lemma Hash_notin_image_Key [simp] :"Hash X ∉ Key ` A"
  ⟨proof⟩
```

```
lemma synth_analz_mono: "G⊆H ⟹ synth (analz(G)) ⊆ synth (analz(H))"
  ⟨proof⟩
```

```
lemma Fake_analz_eq [simp]:
  "X ∈ synth(analz H) ⟹ synth (analz (insert X H)) = synth (analz H)"
```

⟨*proof*⟩

Two generalizations of `analz_insert_eq`

```
lemma gen_analz_insert_eq [rule_format]:
  "X ∈ analz H ⟹ ∀G. H ⊆ G ⟶ analz (insert X G) = analz G"
  ⟨proof⟩

lemma synth_analz_insert_eq:
  "⟦X ∈ synth (analz H); H ⊆ G⟧
     ⟹ (Key K ∈ analz (insert X G)) ⟷ (Key K ∈ analz G)"
⟨proof⟩

lemma Fake_parts_sing:
  "X ∈ synth (analz H) ⟹ parts{X} ⊆ synth (analz H) ∪ parts H"
  ⟨proof⟩

lemmas Fake_parts_sing_imp_Un = Fake_parts_sing [THEN [2] rev_subsetD]

⟨ML⟩

end
```

2 Theory of Events for Security Protocols

```
theory Event imports Message begin

consts  — Initial states of agents — a parameter of the construction
  initState :: "agent ⇒ msg set"

datatype
  event = Says  agent agent msg
        | Gets  agent       msg
        | Notes agent       msg

consts
  bad    :: "agent set"                        — compromised agents
```

Spy has access to his own key for spoof messages, but Server is secure

```
specification (bad)
  Spy_in_bad     [iff]: "Spy ∈ bad"
  Server_not_bad [iff]: "Server ∉ bad"
    ⟨proof⟩

primrec knows :: "agent ⇒ event list ⇒ msg set"
where
  knows_Nil:   "knows A [] = initState A"
| knows_Cons:
    "knows A (ev # evs) =
       (if A = Spy then
        (case ev of
           Says A' B X ⇒ insert X (knows Spy evs)
         | Gets A' X ⇒ knows Spy evs
         | Notes A' X  ⇒
```

```
          if A' ∈ bad then insert X (knows Spy evs) else knows Spy evs)
     else
     (case ev of
        Says A' B X ⇒
          if A'=A then insert X (knows A evs) else knows A evs
      | Gets A' X    ⇒
          if A'=A then insert X (knows A evs) else knows A evs
      | Notes A' X    ⇒
          if A'=A then insert X (knows A evs) else knows A evs))"
```

The constant "spies" is retained for compatibility's sake

abbreviation *(input)*
spies :: "event list ⇒ msg set" **where**
"spies ≡ knows Spy"

Set of items that might be visible to somebody: complement of the set of fresh items

primrec *used :: "event list ⇒ msg set"*
where

```
  used_Nil:   "used []         = (UN B. parts (initState B))"
| used_Cons:  "used (ev # evs) =
                     (case ev of
                        Says A B X ⇒ parts {X} ∪ used evs
                      | Gets A X   ⇒ used evs
                      | Notes A X  ⇒ parts {X} ∪ used evs)"
```

— The case for *Gets* seems anomalous, but *Gets* always follows *Says* in real protocols. Seems difficult to change. See *Gets_correct* in theory *Guard/Extensions.thy*.

lemma *Notes_imp_used: "Notes A X ∈ set evs ⟹ X ∈ used evs"*
⟨*proof*⟩

lemma *Says_imp_used: "Says A B X ∈ set evs ⟹ X ∈ used evs"*
⟨*proof*⟩

2.1 Function *knows*

lemmas *parts_insert_knows_A = parts_insert [of _ "knows A evs"]* **for** *A evs*

lemma *knows_Spy_Says [simp]:*
"knows Spy (Says A B X # evs) = insert X (knows Spy evs)"
⟨*proof*⟩

Letting the Spy see "bad" agents' notes avoids redundant case-splits on whether *A = Spy* and whether *A ∈ bad*

lemma *knows_Spy_Notes [simp]:*
"knows Spy (Notes A X # evs) =
(if A∈bad then insert X (knows Spy evs) else knows Spy evs)"
⟨*proof*⟩

lemma *knows_Spy_Gets [simp]: "knows Spy (Gets A X # evs) = knows Spy evs"*
⟨*proof*⟩

lemma *knows_Spy_subset_knows_Spy_Says:*

"knows Spy evs ⊆ knows Spy (Says A B X # evs)"
⟨*proof*⟩

lemma *knows_Spy_subset_knows_Spy_Notes:*
"knows Spy evs ⊆ knows Spy (Notes A X # evs)"
⟨*proof*⟩

lemma *knows_Spy_subset_knows_Spy_Gets:*
"knows Spy evs ⊆ knows Spy (Gets A X # evs)"
⟨*proof*⟩

Spy sees what is sent on the traffic

lemma *Says_imp_knows_Spy:*
"Says A B X ∈ set evs ⟹ X ∈ knows Spy evs"
⟨*proof*⟩

lemma *Notes_imp_knows_Spy [rule_format]:*
"Notes A X ∈ set evs ⟹ A ∈ bad ⟹ X ∈ knows Spy evs"
⟨*proof*⟩

Elimination rules: derive contradictions from old Says events containing items known to be fresh

lemmas *Says_imp_parts_knows_Spy =*
Says_imp_knows_Spy [THEN parts.Inj, elim_format]

lemmas *knows_Spy_partsEs =*
Says_imp_parts_knows_Spy parts.Body [elim_format]

lemmas *Says_imp_analz_Spy = Says_imp_knows_Spy [THEN analz.Inj]*

Compatibility for the old "spies" function

lemmas *spies_partsEs = knows_Spy_partsEs*
lemmas *Says_imp_spies = Says_imp_knows_Spy*
lemmas *parts_insert_spies = parts_insert_knows_A [of _ Spy]*

2.2 Knowledge of Agents

lemma *knows_subset_knows_Says: "knows A evs ⊆ knows A (Says A' B X # evs)"*
⟨*proof*⟩

lemma *knows_subset_knows_Notes: "knows A evs ⊆ knows A (Notes A' X # evs)"*
⟨*proof*⟩

lemma *knows_subset_knows_Gets: "knows A evs ⊆ knows A (Gets A' X # evs)"*
⟨*proof*⟩

Agents know what they say

lemma *Says_imp_knows [rule_format]: "Says A B X ∈ set evs ⟹ X ∈ knows A evs"*
⟨*proof*⟩

Agents know what they note

lemma *Notes_imp_knows [rule_format]: "Notes A X ∈ set evs ⟹ X ∈ knows A evs"*
⟨*proof*⟩

Agents know what they receive

lemma *Gets_imp_knows_agents [rule_format]:*
"A ≠ Spy ⟹ Gets A X ∈ set evs ⟹ X ∈ knows A evs"
⟨*proof*⟩

What agents DIFFERENT FROM Spy know was either said, or noted, or got, or known initially

lemma *knows_imp_Says_Gets_Notes_initState:*
"⟦X ∈ knows A evs; A ≠ Spy⟧ ⟹
∃B. Says A B X ∈ set evs ∨ Gets A X ∈ set evs ∨ Notes A X ∈ set evs ∨ X ∈ initState A"
⟨*proof*⟩

What the Spy knows – for the time being – was either said or noted, or known initially

lemma *knows_Spy_imp_Says_Notes_initState:*
"X ∈ knows Spy evs ⟹
∃A B. Says A B X ∈ set evs ∨ Notes A X ∈ set evs ∨ X ∈ initState Spy"
⟨*proof*⟩

lemma *parts_knows_Spy_subset_used: "parts (knows Spy evs) ⊆ used evs"*
⟨*proof*⟩

lemmas *usedI = parts_knows_Spy_subset_used [THEN subsetD, intro]*

lemma *initState_into_used: "X ∈ parts (initState B) ⟹ X ∈ used evs"*
⟨*proof*⟩

New simprules to replace the primitive ones for `used` and `knows`

lemma *used_Says [simp]: "used (Says A B X # evs) = parts{X} ∪ used evs"*
⟨*proof*⟩

lemma *used_Notes [simp]: "used (Notes A X # evs) = parts{X} ∪ used evs"*
⟨*proof*⟩

lemma *used_Gets [simp]: "used (Gets A X # evs) = used evs"*
⟨*proof*⟩

lemma *used_nil_subset: "used [] ⊆ used evs"*
⟨*proof*⟩

NOTE REMOVAL: the laws above are cleaner, as they don't involve "case"

declare *knows_Cons [simp del]*
used_Nil [simp del] used_Cons [simp del]

For proving theorems of the form *X ∉ analz (knows Spy evs) ⟶ P* New events added by induction to "evs" are discarded. Provided this information isn't needed, the proof will be much shorter, since it will omit complicated reasoning about `analz`.

```
lemmas analz_mono_contra =
       knows_Spy_subset_knows_Spy_Says [THEN analz_mono, THEN contra_subsetD]
       knows_Spy_subset_knows_Spy_Notes [THEN analz_mono, THEN contra_subsetD]
       knows_Spy_subset_knows_Spy_Gets [THEN analz_mono, THEN contra_subsetD]
```

```
lemma knows_subset_knows_Cons: "knows A evs ⊆ knows A (e # evs)"
  ⟨proof⟩
```

```
lemma initState_subset_knows: "initState A ⊆ knows A evs"
  ⟨proof⟩
```

For proving `new_keys_not_used`

```
lemma keysFor_parts_insert:
     "⟦K ∈ keysFor (parts (insert X G));  X ∈ synth (analz H)⟧
      ⟹ K ∈ keysFor (parts (G ∪ H)) | Key (invKey K) ∈ parts H"
⟨proof⟩
```

```
lemmas analz_impI = impI [where P = "Y ∉ analz (knows Spy evs)"] for Y evs
```

⟨ML⟩

Useful for case analysis on whether a hash is a spoof or not

```
lemmas syan_impI = impI [where P = "Y ∉ synth (analz (knows Spy evs))"]
for Y evs
```

⟨ML⟩

```
end
```

```
theory Public
imports Event
begin
```

```
lemma invKey_K: "K ∈ symKeys ⟹ invKey K = K"
⟨proof⟩
```

2.3 Asymmetric Keys

```
datatype keymode = Signature | Encryption
```

```
consts
  publicKey :: "[keymode,agent] ⇒ key"
```

```
abbreviation
  pubEK :: "agent ⇒ key" where
  "pubEK == publicKey Encryption"
```

```
abbreviation
  pubSK :: "agent ⇒ key" where
  "pubSK == publicKey Signature"
```

abbreviation
privateKey :: "[keymode, agent] ⇒ key" **where**
"privateKey b A == invKey (publicKey b A)"

abbreviation

priEK :: "agent ⇒ key" **where**
"priEK A == privateKey Encryption A"

abbreviation
priSK :: "agent ⇒ key" **where**
"priSK A == privateKey Signature A"

These abbreviations give backward compatibility. They represent the simple situation where the signature and encryption keys are the same.

abbreviation *(input)*
pubK :: "agent ⇒ key" **where**
"pubK A == pubEK A"

abbreviation *(input)*
priK :: "agent ⇒ key" **where**
"priK A == invKey (pubEK A)"

By freeness of agents, no two agents have the same key. Since *True* ≠ *False*, no agent has identical signing and encryption keys

specification *(publicKey)*
injective_publicKey:
"publicKey b A = publicKey c A' ⟹ b=c ∧ A=A'"
⟨*proof*⟩

axiomatization where

privateKey_neq_publicKey [iff]: "privateKey b A ≠ publicKey c A'"

lemmas *publicKey_neq_privateKey = privateKey_neq_publicKey [THEN not_sym]*
declare *publicKey_neq_privateKey [iff]*

2.4 Basic properties of *pubEK* and *priEK*

lemma *publicKey_inject [iff]: "(publicKey b A = publicKey c A') = (b=c ∧ A=A')"*
⟨*proof*⟩

lemma *not_symKeys_pubK [iff]: "publicKey b A ∉ symKeys"*
⟨*proof*⟩

lemma *not_symKeys_priK [iff]: "privateKey b A ∉ symKeys"*
⟨*proof*⟩

lemma *symKey_neq_priEK: "K ∈ symKeys ⟹ K ≠ priEK A"*
⟨*proof*⟩

lemma *symKeys_neq_imp_neq: "(K ∈ symKeys) ≠ (K' ∈ symKeys) ⟹ K ≠ K'"*
⟨*proof*⟩

lemma *symKeys_invKey_iff [iff]: "(invKey K ∈ symKeys) = (K ∈ symKeys)"*
⟨*proof*⟩

lemma *analz_symKeys_Decrypt:*
"⟦Crypt K X ∈ analz H; K ∈ symKeys; Key K ∈ analz H⟧
⟹ X ∈ analz H"
⟨*proof*⟩

2.5 "Image" equations that hold for injective functions

lemma *invKey_image_eq [simp]: "(invKey x ∈ invKey`A) = (x ∈ A)"*
⟨*proof*⟩

lemma *publicKey_image_eq [simp]:*
"(publicKey b x ∈ publicKey c ` AA) = (b=c ∧ x ∈ AA)"
⟨*proof*⟩

lemma *privateKey_notin_image_publicKey [simp]: "privateKey b x ∉ publicKey c ` AA"*
⟨*proof*⟩

lemma *privateKey_image_eq [simp]:*
"(privateKey b A ∈ invKey ` publicKey c ` AS) = (b=c ∧ A∈AS)"
⟨*proof*⟩

lemma *publicKey_notin_image_privateKey [simp]: "publicKey b A ∉ invKey ` publicKey c ` AS"*
⟨*proof*⟩

2.6 Symmetric Keys

For some protocols, it is convenient to equip agents with symmetric as well as asymmetric keys. The theory *Shared* assumes that all keys are symmetric.

consts
shrK :: "agent => key" — long-term shared keys

specification *(shrK)*
inj_shrK: "inj shrK"
— No two agents have the same long-term key
⟨*proof*⟩

axiomatization where
sym_shrK [iff]: "shrK X ∈ symKeys" — All shared keys are symmetric

Injectiveness: Agents' long-term keys are distinct.

lemmas *shrK_injective = inj_shrK [THEN inj_eq]*
declare *shrK_injective [iff]*

lemma *invKey_shrK [simp]: "invKey (shrK A) = shrK A"*

⟨*proof*⟩

lemma *analz_shrK_Decrypt:*
"⟦Crypt (shrK A) X ∈ analz H; Key(shrK A) ∈ analz H⟧ ⟹ X ∈ analz H"
⟨*proof*⟩

lemma *analz_Decrypt':*
"⟦Crypt K X ∈ analz H; K ∈ symKeys; Key K ∈ analz H⟧ ⟹ X ∈ analz H"
⟨*proof*⟩

lemma *priK_neq_shrK [iff]: "shrK A ≠ privateKey b C"*
⟨*proof*⟩

lemmas *shrK_neq_priK = priK_neq_shrK [THEN not_sym]*
declare *shrK_neq_priK [simp]*

lemma *pubK_neq_shrK [iff]: "shrK A ≠ publicKey b C"*
⟨*proof*⟩

lemmas *shrK_neq_pubK = pubK_neq_shrK [THEN not_sym]*
declare *shrK_neq_pubK [simp]*

lemma *priEK_noteq_shrK [simp]: "priEK A ≠ shrK B"*
⟨*proof*⟩

lemma *publicKey_notin_image_shrK [simp]: "publicKey b x ∉ shrK ‘ AA"*
⟨*proof*⟩

lemma *privateKey_notin_image_shrK [simp]: "privateKey b x ∉ shrK ‘ AA"*
⟨*proof*⟩

lemma *shrK_notin_image_publicKey [simp]: "shrK x ∉ publicKey b ‘ AA"*
⟨*proof*⟩

lemma *shrK_notin_image_privateKey [simp]: "shrK x ∉ invKey ‘ publicKey b ‘ AA"*
⟨*proof*⟩

lemma *shrK_image_eq [simp]: "(shrK x ∈ shrK ‘ AA) = (x ∈ AA)"*
⟨*proof*⟩

For some reason, moving this up can make some proofs loop!

declare *invKey_K [simp]*

2.7 Initial States of Agents

Note: for all practical purposes, all that matters is the initial knowledge of the Spy. All other agents are automata, merely following the protocol.

overloading
initState ≡ initState
begin

```
primrec initState where

  initState_Server:
    "initState Server     =
       {Key (priEK Server), Key (priSK Server)} ∪
       (Key ‘ range pubEK) ∪ (Key ‘ range pubSK) ∪ (Key ‘ range shrK)"

| initState_Friend:
    "initState (Friend i) =
       {Key (priEK(Friend i)), Key (priSK(Friend i)), Key (shrK(Friend i))}
∪
       (Key ‘ range pubEK) ∪ (Key ‘ range pubSK)"

| initState_Spy:
    "initState Spy        =
       (Key ‘ invKey ‘ pubEK ‘ bad) ∪ (Key ‘ invKey ‘ pubSK ‘ bad) ∪
       (Key ‘ shrK ‘ bad) ∪
       (Key ‘ range pubEK) ∪ (Key ‘ range pubSK)"

end
```

These lemmas allow reasoning about `used evs` rather than `knows Spy evs`, which is useful when there are private Notes. Because they depend upon the definition of `initState`, they cannot be moved up.

```
lemma used_parts_subset_parts [rule_format]:
    "∀X ∈ used evs. parts {X} ⊆ used evs"
⟨proof⟩

lemma MPair_used_D: "{|X,Y|} ∈ used H ⟹ X ∈ used H ∧ Y ∈ used H"
⟨proof⟩
```

There was a similar theorem in Event.thy, so perhaps this one can be moved up if proved directly by induction.

```
lemma MPair_used [elim!]:
    "⟦{|X,Y|} ∈ used H;
       ⟦X ∈ used H; Y ∈ used H⟧ ⟹ P⟧
     ⟹ P"
⟨proof⟩
```

Rewrites should not refer to `initState (Friend i)` because that expression is not in normal form.

```
lemma keysFor_parts_initState [simp]: "keysFor (parts (initState C)) = {}"
⟨proof⟩

lemma Crypt_notin_initState: "Crypt K X ∉ parts (initState B)"
⟨proof⟩

lemma Crypt_notin_used_empty [simp]: "Crypt K X ∉ used []"
⟨proof⟩
```

lemma *shrK_in_initState [iff]: "Key (shrK A) ∈ initState A"*
⟨*proof*⟩

lemma *shrK_in_knows [iff]: "Key (shrK A) ∈ knows A evs"*
⟨*proof*⟩

lemma *shrK_in_used [iff]: "Key (shrK A) ∈ used evs"*
⟨*proof*⟩

lemma *Key_not_used [simp]: "Key K ∉ used evs ⟹ K ∉ range shrK"*
⟨*proof*⟩

lemma *shrK_neq: "Key K ∉ used evs ⟹ shrK B ≠ K"*
⟨*proof*⟩

lemmas *neq_shrK = shrK_neq [THEN not_sym]*
declare *neq_shrK [simp]*

2.8 Function *knows Spy*

lemma *not_SignatureE [elim!]: "b ≠ Signature ⟹ b = Encryption"*
⟨*proof*⟩

Agents see their own private keys!

lemma *priK_in_initState [iff]: "Key (privateKey b A) ∈ initState A"*
⟨*proof*⟩

Agents see all public keys!

lemma *publicKey_in_initState [iff]: "Key (publicKey b A) ∈ initState B"*
⟨*proof*⟩

All public keys are visible

lemma *spies_pubK [iff]: "Key (publicKey b A) ∈ spies evs"*
⟨*proof*⟩

lemmas *analz_spies_pubK = spies_pubK [THEN analz.Inj]*
declare *analz_spies_pubK [iff]*

Spy sees private keys of bad agents!

lemma *Spy_spies_bad_privateKey [intro!]:*
"A ∈ bad ⟹ Key (privateKey b A) ∈ spies evs"
⟨*proof*⟩

Spy sees long-term shared keys of bad agents!

lemma *Spy_spies_bad_shrK [intro!]:*
"A ∈ bad ⟹ Key (shrK A) ∈ spies evs"
⟨*proof*⟩

lemma *publicKey_into_used [iff] :"Key (publicKey b A) ∈ used evs"*

⟨*proof*⟩

lemma *privateKey_into_used [iff]: "Key (privateKey b A) ∈ used evs"*
⟨*proof*⟩

lemma *Crypt_Spy_analz_bad:*
"⟦Crypt (shrK A) X ∈ analz (knows Spy evs); A ∈ bad⟧
⟹ X ∈ analz (knows Spy evs)"
⟨*proof*⟩

2.9 Fresh Nonces

lemma *Nonce_notin_initState [iff]: "Nonce N ∉ parts (initState B)"*
⟨*proof*⟩

lemma *Nonce_notin_used_empty [simp]: "Nonce N ∉ used []"*
⟨*proof*⟩

2.10 Supply fresh nonces for possibility theorems

In any trace, there is an upper bound N on the greatest nonce in use

lemma *Nonce_supply_lemma: "∃N. ∀n. N≤n ⟶ Nonce n ∉ used evs"*
⟨*proof*⟩

lemma *Nonce_supply1: "∃N. Nonce N ∉ used evs"*
⟨*proof*⟩

lemma *Nonce_supply: "Nonce (SOME N. Nonce N ∉ used evs) ∉ used evs"*
⟨*proof*⟩

2.11 Specialized Rewriting for Theorems About *analz* and Image

lemma *insert_Key_singleton: "insert (Key K) H = Key ‘ {K} ∪ H"*
⟨*proof*⟩

lemma *insert_Key_image: "insert (Key K) (Key‘KK ∪ C) = Key ‘ (insert K KK) ∪ C"*
⟨*proof*⟩

lemma *Crypt_imp_keysFor :"⟦Crypt K X ∈ H; K ∈ symKeys⟧ ⟹ K ∈ keysFor H"*
⟨*proof*⟩

Lemma for the trivial direction of the if-and-only-if of the Session Key Compromise Theorem

lemma *analz_image_freshK_lemma:*
"(Key K ∈ analz (Key‘nE ∪ H)) ⟶ (K ∈ nE | Key K ∈ analz H) ⟹
(Key K ∈ analz (Key‘nE ∪ H)) = (K ∈ nE | Key K ∈ analz H)"
⟨*proof*⟩

lemmas *analz_image_freshK_simps* =
simp_thms mem_simps — these two allow its use with *only:*
disj_comms
image_insert [THEN sym] image_Un [THEN sym] empty_subsetI insert_subset
analz_insert_eq Un_upper2 [THEN analz_mono, THEN subsetD]
insert_Key_singleton
Key_not_used insert_Key_image Un_assoc [THEN sym]

⟨*ML*⟩

2.12 Specialized Methods for Possibility Theorems

⟨*ML*⟩

end

3 Needham-Schroeder Shared-Key Protocol

theory *NS_Shared* **imports** *Public* **begin**

From page 247 of Burrows, Abadi and Needham (1989). A Logic of Authentication. Proc. Royal Soc. 426

definition

```
Issues :: "[agent, agent, msg, event list] ⇒ bool"
          (‹_ Issues _ with _ on _›) where
 "A Issues B with X on evs =
    (∃ Y. Says A B Y ∈ set evs ∧ X ∈ parts {Y} ∧
      X ∉ parts (spies (takeWhile (λz. z ≠ Says A B Y) (rev evs))))"
```

inductive_set *ns_shared* :: *"event list set"*
where

```
Nil:  "[] ∈ ns_shared"

| Fake: "⟦evsf ∈ ns_shared;  X ∈ synth (analz (spies evsf))⟧
         ⟹ Says Spy B X # evsf ∈ ns_shared"


| NS1:  "⟦evs1 ∈ ns_shared;  Nonce NA ∉ used evs1⟧
         ⟹ Says A Server ⦃Agent A, Agent B, Nonce NA⦄ # evs1  ∈  ns_shared"


| NS2:  "⟦evs2 ∈ ns_shared;  Key KAB ∉ used evs2;  KAB ∈ symKeys;
          Says A' Server ⦃Agent A, Agent B, Nonce NA⦄ ∈ set evs2⟧
         ⟹ Says Server A
               (Crypt (shrK A)
                  ⦃Nonce NA, Agent B, Key KAB,
                     (Crypt (shrK B) ⦃Key KAB, Agent A⦄)⦄)
               # evs2 ∈ ns_shared"
```

```
| NS3:  "⟦evs3 ∈ ns_shared;  A ≠ Server;
          Says S A (Crypt (shrK A) {|Nonce NA, Agent B, Key K, X|}) ∈ set evs3;
          Says A Server {|Agent A, Agent B, Nonce NA|} ∈ set evs3⟧
         ⟹ Says A B X # evs3 ∈ ns_shared"


| NS4:  "⟦evs4 ∈ ns_shared;  Nonce NB ∉ used evs4;  K ∈ symKeys;
          Says A' B (Crypt (shrK B) {|Key K, Agent A|}) ∈ set evs4⟧
         ⟹ Says B A (Crypt K (Nonce NB)) # evs4 ∈ ns_shared"


| NS5:  "⟦evs5 ∈ ns_shared;  K ∈ symKeys;
          Says B' A (Crypt K (Nonce NB)) ∈ set evs5;
          Says S  A (Crypt (shrK A) {|Nonce NA, Agent B, Key K, X|})
            ∈ set evs5⟧
         ⟹ Says A B (Crypt K {|Nonce NB, Nonce NB|}) # evs5 ∈ ns_shared"


| Oops: "⟦evso ∈ ns_shared;  Says B A (Crypt K (Nonce NB)) ∈ set evso;
          Says Server A (Crypt (shrK A) {|Nonce NA, Agent B, Key K, X|})
              ∈ set evso⟧
         ⟹ Notes Spy {|Nonce NA, Nonce NB, Key K|} # evso ∈ ns_shared"
```

declare *Says_imp_knows_Spy [THEN parts.Inj, dest]*
declare *parts.Body [dest]*
declare *Fake_parts_insert_in_Un [dest]*
declare *analz_into_parts [dest]*

A "possibility property": there are traces that reach the end

```
lemma "⟦A ≠ Server; Key K ∉ used []; K ∈ symKeys⟧
       ⟹ ∃N. ∃evs ∈ ns_shared.
                    Says A B (Crypt K {|Nonce N, Nonce N|}) ∈ set evs"
```
⟨*proof*⟩

3.1 Inductive proofs about ns_shared

3.1.1 Forwarding lemmas, to aid simplification

For reasoning about the encrypted portion of message NS3

lemma *NS3_msg_in_parts_spies:*
"Says S A (Crypt KA {|N, B, K, X|}) ∈ set evs ⟹ X ∈ parts (spies evs)"
⟨*proof*⟩

For reasoning about the Oops message

lemma *Oops_parts_spies:*
"Says Server A (Crypt (shrK A) {|NA, B, K, X|}) ∈ set evs
⟹ K ∈ parts (spies evs)"
⟨*proof*⟩

Theorems of the form *X ∉ parts (knows Spy evs)* imply that NOBODY sends messages containing *X*

Spy never sees another agent's shared key! (unless it's bad at start)

lemma *Spy_see_shrK [simp]:*
```
"evs ∈ ns_shared ⟹ (Key (shrK A) ∈ parts (spies evs)) = (A ∈ bad)"
```
⟨*proof*⟩

lemma *Spy_analz_shrK [simp]:*
```
"evs ∈ ns_shared ⟹ (Key (shrK A) ∈ analz (spies evs)) = (A ∈ bad)"
```
⟨*proof*⟩

Nobody can have used non-existent keys!

lemma *new_keys_not_used [simp]:*
```
"⟦Key K ∉ used evs; K ∈ symKeys; evs ∈ ns_shared⟧
 ⟹ K ∉ keysFor (parts (spies evs))"
```
⟨*proof*⟩

3.1.2 Lemmas concerning the form of items passed in messages

Describes the form of K, X and K' when the Server sends this message.

lemma *Says_Server_message_form:*
```
"⟦Says Server A (Crypt K' {|N, Agent B, Key K, X|}) ∈ set evs;
  evs ∈ ns_shared⟧
 ⟹ K ∉ range shrK ∧
     X = (Crypt (shrK B) {|Key K, Agent A|}) ∧
     K' = shrK A"
```
⟨*proof*⟩

If the encrypted message appears then it originated with the Server

lemma *A_trusts_NS2:*
```
"⟦Crypt (shrK A) {|NA, Agent B, Key K, X|} ∈ parts (spies evs);
  A ∉ bad;  evs ∈ ns_shared⟧
 ⟹ Says Server A (Crypt (shrK A) {|NA, Agent B, Key K, X|}) ∈ set evs"
```
⟨*proof*⟩

lemma *cert_A_form:*
```
"⟦Crypt (shrK A) {|NA, Agent B, Key K, X|} ∈ parts (spies evs);
  A ∉ bad;  evs ∈ ns_shared⟧
 ⟹ K ∉ range shrK ∧  X = (Crypt (shrK B) {|Key K, Agent A|})"
```
⟨*proof*⟩

EITHER describes the form of X when the following message is sent, OR reduces it to the Fake case. Use *Says_Server_message_form* if applicable.

lemma *Says_S_message_form:*
```
"⟦Says S A (Crypt (shrK A) {|Nonce NA, Agent B, Key K, X|}) ∈ set evs;
  evs ∈ ns_shared⟧
 ⟹ (K ∉ range shrK ∧ X = (Crypt (shrK B) {|Key K, Agent A|}))
    ∨ X ∈ analz (spies evs)"
```
⟨*proof*⟩

NOT useful in this form, but it says that session keys are not used to encrypt messages containing other keys, in the actual protocol. We require that agents should behave like this subsequently also.

lemma
```
"⟦evs ∈ ns_shared;  Kab ∉ range shrK⟧ ⟹
 (Crypt KAB X) ∈ parts (spies evs) ∧
```

```
        Key K ∈ parts {X} ⟶ Key K ∈ parts (spies evs)"
```
⟨proof⟩

3.1.3 Session keys are not used to encrypt other session keys

The equality makes the induction hypothesis easier to apply

lemma *analz_image_freshK [rule_format]:*
```
 "evs ∈ ns_shared ⟹
   ∀K KK. KK ⊆ - (range shrK) ⟶
            (Key K ∈ analz (Key`KK ∪ (spies evs))) =
            (K ∈ KK ∨ Key K ∈ analz (spies evs))"
```
⟨proof⟩

lemma *analz_insert_freshK:*
```
     "⟦evs ∈ ns_shared;  KAB ∉ range shrK⟧ ⟹
       (Key K ∈ analz (insert (Key KAB) (spies evs))) =
       (K = KAB ∨ Key K ∈ analz (spies evs))"
```
⟨proof⟩

3.1.4 The session key K uniquely identifies the message

In messages of this form, the session key uniquely identifies the rest

lemma *unique_session_keys:*
```
     "⟦Says Server A (Crypt (shrK A) ⦃NA, Agent B, Key K, X⦄) ∈ set evs;
       Says Server A' (Crypt (shrK A') ⦃NA', Agent B', Key K, X'⦄) ∈ set evs;
       evs ∈ ns_shared⟧ ⟹ A=A' ∧ NA=NA' ∧ B=B' ∧ X = X'"
```
⟨proof⟩

3.1.5 Crucial secrecy property: Spy doesn't see the keys sent in NS2

Beware of `[rule_format]` and the universal quantifier!

lemma *secrecy_lemma:*
```
     "⟦Says Server A (Crypt (shrK A) ⦃NA, Agent B, Key K,
                                       Crypt (shrK B) ⦃Key K, Agent A⦄⦄)
            ∈ set evs;
        A ∉ bad;  B ∉ bad;  evs ∈ ns_shared⟧
      ⟹ (∀NB. Notes Spy ⦃NA, NB, Key K⦄ ∉ set evs) ⟶
        Key K ∉ analz (spies evs)"
```
⟨proof⟩

Final version: Server's message in the most abstract form

lemma *Spy_not_see_encrypted_key:*
```
     "⟦Says Server A (Crypt K' ⦃NA, Agent B, Key K, X⦄) ∈ set evs;
       ∀NB. Notes Spy ⦃NA, NB, Key K⦄ ∉ set evs;
       A ∉ bad;  B ∉ bad;  evs ∈ ns_shared⟧
      ⟹ Key K ∉ analz (spies evs)"
```
⟨proof⟩

3.2 Guarantees available at various stages of protocol

If the encrypted message appears then it originated with the Server

lemma *B_trusts_NS3:*
```
"⟦Crypt (shrK B) {|Key K, Agent A|} ∈ parts (spies evs);
  B ∉ bad;  evs ∈ ns_shared⟧
 ⟹ ∃NA. Says Server A
         (Crypt (shrK A) {|NA, Agent B, Key K,
                           Crypt (shrK B) {|Key K, Agent A|}|})
        ∈ set evs"
```
⟨*proof*⟩

lemma *A_trusts_NS4_lemma [rule_format]:*
```
"evs ∈ ns_shared ⟹
   Key K ∉ analz (spies evs) ⟶
   Says Server A (Crypt (shrK A) {|NA, Agent B, Key K, X|}) ∈ set evs ⟶
   Crypt K (Nonce NB) ∈ parts (spies evs) ⟶
   Says B A (Crypt K (Nonce NB)) ∈ set evs"
```
⟨*proof*⟩

This version no longer assumes that K is secure

lemma *A_trusts_NS4:*
```
"⟦Crypt K (Nonce NB) ∈ parts (spies evs);
  Crypt (shrK A) {|NA, Agent B, Key K, X|} ∈ parts (spies evs);
  ∀NB. Notes Spy {|NA, NB, Key K|} ∉ set evs;
  A ∉ bad;  B ∉ bad;  evs ∈ ns_shared⟧
 ⟹ Says B A (Crypt K (Nonce NB)) ∈ set evs"
```
⟨*proof*⟩

If the session key has been used in NS4 then somebody has forwarded component X in some instance of NS4. Perhaps an interesting property, but not needed (after all) for the proofs below.

theorem *NS4_implies_NS3 [rule_format]:*
```
"evs ∈ ns_shared ⟹
   Key K ∉ analz (spies evs) ⟶
   Says Server A (Crypt (shrK A) {|NA, Agent B, Key K, X|}) ∈ set evs ⟶
   Crypt K (Nonce NB) ∈ parts (spies evs) ⟶
   (∃A'. Says A' B X ∈ set evs)"
```
⟨*proof*⟩

lemma *B_trusts_NS5_lemma [rule_format]:*
```
"⟦B ∉ bad;  evs ∈ ns_shared⟧ ⟹
   Key K ∉ analz (spies evs) ⟶
   Says Server A
        (Crypt (shrK A) {|NA, Agent B, Key K,
                          Crypt (shrK B) {|Key K, Agent A|}|}) ∈ set evs ⟶
   Crypt K {|Nonce NB, Nonce NB|} ∈ parts (spies evs) ⟶
   Says A B (Crypt K {|Nonce NB, Nonce NB|}) ∈ set evs"
```
⟨*proof*⟩

Very strong Oops condition reveals protocol's weakness

lemma *B_trusts_NS5:*
```
"⟦Crypt K {|Nonce NB, Nonce NB|} ∈ parts (spies evs);
  Crypt (shrK B) {|Key K, Agent A|} ∈ parts (spies evs);
```

```
  ∀NA NB. Notes Spy {|NA, NB, Key K|} ∉ set evs;
  A ∉ bad;  B ∉ bad;  evs ∈ ns_shared⟧
 ⟹ Says A B (Crypt K {|Nonce NB, Nonce NB|}) ∈ set evs"
```
⟨proof⟩

Unaltered so far wrt original version

3.3 Lemmas for reasoning about predicate "Issues"

lemma *spies_Says_rev: "spies (evs @ [Says A B X]) = insert X (spies evs)"*
⟨proof⟩

lemma *spies_Gets_rev: "spies (evs @ [Gets A X]) = spies evs"*
⟨proof⟩

lemma *spies_Notes_rev: "spies (evs @ [Notes A X]) =*
(if A∈bad then insert X (spies evs) else spies evs)"
⟨proof⟩

lemma *spies_evs_rev: "spies evs = spies (rev evs)"*
⟨proof⟩

lemmas *parts_spies_evs_revD2 = spies_evs_rev [THEN equalityD2, THEN parts_mono]*

lemma *spies_takeWhile: "spies (takeWhile P evs) ⊆ spies evs"*
⟨proof⟩

lemmas *parts_spies_takeWhile_mono = spies_takeWhile [THEN parts_mono]*

3.4 Guarantees of non-injective agreement on the session key, and of key distribution. They also express forms of freshness of certain messages, namely that agents were alive after something happened.

lemma *B_Issues_A:*
```
 "⟦ Says B A (Crypt K (Nonce Nb)) ∈ set evs;
    Key K ∉ analz (spies evs);
    A ∉ bad;  B ∉ bad; evs ∈ ns_shared ⟧
  ⟹ B Issues A with (Crypt K (Nonce Nb)) on evs"
```
⟨proof⟩

Tells A that B was alive after she sent him the session key. The session key must be assumed confidential for this deduction to be meaningful, but that assumption can be relaxed by the appropriate argument.

Precisely, the theorem guarantees (to A) key distribution of the session key to B. It also guarantees (to A) non-injective agreement of B with A on the session key. Both goals are available to A in the sense of Goal Availability.

lemma *A_authenticates_and_keydist_to_B:*
```
 "⟦Crypt K (Nonce NB) ∈ parts (spies evs);
   Crypt (shrK A) {|NA, Agent B, Key K, X|} ∈ parts (spies evs);
   Key K ∉ analz(knows Spy evs);
   A ∉ bad;  B ∉ bad;  evs ∈ ns_shared⟧
```

```
      ⟹ B Issues A with (Crypt K (Nonce NB)) on evs"
```
⟨proof⟩

lemma *A_trusts_NS5:*
```
  "⟦ Crypt K ⦃Nonce NB, Nonce NB⦄ ∈ parts(spies evs);
     Crypt (shrK A) ⦃Nonce NA, Agent B, Key K, X⦄ ∈ parts(spies evs);
     Key K ∉ analz (spies evs);
     A ∉ bad; B ∉ bad; evs ∈ ns_shared ⟧
 ⟹ Says A B (Crypt K ⦃Nonce NB, Nonce NB⦄) ∈ set evs"
```
⟨proof⟩

lemma *A_Issues_B:*
```
     "⟦ Says A B (Crypt K ⦃Nonce NB, Nonce NB⦄) ∈ set evs;
        Key K ∉ analz (spies evs);
        A ∉ bad;  B ∉ bad; evs ∈ ns_shared ⟧
    ⟹ A Issues B with (Crypt K ⦃Nonce NB, Nonce NB⦄) on evs"
```
⟨proof⟩

Tells B that A was alive after B issued NB.

Precisely, the theorem guarantees (to B) key distribution of the session key to A. It also guarantees (to B) non-injective agreement of A with B on the session key. Both goals are available to B in the sense of Goal Availability.

lemma *B_authenticates_and_keydist_to_A:*
```
     "⟦Crypt K ⦃Nonce NB, Nonce NB⦄ ∈ parts (spies evs);
       Crypt (shrK B) ⦃Key K, Agent A⦄ ∈ parts (spies evs);
       Key K ∉ analz (spies evs);
       A ∉ bad;  B ∉ bad;  evs ∈ ns_shared⟧
   ⟹ A Issues B with (Crypt K ⦃Nonce NB, Nonce NB⦄) on evs"
```
⟨proof⟩

end

4 The Kerberos Protocol, BAN Version

theory *Kerberos_BAN* **imports** *Public* **begin**

From page 251 of Burrows, Abadi and Needham (1989). A Logic of Authentication. Proc. Royal Soc. 426

Confidentiality (secrecy) and authentication properties are also given in a termporal version: strong guarantees in a little abstracted - but very realistic - model.

consts

```
    sesKlife   :: nat

    authlife :: nat
```

The ticket should remain fresh for two journeys on the network at least

specification *(sesKlife)*
```
  sesKlife_LB [iff]: "2 ≤ sesKlife"
```
⟨proof⟩

The authenticator only for one journey

```
specification (authlife)
  authlife_LB [iff]:    "authlife ≠ 0"
    ⟨proof⟩

abbreviation
  CT :: "event list ⇒ nat" where
  "CT == length "

abbreviation
  expiredK :: "[nat, event list] ⇒ bool" where
  "expiredK T evs == sesKlife + T < CT evs"

abbreviation
  expiredA :: "[nat, event list] ⇒ bool" where
  "expiredA T evs == authlife + T < CT evs"


definition

  Issues :: "[agent, agent, msg, event list] ⇒ bool"
            (‹_ Issues _ with _ on _›) where
   "A Issues B with X on evs =
      (∃Y. Says A B Y ∈ set evs ∧ X ∈ parts {Y} ∧
        X ∉ parts (spies (takeWhile (λz. z  ≠ Says A B Y) (rev evs))))"

definition

  before :: "[event, event list] ⇒ event list" (‹before _ on _›)
  where "before ev on evs = takeWhile (λz. z ≠ ev) (rev evs)"

definition

  Unique :: "[event, event list] ⇒ bool" (‹Unique _ on _›)
  where "Unique ev on evs = (ev ∉ set (tl (dropWhile (λz. z ≠ ev) evs)))"


inductive_set bankerberos :: "event list set"
 where

   Nil:  "[] ∈ bankerberos"

 | Fake: "⟦ evsf ∈ bankerberos;  X ∈ synth (analz (spies evsf)) ⟧
         ⟹ Says Spy B X # evsf ∈ bankerberos"


 | BK1:  "⟦ evs1 ∈ bankerberos ⟧
         ⟹ Says A Server ⦃Agent A, Agent B⦄ # evs1
               ∈  bankerberos"


 | BK2:  "⟦ evs2 ∈ bankerberos;  Key K ∉ used evs2; K ∈ symKeys;
             Says A' Server ⦃Agent A, Agent B⦄ ∈ set evs2 ⟧
         ⟹ Says Server A
```

```
              (Crypt (shrK A)
                 {|Number (CT evs2), Agent B, Key K,
                   (Crypt (shrK B) {|Number (CT evs2), Agent A, Key K|})|})
              # evs2 ∈ bankerberos"


 | BK3:  "⟦ evs3 ∈ bankerberos;
             Says S A (Crypt (shrK A) {|Number Tk, Agent B, Key K, Ticket|})
               ∈ set evs3;
             Says A Server {|Agent A, Agent B|} ∈ set evs3;
             ¬ expiredK Tk evs3 ⟧
          ⟹ Says A B {|Ticket, Crypt K {|Agent A, Number (CT evs3)|} |}
               # evs3 ∈ bankerberos"


 | BK4:  "⟦ evs4 ∈ bankerberos;
             Says A' B {|(Crypt (shrK B) {|Number Tk, Agent A, Key K|}),
                         (Crypt K {|Agent A, Number Ta|}) |} ∈ set evs4;
             ¬ expiredK Tk evs4;  ¬ expiredA Ta evs4 ⟧
          ⟹ Says B A (Crypt K (Number Ta)) # evs4
               ∈ bankerberos"


 | Oops: "⟦ evso ∈ bankerberos;
         Says Server A (Crypt (shrK A) {|Number Tk, Agent B, Key K, Ticket|})
               ∈ set evso;
             expiredK Tk evso ⟧
          ⟹ Notes Spy {|Number Tk, Key K|} # evso ∈ bankerberos"
```

declare *Says_imp_knows_Spy [THEN parts.Inj, dest]*
declare *parts.Body [dest]*
declare *analz_into_parts [dest]*
declare *Fake_parts_insert_in_Un [dest]*

A "possibility property": there are traces that reach the end.

lemma *"⟦Key K ∉ used []; K ∈ symKeys⟧*
⟹ ∃Timestamp. ∃evs ∈ bankerberos.
Says B A (Crypt K (Number Timestamp))
∈ set evs"
⟨*proof*⟩

4.1 Lemmas for reasoning about predicate "Issues"

lemma *spies_Says_rev: "spies (evs @ [Says A B X]) = insert X (spies evs)"*
⟨*proof*⟩

lemma *spies_Gets_rev: "spies (evs @ [Gets A X]) = spies evs"*
⟨*proof*⟩

lemma *spies_Notes_rev: "spies (evs @ [Notes A X]) =*
(if A∈bad then insert X (spies evs) else spies evs)"
⟨*proof*⟩

lemma *spies_evs_rev: "spies evs = spies (rev evs)"*
⟨*proof*⟩

lemmas *parts_spies_evs_revD2 = spies_evs_rev [THEN equalityD2, THEN parts_mono]*

lemma *spies_takeWhile: "spies (takeWhile P evs) ⊆ spies evs"*
⟨*proof*⟩

lemmas *parts_spies_takeWhile_mono = spies_takeWhile [THEN parts_mono]*

Lemmas for reasoning about predicate "before"

lemma *used_Says_rev: "used (evs @ [Says A B X]) = parts {X} ∪ (used evs)"*
⟨*proof*⟩

lemma *used_Notes_rev: "used (evs @ [Notes A X]) = parts {X} ∪ (used evs)"*
⟨*proof*⟩

lemma *used_Gets_rev: "used (evs @ [Gets B X]) = used evs"*
⟨*proof*⟩

lemma *used_evs_rev: "used evs = used (rev evs)"*
⟨*proof*⟩

lemma *used_takeWhile_used [rule_format]:*
"x ∈ used (takeWhile P X) ⟶ x ∈ used X"
⟨*proof*⟩

lemma *set_evs_rev: "set evs = set (rev evs)"*
⟨*proof*⟩

lemma *takeWhile_void [rule_format]:*
"x ∉ set evs ⟶ takeWhile (λz. z ≠ x) evs = evs"
⟨*proof*⟩

Forwarding Lemma for reasoning about the encrypted portion of message BK3

lemma *BK3_msg_in_parts_spies:*
"Says S A (Crypt KA ⦃Timestamp, B, K, X⦄) ∈ set evs
⟹ X ∈ parts (spies evs)"
⟨*proof*⟩

lemma *Oops_parts_spies:*
"Says Server A (Crypt (shrK A) ⦃Timestamp, B, K, X⦄) ∈ set evs
⟹ K ∈ parts (spies evs)"
⟨*proof*⟩

Spy never sees another agent's shared key! (unless it's bad at start)

lemma *Spy_see_shrK [simp]:*
"evs ∈ bankerberos ⟹ (Key (shrK A) ∈ parts (spies evs)) = (A ∈ bad)"
⟨*proof*⟩

lemma *Spy_analz_shrK [simp]:*
"evs ∈ bankerberos ⟹ (Key (shrK A) ∈ analz (spies evs)) = (A ∈ bad)"

⟨proof⟩

lemma *Spy_see_shrK_D [dest!]:*
```
"⟦ Key (shrK A) ∈ parts (spies evs);
          evs ∈ bankerberos ⟧ ⟹ A∈bad"
```
⟨proof⟩

lemmas *Spy_analz_shrK_D = analz_subset_parts [THEN subsetD, THEN Spy_see_shrK_D, dest!]*

Nobody can have used non-existent keys!

lemma *new_keys_not_used [simp]:*
```
"⟦Key K ∉ used evs; K ∈ symKeys; evs ∈ bankerberos⟧
 ⟹ K ∉ keysFor (parts (spies evs))"
```
⟨proof⟩

4.2 Lemmas concerning the form of items passed in messages

Describes the form of K, X and K' when the Server sends this message.

lemma *Says_Server_message_form:*
```
"⟦ Says Server A (Crypt K' ⦃Number Tk, Agent B, Key K, Ticket⦄)
    ∈ set evs; evs ∈ bankerberos ⟧
 ⟹ K' = shrK A ∧ K ∉ range shrK ∧
    Ticket = (Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄) ∧
    Key K ∉ used(before
            Says Server A (Crypt K' ⦃Number Tk, Agent B, Key K, Ticket⦄)
            on evs) ∧
    Tk = CT(before
            Says Server A (Crypt K' ⦃Number Tk, Agent B, Key K, Ticket⦄)
            on evs)"
```
⟨proof⟩

If the encrypted message appears then it originated with the Server PROVIDED that A is NOT compromised! This allows A to verify freshness of the session key.

lemma *Kab_authentic:*
```
"⟦ Crypt (shrK A) ⦃Number Tk, Agent B, Key K, X⦄
     ∈ parts (spies evs);
    A ∉ bad;  evs ∈ bankerberos ⟧
  ⟹ Says Server A (Crypt (shrK A) ⦃Number Tk, Agent B, Key K, X⦄)
       ∈ set evs"
```
⟨proof⟩

If the TICKET appears then it originated with the Server

FRESHNESS OF THE SESSION KEY to B

lemma *ticket_authentic:*
```
"⟦ Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄ ∈ parts (spies evs);
    B ∉ bad;  evs ∈ bankerberos ⟧
  ⟹ Says Server A
      (Crypt (shrK A) ⦃Number Tk, Agent B, Key K,
```

```
                        Crypt (shrK B) {|Number Tk, Agent A, Key K|}|})
          ∈ set evs"
```
⟨proof⟩

EITHER describes the form of X when the following message is sent, OR reduces it to the Fake case. Use `Says_Server_message_form` if applicable.

lemma *Says_S_message_form:*
```
     "[| Says S A (Crypt (shrK A) {|Number Tk, Agent B, Key K, X|})
           ∈ set evs;
         evs ∈ bankerberos |]
 ⟹ (K ∉ range shrK ∧ X = (Crypt (shrK B) {|Number Tk, Agent A, Key K|}))
          | X ∈ analz (spies evs)"
```
⟨proof⟩

Session keys are not used to encrypt other session keys

lemma *analz_image_freshK [rule_format (no_asm)]:*
```
     "evs ∈ bankerberos ⟹
   ∀K KK. KK ⊆ - (range shrK) ⟶
          (Key K ∈ analz (Key`KK ∪ (spies evs))) =
          (K ∈ KK | Key K ∈ analz (spies evs))"
```
⟨proof⟩

lemma *analz_insert_freshK:*
```
     "[| evs ∈ bankerberos;  KAB ∉ range shrK |] ⟹
      (Key K ∈ analz (insert (Key KAB) (spies evs))) =
      (K = KAB | Key K ∈ analz (spies evs))"
```
⟨proof⟩

The session key K uniquely identifies the message

lemma *unique_session_keys:*
```
     "[| Says Server A
           (Crypt (shrK A) {|Number Tk, Agent B, Key K, X|}) ∈ set evs;
         Says Server A'
          (Crypt (shrK A') {|Number Tk', Agent B', Key K, X'|}) ∈ set evs;
         evs ∈ bankerberos |] ⟹ A=A' ∧ Tk=Tk' ∧ B=B' ∧ X = X'"
```
⟨proof⟩

lemma *Server_Unique:*
```
     "[| Says Server A
            (Crypt (shrK A) {|Number Tk, Agent B, Key K, Ticket|}) ∈ set evs;
        evs ∈ bankerberos |] ⟹
   Unique Says Server A (Crypt (shrK A) {|Number Tk, Agent B, Key K, Ticket|})
   on evs"
```
⟨proof⟩

4.3 Non-temporal guarantees, explicitly relying on non-occurrence of oops events - refined below by temporal guarantees

Non temporal treatment of confidentiality

Lemma: the session key sent in msg BK2 would be lost by oops if the spy could see it!

```
lemma lemma_conf [rule_format (no_asm)]:
     "⟦ A ∉ bad;  B ∉ bad;  evs ∈ bankerberos ⟧
  ⟹ Says Server A
          (Crypt (shrK A) ⦃Number Tk, Agent B, Key K,
                           Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄⦄)
        ∈ set evs ⟶
      Key K ∈ analz (spies evs) ⟶ Notes Spy ⦃Number Tk, Key K⦄ ∈ set evs"
```
⟨*proof*⟩

Confidentiality for the Server: Spy does not see the keys sent in msg BK2 as long as they have not expired.

```
lemma Confidentiality_S:
     "⟦ Says Server A
          (Crypt K' ⦃Number Tk, Agent B, Key K, Ticket⦄) ∈ set evs;
        Notes Spy ⦃Number Tk, Key K⦄ ∉ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ bankerberos
      ⟧ ⟹ Key K ∉ analz (spies evs)"
```
⟨*proof*⟩

Confidentiality for Alice

```
lemma Confidentiality_A:
     "⟦ Crypt (shrK A) ⦃Number Tk, Agent B, Key K, X⦄ ∈ parts (spies evs);
        Notes Spy ⦃Number Tk, Key K⦄ ∉ set evs;
        A ∉ bad;  B ∉ bad;  evs ∈ bankerberos
      ⟧ ⟹ Key K ∉ analz (spies evs)"
```
⟨*proof*⟩

Confidentiality for Bob

```
lemma Confidentiality_B:
     "⟦ Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄
          ∈ parts (spies evs);
        Notes Spy ⦃Number Tk, Key K⦄ ∉ set evs;
        A ∉ bad;  B ∉ bad;  evs ∈ bankerberos
      ⟧ ⟹ Key K ∉ analz (spies evs)"
```
⟨*proof*⟩

Non temporal treatment of authentication

Lemmas `lemma_A` and `lemma_B` in fact are common to both temporal and non-temporal treatments

```
lemma lemma_A [rule_format]:
     "⟦ A ∉ bad; B ∉ bad; evs ∈ bankerberos ⟧
      ⟹
         Key K ∉ analz (spies evs) ⟶
         Says Server A (Crypt (shrK A) ⦃Number Tk, Agent B, Key K, X⦄)
         ∈ set evs ⟶
          Crypt K ⦃Agent A, Number Ta⦄ ∈ parts (spies evs) ⟶
         Says A B ⦃X, Crypt K ⦃Agent A, Number Ta⦄⦄
             ∈ set evs"
```
⟨*proof*⟩

lemma *lemma_B [rule_format]:*
```
"⟦ B ∉ bad;  evs ∈ bankerberos ⟧
 ⟹ Key K ∉ analz (spies evs) ⟶
    Says Server A (Crypt (shrK A) ⦃Number Tk, Agent B, Key K, X⦄)
    ∈ set evs  ⟶
    Crypt K (Number Ta) ∈ parts (spies evs) ⟶
    Says B A (Crypt K (Number Ta)) ∈ set evs"
```
⟨*proof*⟩

The "r" suffix indicates theorems where the confidentiality assumptions are relaxed by the corresponding arguments.

Authentication of A to B

lemma *B_authenticates_A_r:*
```
"⟦ Crypt K ⦃Agent A, Number Ta⦄ ∈ parts (spies evs);
   Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄  ∈ parts (spies evs);
  Notes Spy ⦃Number Tk, Key K⦄ ∉ set evs;
   A ∉ bad;  B ∉ bad;  evs ∈ bankerberos ⟧
 ⟹ Says A B ⦃Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄,
             Crypt K ⦃Agent A, Number Ta⦄⦄ ∈ set evs"
```
⟨*proof*⟩

Authentication of B to A

lemma *A_authenticates_B_r:*
```
"⟦ Crypt K (Number Ta) ∈ parts (spies evs);
  Crypt (shrK A) ⦃Number Tk, Agent B, Key K, X⦄ ∈ parts (spies evs);
  Notes Spy ⦃Number Tk, Key K⦄ ∉ set evs;
  A ∉ bad;  B ∉ bad;  evs ∈ bankerberos ⟧
 ⟹ Says B A (Crypt K (Number Ta)) ∈ set evs"
```
⟨*proof*⟩

lemma *B_authenticates_A:*
```
"⟦ Crypt K ⦃Agent A, Number Ta⦄ ∈ parts (spies evs);
   Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄  ∈ parts (spies evs);
  Key K ∉ analz (spies evs);
   A ∉ bad;  B ∉ bad;  evs ∈ bankerberos ⟧
 ⟹ Says A B ⦃Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄,
             Crypt K ⦃Agent A, Number Ta⦄⦄ ∈ set evs"
```
⟨*proof*⟩

lemma *A_authenticates_B:*
```
"⟦ Crypt K (Number Ta) ∈ parts (spies evs);
  Crypt (shrK A) ⦃Number Tk, Agent B, Key K, X⦄ ∈ parts (spies evs);
  Key K ∉ analz (spies evs);
  A ∉ bad;  B ∉ bad;  evs ∈ bankerberos ⟧
 ⟹ Says B A (Crypt K (Number Ta)) ∈ set evs"
```
⟨*proof*⟩

4.4 Temporal guarantees, relying on a temporal check that insures that no oops event occurred. These are available in the sense of goal availability

Temporal treatment of confidentiality

Lemma: the session key sent in msg BK2 would be EXPIRED if the spy could see it!

lemma *lemma_conf_temporal [rule_format (no_asm)]:*
```
     "⟦ A ∉ bad;  B ∉ bad;  evs ∈ bankerberos ⟧
  ⟹ Says Server A
          (Crypt (shrK A) ⦃Number Tk, Agent B, Key K,
                             Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄⦄)
         ∈ set evs ⟶
      Key K ∈ analz (spies evs) ⟶ expiredK Tk evs"
```
⟨*proof*⟩

Confidentiality for the Server: Spy does not see the keys sent in msg BK2 as long as they have not expired.

lemma *Confidentiality_S_temporal:*
```
     "⟦ Says Server A
          (Crypt K' ⦃Number T, Agent B, Key K, X⦄) ∈ set evs;
         ¬ expiredK T evs;
         A ∉ bad;  B ∉ bad;  evs ∈ bankerberos
      ⟧ ⟹ Key K ∉ analz (spies evs)"
```
⟨*proof*⟩

Confidentiality for Alice

lemma *Confidentiality_A_temporal:*
```
     "⟦ Crypt (shrK A) ⦃Number T, Agent B, Key K, X⦄ ∈ parts (spies evs);
         ¬ expiredK T evs;
         A ∉ bad;  B ∉ bad;  evs ∈ bankerberos
      ⟧ ⟹ Key K ∉ analz (spies evs)"
```
⟨*proof*⟩

Confidentiality for Bob

lemma *Confidentiality_B_temporal:*
```
     "⟦ Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄
          ∈ parts (spies evs);
        ¬ expiredK Tk evs;
        A ∉ bad;  B ∉ bad;  evs ∈ bankerberos
      ⟧ ⟹ Key K ∉ analz (spies evs)"
```
⟨*proof*⟩

Temporal treatment of authentication

Authentication of A to B

lemma *B_authenticates_A_temporal:*
```
     "⟦ Crypt K ⦃Agent A, Number Ta⦄ ∈ parts (spies evs);
         Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄
         ∈ parts (spies evs);
         ¬ expiredK Tk evs;
```

```
          A ∉ bad;  B ∉ bad;  evs ∈ bankerberos ⟧
       ⟹ Says A B ⦃Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄,
                    Crypt K ⦃Agent A, Number Ta⦄⦄ ∈ set evs"
⟨proof⟩
```

Authentication of B to A

```
lemma A_authenticates_B_temporal:
     "⟦ Crypt K (Number Ta) ∈ parts (spies evs);
        Crypt (shrK A) ⦃Number Tk, Agent B, Key K, X⦄
        ∈ parts (spies evs);
        ¬ expiredK Tk evs;
        A ∉ bad;  B ∉ bad;  evs ∈ bankerberos ⟧
      ⟹ Says B A (Crypt K (Number Ta)) ∈ set evs"
⟨proof⟩
```

4.5 Treatment of the key distribution goal using trace inspection. All guarantees are in non-temporal form, hence non available, though their temporal form is trivial to derive. These guarantees also convey a stronger form of authentication - non-injective agreement on the session key

```
lemma B_Issues_A:
     "⟦ Says B A (Crypt K (Number Ta)) ∈ set evs;
        Key K ∉ analz (spies evs);
        A ∉ bad;  B ∉ bad; evs ∈ bankerberos ⟧
      ⟹ B Issues A with (Crypt K (Number Ta)) on evs"
⟨proof⟩
```

```
lemma A_authenticates_and_keydist_to_B:
     "⟦ Crypt K (Number Ta) ∈ parts (spies evs);
       Crypt (shrK A) ⦃Number Tk, Agent B, Key K, X⦄ ∈ parts (spies evs);
        Key K ∉ analz (spies evs);
        A ∉ bad;  B ∉ bad; evs ∈ bankerberos ⟧
      ⟹ B Issues A with (Crypt K (Number Ta)) on evs"
⟨proof⟩
```

```
lemma A_Issues_B:
     "⟦ Says A B ⦃Ticket, Crypt K ⦃Agent A, Number Ta⦄⦄
          ∈ set evs;
        Key K ∉ analz (spies evs);
        A ∉ bad;  B ∉ bad;  evs ∈ bankerberos ⟧
   ⟹ A Issues B with (Crypt K ⦃Agent A, Number Ta⦄) on evs"
⟨proof⟩
```

```
lemma B_authenticates_and_keydist_to_A:
     "⟦ Crypt K ⦃Agent A, Number Ta⦄ ∈ parts (spies evs);
       Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄  ∈ parts (spies evs);
       Key K ∉ analz (spies evs);
       A ∉ bad;  B ∉ bad;  evs ∈ bankerberos ⟧
   ⟹ A Issues B with (Crypt K ⦃Agent A, Number Ta⦄) on evs"
```

⟨*proof*⟩

end

5 The Kerberos Protocol, BAN Version, with Gets event

theory *Kerberos_BAN_Gets* **imports** *Public* **begin**

From page 251 of Burrows, Abadi and Needham (1989). A Logic of Authentication. Proc. Royal Soc. 426

Confidentiality (secrecy) and authentication properties rely on temporal checks: strong guarantees in a little abstracted - but very realistic - model.

consts

```
sesKlife   :: nat

authlife :: nat
```

The ticket should remain fresh for two journeys on the network at least

The Gets event causes longer traces for the protocol to reach its end

specification *(sesKlife)*
```
  sesKlife_LB [iff]: "4 ≤ sesKlife"
```
⟨*proof*⟩

The authenticator only for one journey

The Gets event causes longer traces for the protocol to reach its end

specification *(authlife)*
```
  authlife_LB [iff]:    "2 ≤ authlife"
```
⟨*proof*⟩

abbreviation
```
  CT :: "event list ⇒ nat" where
  "CT == length"
```

abbreviation
```
  expiredK :: "[nat, event list] ⇒ bool" where
  "expiredK T evs == sesKlife + T < CT evs"
```

abbreviation
```
  expiredA :: "[nat, event list] ⇒ bool" where
  "expiredA T evs == authlife + T < CT evs"
```

definition

```
  before :: "[event, event list] ⇒ event list" (‹before _ on _›)
  where "before ev on evs = takeWhile (λz. z ≠ ev) (rev evs)"
```

definition

```
  Unique :: "[event, event list] ⇒ bool" (‹Unique _ on _›)
  where "Unique ev on evs = (ev ∉ set (tl (dropWhile (λz. z ≠ ev) evs)))"
```

```
inductive_set bankerb_gets :: "event list set"
 where

   Nil:  "[] ∈ bankerb_gets"

 | Fake: "⟦ evsf ∈ bankerb_gets;  X ∈ synth (analz (knows Spy evsf)) ⟧
         ⟹ Says Spy B X # evsf ∈ bankerb_gets"

 | Reception: "⟦ evsr∈ bankerb_gets; Says A B X ∈ set evsr ⟧
               ⟹ Gets B X # evsr ∈ bankerb_gets"

 | BK1:  "⟦ evs1 ∈ bankerb_gets ⟧
         ⟹ Says A Server ⦃Agent A, Agent B⦄ # evs1
               ∈  bankerb_gets"


 | BK2:  "⟦ evs2 ∈ bankerb_gets;  Key K ∉ used evs2; K ∈ symKeys;
            Gets Server ⦃Agent A, Agent B⦄ ∈ set evs2 ⟧
         ⟹ Says Server A
              (Crypt (shrK A)
                 ⦃Number (CT evs2), Agent B, Key K,
                  (Crypt (shrK B) ⦃Number (CT evs2), Agent A, Key K⦄)⦄)
              # evs2 ∈ bankerb_gets"


 | BK3:  "⟦ evs3 ∈ bankerb_gets;
            Gets A (Crypt (shrK A) ⦃Number Tk, Agent B, Key K, Ticket⦄)
              ∈ set evs3;
            Says A Server ⦃Agent A, Agent B⦄ ∈ set evs3;
            ¬ expiredK Tk evs3 ⟧
         ⟹ Says A B ⦃Ticket, Crypt K ⦃Agent A, Number (CT evs3)⦄ ⦄
             # evs3 ∈ bankerb_gets"


 | BK4:  "⟦ evs4 ∈ bankerb_gets;
            Gets B ⦃(Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄),
                        (Crypt K ⦃Agent A, Number Ta⦄) ⦄ ∈ set evs4;
            ¬ expiredK Tk evs4;  ¬ expiredA Ta evs4 ⟧
         ⟹ Says B A (Crypt K (Number Ta)) # evs4
               ∈ bankerb_gets"


 | Oops: "⟦ evso ∈ bankerb_gets;
```

```
        Says Server A (Crypt (shrK A) {|Number Tk, Agent B, Key K, Ticket|})
              ∈ set evso;
            expiredK Tk evso ⟧
         ⟹ Notes Spy {|Number Tk, Key K|} # evso ∈ bankerb_gets"
```

declare *Says_imp_knows_Spy [THEN parts.Inj, dest]*
declare *parts.Body [dest]*
declare *analz_into_parts [dest]*
declare *Fake_parts_insert_in_Un [dest]*
declare *knows_Spy_partsEs [elim]*

A "possibility property": there are traces that reach the end.

```
lemma "⟦Key K ∉ used []; K ∈ symKeys⟧
       ⟹ ∃Timestamp. ∃evs ∈ bankerb_gets.
            Says B A (Crypt K (Number Timestamp))
                 ∈ set evs"
```
⟨*proof*⟩

Lemmas about reception invariant: if a message is received it certainly was sent

```
lemma Gets_imp_Says :
     "⟦ Gets B X ∈ set evs; evs ∈ bankerb_gets ⟧ ⟹ ∃A. Says A B X ∈ set
evs"
```
⟨*proof*⟩

```
lemma Gets_imp_knows_Spy:
     "⟦ Gets B X ∈ set evs; evs ∈ bankerb_gets ⟧  ⟹ X ∈ knows Spy evs"
```
⟨*proof*⟩

```
lemma Gets_imp_knows_Spy_parts[dest]:
    "⟦ Gets B X ∈ set evs; evs ∈ bankerb_gets ⟧  ⟹ X ∈ parts (knows Spy
evs)"
```
⟨*proof*⟩

```
lemma Gets_imp_knows:
     "⟦ Gets B X ∈ set evs; evs ∈ bankerb_gets ⟧  ⟹ X ∈ knows B evs"
```
⟨*proof*⟩

```
lemma Gets_imp_knows_analz:
    "⟦ Gets B X ∈ set evs; evs ∈ bankerb_gets ⟧  ⟹ X ∈ analz (knows B evs)"
```
⟨*proof*⟩

Lemmas for reasoning about predicate "before"

lemma *used_Says_rev: "used (evs @ [Says A B X]) = parts {X} ∪ (used evs)"*
⟨*proof*⟩

lemma *used_Notes_rev: "used (evs @ [Notes A X]) = parts {X} ∪ (used evs)"*
⟨*proof*⟩

lemma *used_Gets_rev: "used (evs @ [Gets B X]) = used evs"*
⟨*proof*⟩

lemma *used_evs_rev: "used evs = used (rev evs)"*

⟨proof⟩

lemma *used_takeWhile_used [rule_format]:*
"x ∈ used (takeWhile P X) ⟶ x ∈ used X"
⟨proof⟩

lemma *set_evs_rev: "set evs = set (rev evs)"*
⟨proof⟩

lemma *takeWhile_void [rule_format]:*
"x ∉ set evs ⟶ takeWhile (λz. z ≠ x) evs = evs"
⟨proof⟩

Forwarding Lemma for reasoning about the encrypted portion of message BK3

lemma *BK3_msg_in_parts_knows_Spy:*
"⟦Gets A (Crypt KA ⦃Timestamp, B, K, X⦄) ∈ set evs; evs ∈ bankerb_gets ⟧
⟹ X ∈ parts (knows Spy evs)"
⟨proof⟩

lemma *Oops_parts_knows_Spy:*
"Says Server A (Crypt (shrK A) ⦃Timestamp, B, K, X⦄) ∈ set evs
⟹ K ∈ parts (knows Spy evs)"
⟨proof⟩

Spy never sees another agent's shared key! (unless it's bad at start)

lemma *Spy_see_shrK [simp]:*
"evs ∈ bankerb_gets ⟹ (Key (shrK A) ∈ parts (knows Spy evs)) = (A ∈ bad)"
⟨proof⟩

lemma *Spy_analz_shrK [simp]:*
"evs ∈ bankerb_gets ⟹ (Key (shrK A) ∈ analz (knows Spy evs)) = (A ∈ bad)"
⟨proof⟩

lemma *Spy_see_shrK_D [dest!]:*
"⟦ Key (shrK A) ∈ parts (knows Spy evs);
evs ∈ bankerb_gets ⟧ ⟹ A∈bad"
⟨proof⟩

lemmas *Spy_analz_shrK_D = analz_subset_parts [THEN subsetD, THEN Spy_see_shrK_D, dest!]*

Nobody can have used non-existent keys!

lemma *new_keys_not_used [simp]:*
"⟦Key K ∉ used evs; K ∈ symKeys; evs ∈ bankerb_gets⟧
⟹ K ∉ keysFor (parts (knows Spy evs))"
⟨proof⟩

5.1 Lemmas concerning the form of items passed in messages

Describes the form of K, X and K' when the Server sends this message.

lemma *Says_Server_message_form:*

```
"⟦ Says Server A (Crypt K' ⦃Number Tk, Agent B, Key K, Ticket⦄)
     ∈ set evs; evs ∈ bankerb_gets ⟧
  ⟹ K' = shrK A ∧ K ∉ range shrK ∧
      Ticket = (Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄) ∧
      Key K ∉ used(before
              Says Server A (Crypt K' ⦃Number Tk, Agent B, Key K, Ticket⦄)
              on evs) ∧
      Tk = CT(before
              Says Server A (Crypt K' ⦃Number Tk, Agent B, Key K, Ticket⦄)
              on evs)"
```

⟨*proof*⟩

If the encrypted message appears then it originated with the Server PROVIDED that A is NOT compromised! This allows A to verify freshness of the session key.

lemma *Kab_authentic:*

```
"⟦ Crypt (shrK A) ⦃Number Tk, Agent B, Key K, X⦄
       ∈ parts (knows Spy evs);
     A ∉ bad;  evs ∈ bankerb_gets ⟧
   ⟹ Says Server A (Crypt (shrK A) ⦃Number Tk, Agent B, Key K, X⦄)
         ∈ set evs"
```

⟨*proof*⟩

If the TICKET appears then it originated with the Server

FRESHNESS OF THE SESSION KEY to B

lemma *ticket_authentic:*

```
"⟦ Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄ ∈ parts (knows Spy evs);
     B ∉ bad;  evs ∈ bankerb_gets ⟧
   ⟹ Says Server A
        (Crypt (shrK A) ⦃Number Tk, Agent B, Key K,
                      Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄⦄)
       ∈ set evs"
```

⟨*proof*⟩

EITHER describes the form of X when the following message is sent, OR reduces it to the Fake case. Use `Says_Server_message_form` if applicable.

lemma *Gets_Server_message_form:*

```
"⟦ Gets A (Crypt (shrK A) ⦃Number Tk, Agent B, Key K, X⦄)
        ∈ set evs;
     evs ∈ bankerb_gets ⟧
⟹ (K ∉ range shrK ∧ X = (Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄))
      | X ∈ analz (knows Spy evs)"
```

⟨*proof*⟩

Reliability guarantees: honest agents act as we expect

lemma *BK3_imp_Gets:*

```
  "⟦ Says A B {|Ticket, Crypt K {|Agent A, Number Ta|}|} ∈ set evs;
     A ∉ bad; evs ∈ bankerb_gets ⟧
   ⟹ ∃ Tk. Gets A (Crypt (shrK A) {|Number Tk, Agent B, Key K, Ticket|})
     ∈ set evs"
```
⟨proof⟩

lemma `BK4_imp_Gets:`
```
  "⟦ Says B A (Crypt K (Number Ta)) ∈ set evs;
     B ∉ bad; evs ∈ bankerb_gets ⟧
 ⟹ ∃ Tk. Gets B {|Crypt (shrK B) {|Number Tk, Agent A, Key K|},
                  Crypt K {|Agent A, Number Ta|}|} ∈ set evs"
```
⟨proof⟩

lemma `Gets_A_knows_K:`
```
 "⟦ Gets A (Crypt (shrK A) {|Number Tk, Agent B, Key K, Ticket|}) ∈ set evs;
    evs ∈ bankerb_gets ⟧
⟹ Key K ∈ analz (knows A evs)"
```
⟨proof⟩

lemma `Gets_B_knows_K:`
```
 "⟦ Gets B {|Crypt (shrK B) {|Number Tk, Agent A, Key K|},
            Crypt K {|Agent A, Number Ta|}|} ∈ set evs;
    evs ∈ bankerb_gets ⟧
⟹ Key K ∈ analz (knows B evs)"
```
⟨proof⟩

Session keys are not used to encrypt other session keys

lemma `analz_image_freshK [rule_format (no_asm)]:`
```
     "evs ∈ bankerb_gets ⟹
  ∀K KK. KK ⊆ - (range shrK) ⟶
         (Key K ∈ analz (Key`KK ∪ (knows Spy evs))) =
         (K ∈ KK | Key K ∈ analz (knows Spy evs))"
```
⟨proof⟩

lemma `analz_insert_freshK:`
```
     "⟦ evs ∈ bankerb_gets;  KAB ∉ range shrK ⟧ ⟹
      (Key K ∈ analz (insert (Key KAB) (knows Spy evs))) =
      (K = KAB | Key K ∈ analz (knows Spy evs))"
```
⟨proof⟩

The session key K uniquely identifies the message

lemma `unique_session_keys:`
```
     "⟦ Says Server A
           (Crypt (shrK A) {|Number Tk, Agent B, Key K, X|}) ∈ set evs;
         Says Server A'
          (Crypt (shrK A') {|Number Tk', Agent B', Key K, X'|}) ∈ set evs;
         evs ∈ bankerb_gets ⟧ ⟹ A=A' ∧ Tk=Tk' ∧ B=B' ∧ X = X'"
```
⟨proof⟩

lemma `unique_session_keys_Gets:`
```
     "⟦ Gets A
           (Crypt (shrK A) {|Number Tk, Agent B, Key K, X|}) ∈ set evs;
        Gets A
```

```
        (Crypt (shrK A) {|Number Tk', Agent B', Key K, X'|}) ∈ set evs;
      A ∉ bad; evs ∈ bankerb_gets ⟧ ⟹ Tk=Tk' ∧ B=B' ∧ X = X'"
```
⟨proof⟩

lemma *Server_Unique:*
```
     "⟦ Says Server A
          (Crypt (shrK A) {|Number Tk, Agent B, Key K, Ticket|}) ∈ set evs;
      evs ∈ bankerb_gets ⟧ ⟹
   Unique Says Server A (Crypt (shrK A) {|Number Tk, Agent B, Key K, Ticket|})
   on evs"
```
⟨proof⟩

5.2 Non-temporal guarantees, explicitly relying on non-occurrence of oops events - refined below by temporal guarantees

Non temporal treatment of confidentiality

Lemma: the session key sent in msg BK2 would be lost by oops if the spy could see it!

lemma *lemma_conf [rule_format (no_asm)]:*
```
     "⟦ A ∉ bad;  B ∉ bad;  evs ∈ bankerb_gets ⟧
  ⟹ Says Server A
        (Crypt (shrK A) {|Number Tk, Agent B, Key K,
                          Crypt (shrK B) {|Number Tk, Agent A, Key K|}|})
       ∈ set evs ⟶
      Key K ∈ analz (knows Spy evs) ⟶ Notes Spy {|Number Tk, Key K|} ∈ set
evs"
```
⟨proof⟩

Confidentiality for the Server: Spy does not see the keys sent in msg BK2 as long as they have not expired.

lemma *Confidentiality_S:*
```
     "⟦ Says Server A
        (Crypt K' {|Number Tk, Agent B, Key K, Ticket|}) ∈ set evs;
      Notes Spy {|Number Tk, Key K|} ∉ set evs;
       A ∉ bad;  B ∉ bad;  evs ∈ bankerb_gets
     ⟧ ⟹ Key K ∉ analz (knows Spy evs)"
```
⟨proof⟩

Confidentiality for Alice

lemma *Confidentiality_A:*
```
     "⟦ Crypt (shrK A) {|Number Tk, Agent B, Key K, X|} ∈ parts (knows Spy evs);
      Notes Spy {|Number Tk, Key K|} ∉ set evs;
      A ∉ bad;  B ∉ bad;  evs ∈ bankerb_gets
     ⟧ ⟹ Key K ∉ analz (knows Spy evs)"
```
⟨proof⟩

Confidentiality for Bob

lemma *Confidentiality_B:*
```
     "⟦ Crypt (shrK B) {|Number Tk, Agent A, Key K|}
```

```
        ∈ parts (knows Spy evs);
      Notes Spy {|Number Tk, Key K|} ∉ set evs;
      A ∉ bad;  B ∉ bad;  evs ∈ bankerb_gets
    ⟧ ⟹ Key K ∉ analz (knows Spy evs)"
```

⟨proof⟩

Non temporal treatment of authentication

Lemmas `lemma_A` and `lemma_B` in fact are common to both temporal and non-temporal treatments

```
lemma lemma_A [rule_format]:
     "⟦ A ∉ bad; B ∉ bad; evs ∈ bankerb_gets ⟧
      ⟹
         Key K ∉ analz (knows Spy evs) ⟶
         Says Server A (Crypt (shrK A) {|Number Tk, Agent B, Key K, X|})
         ∈ set evs ⟶
          Crypt K {|Agent A, Number Ta|} ∈ parts (knows Spy evs) ⟶
         Says A B {|X, Crypt K {|Agent A, Number Ta|}|}
             ∈ set evs"
```

⟨proof⟩

```
lemma lemma_B [rule_format]:
     "⟦ B ∉ bad;  evs ∈ bankerb_gets ⟧
      ⟹ Key K ∉ analz (knows Spy evs) ⟶
          Says Server A (Crypt (shrK A) {|Number Tk, Agent B, Key K, X|})
          ∈ set evs ⟶
          Crypt K (Number Ta) ∈ parts (knows Spy evs) ⟶
          Says B A (Crypt K (Number Ta)) ∈ set evs"
```

⟨proof⟩

The "r" suffix indicates theorems where the confidentiality assumptions are relaxed by the corresponding arguments.

Authentication of A to B

```
lemma B_authenticates_A_r:
     "⟦ Crypt K {|Agent A, Number Ta|} ∈ parts (knows Spy evs);
         Crypt (shrK B) {|Number Tk, Agent A, Key K|}  ∈ parts (knows Spy evs);
        Notes Spy {|Number Tk, Key K|} ∉ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ bankerb_gets ⟧
      ⟹ Says A B {|Crypt (shrK B) {|Number Tk, Agent A, Key K|},
                    Crypt K {|Agent A, Number Ta|}|} ∈ set evs"
```

⟨proof⟩

Authentication of B to A

```
lemma A_authenticates_B_r:
     "⟦ Crypt K (Number Ta) ∈ parts (knows Spy evs);
        Crypt (shrK A) {|Number Tk, Agent B, Key K, X|} ∈ parts (knows Spy evs);
        Notes Spy {|Number Tk, Key K|} ∉ set evs;
        A ∉ bad;  B ∉ bad;  evs ∈ bankerb_gets ⟧
      ⟹ Says B A (Crypt K (Number Ta)) ∈ set evs"
```

⟨proof⟩

```
lemma B_authenticates_A:
     "⟦ Crypt K {|Agent A, Number Ta|} ∈ parts (spies evs);
```

```
         Crypt (shrK B) {|Number Tk, Agent A, Key K|}  ∈ parts (spies evs);
        Key K ∉ analz (spies evs);
         A ∉ bad;  B ∉ bad;  evs ∈ bankerb_gets ⟧
      ⟹ Says A B {|Crypt (shrK B) {|Number Tk, Agent A, Key K|},
                    Crypt K {|Agent A, Number Ta|}|} ∈ set evs"
```
⟨*proof*⟩

lemma *A_authenticates_B:*
```
     "⟦ Crypt K (Number Ta) ∈ parts (spies evs);
        Crypt (shrK A) {|Number Tk, Agent B, Key K, X|} ∈ parts (spies evs);
        Key K ∉ analz (spies evs);
        A ∉ bad;  B ∉ bad;  evs ∈ bankerb_gets ⟧
      ⟹ Says B A (Crypt K (Number Ta)) ∈ set evs"
```
⟨*proof*⟩

5.3 Temporal guarantees, relying on a temporal check that insures that no oops event occurred. These are available in the sense of goal availability

Temporal treatment of confidentiality

Lemma: the session key sent in msg BK2 would be EXPIRED if the spy could see it!

lemma *lemma_conf_temporal [rule_format (no_asm)]:*
```
     "⟦ A ∉ bad;  B ∉ bad;  evs ∈ bankerb_gets ⟧
  ⟹ Says Server A
          (Crypt (shrK A) {|Number Tk, Agent B, Key K,
                            Crypt (shrK B) {|Number Tk, Agent A, Key K|}|})
         ∈ set evs ⟶
      Key K ∈ analz (knows Spy evs) ⟶ expiredK Tk evs"
```
⟨*proof*⟩

Confidentiality for the Server: Spy does not see the keys sent in msg BK2 as long as they have not expired.

lemma *Confidentiality_S_temporal:*
```
     "⟦ Says Server A
          (Crypt K' {|Number T, Agent B, Key K, X|}) ∈ set evs;
         ¬ expiredK T evs;
         A ∉ bad;  B ∉ bad;  evs ∈ bankerb_gets
      ⟧ ⟹ Key K ∉ analz (knows Spy evs)"
```
⟨*proof*⟩

Confidentiality for Alice

lemma *Confidentiality_A_temporal:*
```
     "⟦ Crypt (shrK A) {|Number T, Agent B, Key K, X|} ∈ parts (knows Spy evs);
         ¬ expiredK T evs;
         A ∉ bad;  B ∉ bad;  evs ∈ bankerb_gets
      ⟧ ⟹ Key K ∉ analz (knows Spy evs)"
```
⟨*proof*⟩

Confidentiality for Bob

lemma *Confidentiality_B_temporal:*

```
"⟦ Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄
     ∈ parts (knows Spy evs);
   ¬ expiredK Tk evs;
   A ∉ bad;  B ∉ bad;  evs ∈ bankerb_gets
 ⟧ ⟹ Key K ∉ analz (knows Spy evs)"
```
⟨proof⟩

Temporal treatment of authentication

Authentication of A to B

lemma *B_authenticates_A_temporal:*
```
"⟦ Crypt K ⦃Agent A, Number Ta⦄ ∈ parts (knows Spy evs);
    Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄
    ∈ parts (knows Spy evs);
    ¬ expiredK Tk evs;
    A ∉ bad;  B ∉ bad;  evs ∈ bankerb_gets ⟧
 ⟹ Says A B ⦃Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄,
              Crypt K ⦃Agent A, Number Ta⦄⦄ ∈ set evs"
```
⟨proof⟩

Authentication of B to A

lemma *A_authenticates_B_temporal:*
```
"⟦ Crypt K (Number Ta) ∈ parts (knows Spy evs);
    Crypt (shrK A) ⦃Number Tk, Agent B, Key K, X⦄
    ∈ parts (knows Spy evs);
    ¬ expiredK Tk evs;
    A ∉ bad;  B ∉ bad;  evs ∈ bankerb_gets ⟧
 ⟹ Says B A (Crypt K (Number Ta)) ∈ set evs"
```
⟨proof⟩

5.4 Combined guarantees of key distribution and non-injective agreement on the session keys

lemma *B_authenticates_and_keydist_to_A:*
```
"⟦ Gets B ⦃Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄,
           Crypt K ⦃Agent A, Number Ta⦄⦄ ∈ set evs;
   Key K ∉ analz (spies evs);
   A ∉ bad;  B ∉ bad;  evs ∈ bankerb_gets ⟧
 ⟹ Says A B ⦃Crypt (shrK B) ⦃Number Tk, Agent A, Key K⦄,
             Crypt K ⦃Agent A, Number Ta⦄⦄ ∈ set evs
 ∧  Key K ∈ analz (knows A evs)"
```
⟨proof⟩

lemma *A_authenticates_and_keydist_to_B:*
```
"⟦ Gets A (Crypt (shrK A) ⦃Number Tk, Agent B, Key K, Ticket⦄) ∈ set
evs;
   Gets A (Crypt K (Number Ta)) ∈ set evs;
   Key K ∉ analz (spies evs);
   A ∉ bad;  B ∉ bad;  evs ∈ bankerb_gets ⟧
 ⟹ Says B A (Crypt K (Number Ta)) ∈ set evs
 ∧  Key K ∈ analz (knows B evs)"
```
⟨proof⟩

end

6 The Kerberos Protocol, Version IV

theory *KerberosIV* **imports** *Public* **begin**

The "u" prefix indicates theorems referring to an updated version of the protocol. The "r" suffix indicates theorems where the confidentiality assumptions are relaxed by the corresponding arguments.

```
abbreviation
  Kas :: agent where "Kas == Server"

abbreviation
  Tgs :: agent where "Tgs == Friend 0"


axiomatization where
  Tgs_not_bad [iff]: "Tgs ∉ bad"
```
— Tgs is secure — we already know that Kas is secure

```
definition

    authKeys :: "event list ⇒ key set" where
    "authKeys evs = {authK. ∃A Peer Ta. Says Kas A
                          (Crypt (shrK A) {|Key authK, Agent Peer, Number Ta,
               (Crypt (shrK Peer) {|Agent A, Agent Peer, Key authK, Number
Ta|})
                  |}) ∈ set evs}"

definition

  Issues :: "[agent, agent, msg, event list] ⇒ bool"
           (‹_ Issues _ with _ on _› [50, 0, 0, 50] 50) where
   "(A Issues B with X on evs) =
      (∃Y. Says A B Y ∈ set evs ∧ X ∈ parts {Y} ∧
        X ∉ parts (spies (takeWhile (λz. z ≠ Says A B Y) (rev evs))))"

definition

  before :: "[event, event list] ⇒ event list" (‹before _ on _› [0, 50] 50)
  where "(before ev on evs) = takeWhile (λz. z ≠ ev) (rev evs)"

definition

  Unique :: "[event, event list] ⇒ bool" (‹Unique _ on _› [0, 50] 50)
  where "(Unique ev on evs) = (ev ∉ set (tl (dropWhile (λz. z ≠ ev) evs)))"


consts
```

authKlife :: nat

servKlife :: nat

authlife :: nat

replylife :: nat

specification *(authKlife)*
authKlife_LB [iff]: "2 ≤ authKlife"
⟨*proof*⟩

specification *(servKlife)*
servKlife_LB [iff]: "2 + authKlife ≤ servKlife"
⟨*proof*⟩

specification *(authlife)*
authlife_LB [iff]: "Suc 0 ≤ authlife"
⟨*proof*⟩

specification *(replylife)*
replylife_LB [iff]: "Suc 0 ≤ replylife"
⟨*proof*⟩

abbreviation

CT :: "event list ⇒ nat" **where**
"CT == length"

abbreviation
expiredAK :: "[nat, event list] ⇒ bool" **where**
"expiredAK Ta evs == authKlife + Ta < CT evs"

abbreviation
expiredSK :: "[nat, event list] ⇒ bool" **where**
"expiredSK Ts evs == servKlife + Ts < CT evs"

abbreviation
expiredA :: "[nat, event list] ⇒ bool" **where**
"expiredA T evs == authlife + T < CT evs"

abbreviation
valid :: "[nat, nat] ⇒ bool" (‹valid _ wrt _› [0, 50] 50) **where**
"valid T1 wrt T2 == T1 ≤ replylife + T2"

definition *AKcryptSK :: "[key, key, event list] ⇒ bool"* **where**
"AKcryptSK authK servK evs ==

```
    ∃A B Ts.
      Says Tgs A (Crypt authK
                    {|Key servK, Agent B, Number Ts,
                      Crypt (shrK B) {|Agent A, Agent B, Key servK, Number
Ts|} |})
        ∈ set evs"
```

inductive_set *kerbIV :: "event list set"*
where

```
  Nil:  "[] ∈ kerbIV"

 | Fake: "⟦ evsf ∈ kerbIV;  X ∈ synth (analz (spies evsf)) ⟧
         ⟹ Says Spy B X  # evsf ∈ kerbIV"


 | K1:   "⟦ evs1 ∈ kerbIV ⟧
         ⟹ Says A Kas {|Agent A, Agent Tgs, Number (CT evs1)|} # evs1
         ∈ kerbIV"


 | K2:  "⟦ evs2 ∈ kerbIV; Key authK ∉ used evs2; authK ∈ symKeys;
           Says A' Kas {|Agent A, Agent Tgs, Number T1|} ∈ set evs2 ⟧
         ⟹ Says Kas A
               (Crypt (shrK A) {|Key authK, Agent Tgs, Number (CT evs2),
                     (Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK,
                         Number (CT evs2)|})|}) # evs2 ∈ kerbIV"


 | K3:  "⟦ evs3 ∈ kerbIV;
           Says A Kas {|Agent A, Agent Tgs, Number T1|} ∈ set evs3;
           Says Kas' A (Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta,
             authTicket|}) ∈ set evs3;
           valid Ta wrt T1
        ⟧
         ⟹ Says A Tgs {|authTicket,
                          (Crypt authK {|Agent A, Number (CT evs3)|}),
                          Agent B|} # evs3 ∈ kerbIV"


 | K4:  "⟦ evs4 ∈ kerbIV; Key servK ∉ used evs4; servK ∈ symKeys;
           B ≠ Tgs;  authK ∈ symKeys;
           Says A' Tgs {|
```

```
             (Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK,
                                 Number Ta|}),
             (Crypt authK {|Agent A, Number T2|}), Agent B|}
                ∈ set evs4;
           ¬ expiredAK Ta evs4;
           ¬ expiredA T2 evs4;
           servKlife + (CT evs4) ≤ authKlife + Ta
        ⟧
         ⟹ Says Tgs A
               (Crypt authK {|Key servK, Agent B, Number (CT evs4),
                              Crypt (shrK B) {|Agent A, Agent B, Key servK,
                                               Number (CT evs4)|} |})
               # evs4 ∈ kerbIV"




| K5:  "⟦ evs5 ∈ kerbIV; authK ∈ symKeys; servK ∈ symKeys;
           Says A Tgs
               {|authTicket, Crypt authK {|Agent A, Number T2|},
                 Agent B|}
             ∈ set evs5;
           Says Tgs' A
             (Crypt authK {|Key servK, Agent B, Number Ts, servTicket|})
                ∈ set evs5;
           valid Ts wrt T2 ⟧
         ⟹ Says A B {|servTicket,
                        Crypt servK {|Agent A, Number (CT evs5)|} |}
              # evs5 ∈ kerbIV"




 | K6:  "⟦ evs6 ∈ kerbIV;
           Says A' B {|
             (Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|}),
             (Crypt servK {|Agent A, Number T3|})|}
           ∈ set evs6;
           ¬ expiredSK Ts evs6;
           ¬ expiredA T3 evs6
        ⟧
         ⟹ Says B A (Crypt servK (Number T3))
              # evs6 ∈ kerbIV"




| Oops1: "⟦ evsO1 ∈ kerbIV;  A ≠ Spy;
             Says Kas A
               (Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta,
                                  authTicket|})  ∈ set evsO1;
```

```
              expiredAK Ta evsO1 ⟧
          ⟹ Says A Spy {|Agent A, Agent Tgs, Number Ta, Key authK|}
               # evsO1 ∈ kerbIV"
```

```
| Oops2: "⟦ evsO2 ∈ kerbIV;  A ≠ Spy;
              Says Tgs A
                (Crypt authK {|Key servK, Agent B, Number Ts, servTicket|})
                  ∈ set evsO2;
              expiredSK Ts evsO2 ⟧
          ⟹ Says A Spy {|Agent A, Agent B, Number Ts, Key servK|}
               # evsO2 ∈ kerbIV"
```

declare *Says_imp_knows_Spy [THEN parts.Inj, dest]*
declare *parts.Body [dest]*
declare *analz_into_parts [dest]*
declare *Fake_parts_insert_in_Un [dest]*

6.1 Lemmas about lists, for reasoning about Issues

lemma *spies_Says_rev: "spies (evs @ [Says A B X]) = insert X (spies evs)"*
⟨*proof*⟩

lemma *spies_Gets_rev: "spies (evs @ [Gets A X]) = spies evs"*
⟨*proof*⟩

lemma *spies_Notes_rev: "spies (evs @ [Notes A X]) =*
(if A∈bad then insert X (spies evs) else spies evs)"
⟨*proof*⟩

lemma *spies_evs_rev: "spies evs = spies (rev evs)"*
⟨*proof*⟩

lemmas *parts_spies_evs_revD2 = spies_evs_rev [THEN equalityD2, THEN parts_mono]*

lemma *spies_takeWhile: "spies (takeWhile P evs) ⊆ spies evs"*
⟨*proof*⟩

lemmas *parts_spies_takeWhile_mono = spies_takeWhile [THEN parts_mono]*

6.2 Lemmas about *authKeys*

lemma *authKeys_empty: "authKeys [] = {}"*
⟨*proof*⟩

lemma *authKeys_not_insert:*

```
 "(∀A Ta akey Peer.
   ev ≠ Says Kas A (Crypt (shrK A) {|akey, Agent Peer, Ta,
              (Crypt (shrK Peer) {|Agent A, Agent Peer, akey, Ta|})|}))
       ⟹ authKeys (ev # evs) = authKeys evs"
```

⟨proof⟩

lemma *authKeys_insert:*
```
"authKeys
   (Says Kas A (Crypt (shrK A) {|Key K, Agent Peer, Number Ta,
    (Crypt (shrK Peer) {|Agent A, Agent Peer, Key K, Number Ta|})|}) # evs)
     = insert K (authKeys evs)"
```
⟨proof⟩

lemma *authKeys_simp:*
```
"K ∈ authKeys
  (Says Kas A (Crypt (shrK A) {|Key K', Agent Peer, Number Ta,
   (Crypt (shrK Peer) {|Agent A, Agent Peer, Key K', Number Ta|})|}) # evs)
     ⟹ K = K' | K ∈ authKeys evs"
```
⟨proof⟩

lemma *authKeysI:*
```
"Says Kas A (Crypt (shrK A) {|Key K, Agent Tgs, Number Ta,
   (Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key K, Number Ta|})|}) ∈ set evs
     ⟹ K ∈ authKeys evs"
```
⟨proof⟩

lemma *authKeys_used: "K ∈ authKeys evs ⟹ Key K ∈ used evs"*
⟨proof⟩

6.3 Forwarding Lemmas

–For reasoning about the encrypted portion of message K3–

lemma *K3_msg_in_parts_spies:*
```
"Says Kas' A (Crypt KeyA {|authK, Peer, Ta, authTicket|})
          ∈ set evs ⟹ authTicket ∈ parts (spies evs)"
```
⟨proof⟩

lemma *Oops_range_spies1:*
```
"⟦ Says Kas A (Crypt KeyA {|Key authK, Peer, Ta, authTicket|})
      ∈ set evs ;
    evs ∈ kerbIV ⟧ ⟹ authK ∉ range shrK ∧ authK ∈ symKeys"
```
⟨proof⟩

–For reasoning about the encrypted portion of message K5–

lemma *K5_msg_in_parts_spies:*
```
"Says Tgs' A (Crypt authK {|servK, Agent B, Ts, servTicket|})
          ∈ set evs ⟹ servTicket ∈ parts (spies evs)"
```
⟨proof⟩

lemma *Oops_range_spies2:*
```
"⟦ Says Tgs A (Crypt authK {|Key servK, Agent B, Ts, servTicket|})
      ∈ set evs ;
    evs ∈ kerbIV ⟧ ⟹ servK ∉ range shrK ∧ servK ∈ symKeys"
```
⟨proof⟩

lemma *Says_ticket_parts:*
```
"Says S A (Crypt K {|SesKey, B, TimeStamp, Ticket|}) ∈ set evs
```

```
    ⟹ Ticket ∈ parts (spies evs)"
```
⟨*proof*⟩

lemma *Spy_see_shrK [simp]:*
```
    "evs ∈ kerbIV ⟹ (Key (shrK A) ∈ parts (spies evs)) = (A ∈ bad)"
```
⟨*proof*⟩

lemma *Spy_analz_shrK [simp]:*
```
    "evs ∈ kerbIV ⟹ (Key (shrK A) ∈ analz (spies evs)) = (A ∈ bad)"
```
⟨*proof*⟩

lemma *Spy_see_shrK_D [dest!]:*
```
    "⟦ Key (shrK A) ∈ parts (spies evs);  evs ∈ kerbIV ⟧ ⟹ A∈bad"
```
⟨*proof*⟩

lemmas *Spy_analz_shrK_D = analz_subset_parts [THEN subsetD, THEN Spy_see_shrK_D, dest!]*

Nobody can have used non-existent keys!

lemma *new_keys_not_used [simp]:*
```
   "⟦Key K ∉ used evs; K ∈ symKeys; evs ∈ kerbIV⟧
    ⟹ K ∉ keysFor (parts (spies evs))"
```
⟨*proof*⟩

lemma *new_keys_not_analzd:*
```
 "⟦evs ∈ kerbIV; K ∈ symKeys; Key K ∉ used evs⟧
  ⟹ K ∉ keysFor (analz (spies evs))"
```
⟨*proof*⟩

6.4 Lemmas for reasoning about predicate "before"

lemma *used_Says_rev: "used (evs @ [Says A B X]) = parts {X} ∪ (used evs)"*
⟨*proof*⟩

lemma *used_Notes_rev: "used (evs @ [Notes A X]) = parts {X} ∪ (used evs)"*
⟨*proof*⟩

lemma *used_Gets_rev: "used (evs @ [Gets B X]) = used evs"*
⟨*proof*⟩

lemma *used_evs_rev: "used evs = used (rev evs)"*
⟨*proof*⟩

lemma *used_takeWhile_used [rule_format]:*
```
     "x ∈ used (takeWhile P X) ⟶ x ∈ used X"
```
⟨*proof*⟩

lemma *set_evs_rev: "set evs = set (rev evs)"*
⟨*proof*⟩

lemma *takeWhile_void [rule_format]:*
```
     "x ∉ set evs ⟶ takeWhile (λz. z ≠ x) evs = evs"
```

⟨*proof*⟩

6.5 Regularity Lemmas

These concern the form of items passed in messages

Describes the form of all components sent by Kas

lemma *Says_Kas_message_form:*
```
     "⟦ Says Kas A (Crypt K ⦃Key authK, Agent Peer, Number Ta, authTicket⦄)
           ∈ set evs;
         evs ∈ kerbIV ⟧ ⟹
  K = shrK A ∧ Peer = Tgs ∧
  authK ∉ range shrK ∧ authK ∈ authKeys evs ∧ authK ∈ symKeys ∧
  authTicket = (Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number Ta⦄)
∧
  Key authK ∉ used(before
           Says Kas A (Crypt K ⦃Key authK, Agent Peer, Number Ta, authTicket⦄)
                   on evs) ∧
  Ta = CT (before
           Says Kas A (Crypt K ⦃Key authK, Agent Peer, Number Ta, authTicket⦄)
           on evs)"
```
⟨*proof*⟩

lemma *SesKey_is_session_key:*
```
     "⟦ Crypt (shrK Tgs_B) ⦃Agent A, Agent Tgs_B, Key SesKey, Number T⦄
            ∈ parts (spies evs); Tgs_B ∉ bad;
         evs ∈ kerbIV ⟧
      ⟹ SesKey ∉ range shrK"
```
⟨*proof*⟩

lemma *authTicket_authentic:*
```
     "⟦ Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number Ta⦄
           ∈ parts (spies evs);
         evs ∈ kerbIV ⟧
      ⟹ Says Kas A (Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta,
                 Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number Ta⦄⦄)
            ∈ set evs"
```
⟨*proof*⟩

lemma *authTicket_crypt_authK:*
```
     "⟦ Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number Ta⦄
           ∈ parts (spies evs);
         evs ∈ kerbIV ⟧
      ⟹ authK ∈ authKeys evs"
```
⟨*proof*⟩

Describes the form of servK, servTicket and authK sent by Tgs

lemma *Says_Tgs_message_form:*
```
     "⟦ Says Tgs A (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄)
           ∈ set evs;
```

```
         evs ∈ kerbIV ⟧
  ⟹ B ≠ Tgs ∧
      authK ∉ range shrK ∧ authK ∈ authKeys evs ∧ authK ∈ symKeys ∧
      servK ∉ range shrK ∧ servK ∉ authKeys evs ∧ servK ∈ symKeys ∧
      servTicket = (Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄)
∧
      Key servK ∉ used (before
        Says Tgs A (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄)
                        on evs) ∧
      Ts = CT(before
        Says Tgs A (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄)
              on evs) "
```
⟨*proof*⟩

lemma *authTicket_form:*
```
     "⟦ Crypt (shrK A) ⦃Key authK, Agent Tgs, Ta, authTicket⦄
           ∈ parts (spies evs);
         A ∉ bad;
         evs ∈ kerbIV ⟧
    ⟹ authK ∉ range shrK ∧ authK ∈ symKeys ∧
        authTicket = Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Ta⦄"
```
⟨*proof*⟩

This form holds also over an authTicket, but is not needed below.

lemma *servTicket_form:*
```
     "⟦ Crypt authK ⦃Key servK, Agent B, Ts, servTicket⦄
              ∈ parts (spies evs);
            Key authK ∉ analz (spies evs);
            evs ∈ kerbIV ⟧
         ⟹ servK ∉ range shrK ∧ servK ∈ symKeys ∧
    (∃A. servTicket = Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Ts⦄)"
```
⟨*proof*⟩

Essentially the same as `authTicket_form`

lemma *Says_kas_message_form:*
```
     "⟦ Says Kas' A (Crypt (shrK A)
              ⦃Key authK, Agent Tgs, Ta, authTicket⦄) ∈ set evs;
         evs ∈ kerbIV ⟧
      ⟹ authK ∉ range shrK ∧ authK ∈ symKeys ∧
          authTicket =
                  Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Ta⦄
          | authTicket ∈ analz (spies evs)"
```
⟨*proof*⟩

lemma *Says_tgs_message_form:*
```
 "⟦ Says Tgs' A (Crypt authK ⦃Key servK, Agent B, Ts, servTicket⦄)
       ∈ set evs;  authK ∈ symKeys;
     evs ∈ kerbIV ⟧
  ⟹ servK ∉ range shrK ∧
      (∃A. servTicket =
              Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Ts⦄)
       | servTicket ∈ analz (spies evs)"
```
⟨*proof*⟩

6.6 Authenticity theorems: confirm origin of sensitive messages

lemma *authK_authentic:*
```
"⟦ Crypt (shrK A) ⦃Key authK, Peer, Ta, authTicket⦄
      ∈ parts (spies evs);
    A ∉ bad;  evs ∈ kerbIV ⟧
 ⟹ Says Kas A (Crypt (shrK A) ⦃Key authK, Peer, Ta, authTicket⦄)
       ∈ set evs"
```
⟨*proof*⟩

If a certain encrypted message appears then it originated with Tgs

lemma *servK_authentic:*
```
"⟦ Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄
      ∈ parts (spies evs);
    Key authK ∉ analz (spies evs);
    authK ∉ range shrK;
    evs ∈ kerbIV ⟧
⟹ ∃A. Says Tgs A (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄)
    ∈ set evs"
```
⟨*proof*⟩

lemma *servK_authentic_bis:*
```
"⟦ Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄
      ∈ parts (spies evs);
    Key authK ∉ analz (spies evs);
    B ≠ Tgs;
    evs ∈ kerbIV ⟧
⟹ ∃A. Says Tgs A (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄)
    ∈ set evs"
```
⟨*proof*⟩

Authenticity of servK for B

lemma *servTicket_authentic_Tgs:*
```
"⟦ Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
      ∈ parts (spies evs); B ≠ Tgs;  B ∉ bad;
    evs ∈ kerbIV ⟧
⟹ ∃authK.
    Says Tgs A (Crypt authK ⦃Key servK, Agent B, Number Ts,
                Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄⦄)
    ∈ set evs"
```
⟨*proof*⟩

Anticipated here from next subsection

lemma *K4_imp_K2:*
```
"⟦ Says Tgs A (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄)
    ∈ set evs;  evs ∈ kerbIV⟧
  ⟹ ∃Ta. Says Kas A
      (Crypt (shrK A)
       ⦃Key authK, Agent Tgs, Number Ta,
         Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number Ta⦄⦄)
     ∈ set evs"
```
⟨*proof*⟩

Anticipated here from next subsection

lemma *u_K4_imp_K2:*
```
"⟦ Says Tgs A (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄)
     ∈ set evs; evs ∈ kerbIV⟧
   ⟹ ∃Ta. (Says Kas A (Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta,
          Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number Ta⦄⦄)
            ∈ set evs
          ∧ servKlife + Ts ≤ authKlife + Ta)"
```
⟨*proof*⟩

lemma *servTicket_authentic_Kas:*
```
     "⟦ Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
           ∈ parts (spies evs);  B ≠ Tgs;  B ∉ bad;
         evs ∈ kerbIV ⟧
  ⟹ ∃authK Ta.
       Says Kas A
         (Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta,
            Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number Ta⦄⦄)
        ∈ set evs"
```
⟨*proof*⟩

lemma *u_servTicket_authentic_Kas:*
```
     "⟦ Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
           ∈ parts (spies evs);  B ≠ Tgs;  B ∉ bad;
         evs ∈ kerbIV ⟧
  ⟹ ∃authK Ta. Says Kas A (Crypt(shrK A) ⦃Key authK, Agent Tgs, Number Ta,
           Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number Ta⦄⦄)
             ∈ set evs
           ∧ servKlife + Ts ≤ authKlife + Ta"
```
⟨*proof*⟩

lemma *servTicket_authentic:*
```
     "⟦ Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
           ∈ parts (spies evs);  B ≠ Tgs;  B ∉ bad;
         evs ∈ kerbIV ⟧
 ⟹ ∃Ta authK.
     Says Kas A (Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta,
                   Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number
Ta⦄⦄)
       ∈ set evs
     ∧ Says Tgs A (Crypt authK ⦃Key servK, Agent B, Number Ts,
                   Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄⦄)
       ∈ set evs"
```
⟨*proof*⟩

lemma *u_servTicket_authentic:*
```
     "⟦ Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
           ∈ parts (spies evs);  B ≠ Tgs;  B ∉ bad;
         evs ∈ kerbIV ⟧
 ⟹ ∃Ta authK.
     (Says Kas A (Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta,
                   Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number
Ta⦄⦄)
       ∈ set evs
```

```
    ∧ Says Tgs A (Crypt authK {|Key servK, Agent B, Number Ts,
                   Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|}|})
      ∈ set evs
    ∧ servKlife + Ts ≤ authKlife + Ta)"
⟨proof⟩
```

lemma *u_NotexpiredSK_NotexpiredAK:*

```
    "⟦ ¬ expiredSK Ts evs; servKlife + Ts ≤ authKlife + Ta ⟧
     ⟹ ¬ expiredAK Ta evs"
  ⟨proof⟩
```

6.7 Reliability: friendly agents send something if something else happened

lemma *K3_imp_K2:*

```
    "⟦ Says A Tgs
          {|authTicket, Crypt authK {|Agent A, Number T2|}, Agent B|}
         ∈ set evs;
       A ∉ bad;  evs ∈ kerbIV ⟧
     ⟹ ∃Ta. Says Kas A (Crypt (shrK A)
                   {|Key authK, Agent Tgs, Number Ta, authTicket|})
                 ∈ set evs"
⟨proof⟩
```

Anticipated here from next subsection. An authK is encrypted by one and only one Shared key. A servK is encrypted by one and only one authK.

lemma *Key_unique_SesKey:*

```
    "⟦ Crypt K  {|Key SesKey,  Agent B, T, Ticket|}
         ∈ parts (spies evs);
       Crypt K' {|Key SesKey,  Agent B', T', Ticket'|}
         ∈ parts (spies evs);  Key SesKey ∉ analz (spies evs);
       evs ∈ kerbIV ⟧
     ⟹ K=K' ∧ B=B' ∧ T=T' ∧ Ticket=Ticket'"
⟨proof⟩
```

lemma *Tgs_authenticates_A:*

```
  "⟦  Crypt authK {|Agent A, Number T2|} ∈ parts (spies evs);
      Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Number Ta|}
           ∈ parts (spies evs);
      Key authK ∉ analz (spies evs); A ∉ bad; evs ∈ kerbIV ⟧
 ⟹ ∃ B. Says A Tgs {|
          Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Number Ta|},
          Crypt authK {|Agent A, Number T2|}, Agent B |} ∈ set evs"
⟨proof⟩
```

lemma *Says_K5:*

```
    "⟦ Crypt servK {|Agent A, Number T3|} ∈ parts (spies evs);
       Says Tgs A (Crypt authK {|Key servK, Agent B, Number Ts,
                                  servTicket|}) ∈ set evs;
       Key servK ∉ analz (spies evs);
       A ∉ bad; B ∉ bad; evs ∈ kerbIV ⟧
 ⟹ Says A B {|servTicket, Crypt servK {|Agent A, Number T3|}|} ∈ set evs"
⟨proof⟩
```

Anticipated here from next subsection

```
lemma unique_CryptKey:
     "⟦ Crypt (shrK B)  ⦃Agent A,  Agent B,  Key SesKey, T⦄
          ∈ parts (spies evs);
        Crypt (shrK B') ⦃Agent A', Agent B', Key SesKey, T'⦄
          ∈ parts (spies evs);  Key SesKey ∉ analz (spies evs);
        evs ∈ kerbIV ⟧
      ⟹ A=A' ∧ B=B' ∧ T=T'"
⟨proof⟩
```

```
lemma Says_K6:
     "⟦ Crypt servK (Number T3) ∈ parts (spies evs);
        Says Tgs A (Crypt authK ⦃Key servK, Agent B, Number Ts,
                                    servTicket⦄) ∈ set evs;
        Key servK ∉ analz (spies evs);
        A ∉ bad; B ∉ bad; evs ∈ kerbIV ⟧
      ⟹ Says B A (Crypt servK (Number T3)) ∈ set evs"
⟨proof⟩
```

Needs a unicity theorem, hence moved here

```
lemma servK_authentic_ter:
 "⟦ Says Kas A
    (Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta, authTicket⦄) ∈ set evs;
     Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄
       ∈ parts (spies evs);
     Key authK ∉ analz (spies evs);
     evs ∈ kerbIV ⟧
 ⟹ Says Tgs A (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄)
       ∈ set evs"
⟨proof⟩
```

6.8 Unicity Theorems

The session key, if secure, uniquely identifies the Ticket whether authTicket or servTicket. As a matter of fact, one can read also Tgs in the place of B.

```
lemma unique_authKeys:
     "⟦ Says Kas A
              (Crypt Ka ⦃Key authK, Agent Tgs, Ta, X⦄) ∈ set evs;
        Says Kas A'
              (Crypt Ka' ⦃Key authK, Agent Tgs, Ta', X'⦄) ∈ set evs;
        evs ∈ kerbIV ⟧ ⟹ A=A' ∧ Ka=Ka' ∧ Ta=Ta' ∧ X=X'"
⟨proof⟩
```

servK uniquely identifies the message from Tgs

```
lemma unique_servKeys:
     "⟦ Says Tgs A
              (Crypt K ⦃Key servK, Agent B, Ts, X⦄) ∈ set evs;
        Says Tgs A'
              (Crypt K' ⦃Key servK, Agent B', Ts', X'⦄) ∈ set evs;
        evs ∈ kerbIV ⟧ ⟹ A=A' ∧ B=B' ∧ K=K' ∧ Ts=Ts' ∧ X=X'"
⟨proof⟩
```

Revised unicity theorems

lemma *Kas_Unique:*
```
     "⟦ Says Kas A
              (Crypt Ka ⦃Key authK, Agent Tgs, Ta, authTicket⦄) ∈ set evs;
        evs ∈ kerbIV ⟧ ⟹
   Unique (Says Kas A (Crypt Ka ⦃Key authK, Agent Tgs, Ta, authTicket⦄))
   on evs"
```
⟨*proof*⟩

lemma *Tgs_Unique:*
```
     "⟦ Says Tgs A
              (Crypt authK ⦃Key servK, Agent B, Ts, servTicket⦄) ∈ set evs;
        evs ∈ kerbIV ⟧ ⟹
  Unique (Says Tgs A (Crypt authK ⦃Key servK, Agent B, Ts, servTicket⦄))
  on evs"
```
⟨*proof*⟩

6.9 Lemmas About the Predicate *AKcryptSK*

lemma *not_AKcryptSK_Nil [iff]: "¬ AKcryptSK authK servK []"*
⟨*proof*⟩

lemma *AKcryptSKI:*
```
 "⟦ Says Tgs A (Crypt authK ⦃Key servK, Agent B, Number Ts, X ⦄) ∈ set evs;
     evs ∈ kerbIV ⟧ ⟹ AKcryptSK authK servK evs"
```
⟨*proof*⟩

lemma *AKcryptSK_Says [simp]:*
```
   "AKcryptSK authK servK (Says S A X # evs) =
      (Tgs = S ∧
       (∃B Ts. X = Crypt authK
                 ⦃Key servK, Agent B, Number Ts,
                   Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
⦄)
      | AKcryptSK authK servK evs)"
```
⟨*proof*⟩

lemma *Auth_fresh_not_AKcryptSK:*
```
     "⟦ Key authK ∉ used evs; evs ∈ kerbIV ⟧
      ⟹ ¬ AKcryptSK authK servK evs"
```
⟨*proof*⟩

lemma *Serv_fresh_not_AKcryptSK:*
```
 "Key servK ∉ used evs ⟹ ¬ AKcryptSK authK servK evs"
```
⟨*proof*⟩

lemma *authK_not_AKcryptSK:*
```
     "⟦ Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, tk⦄
           ∈ parts (spies evs);  evs ∈ kerbIV ⟧
      ⟹ ¬ AKcryptSK K authK evs"
```
⟨*proof*⟩

A secure serverkey cannot have been used to encrypt others

```
lemma servK_not_AKcryptSK:
 "⟦ Crypt (shrK B) ⦃Agent A, Agent B, Key SK, Number Ts⦄ ∈ parts (spies evs);
     Key SK ∉ analz (spies evs);  SK ∈ symKeys;
     B ≠ Tgs;  evs ∈ kerbIV ⟧
  ⟹ ¬ AKcryptSK SK K evs"
⟨proof⟩
```

Long term keys are not issued as servKeys

```
lemma shrK_not_AKcryptSK:
     "evs ∈ kerbIV ⟹ ¬ AKcryptSK K (shrK A) evs"
⟨proof⟩
```

The Tgs message associates servK with authK and therefore not with any other key authK.

```
lemma Says_Tgs_AKcryptSK:
     "⟦ Says Tgs A (Crypt authK ⦃Key servK, Agent B, Number Ts, X ⦄)
           ∈ set evs;
         authK' ≠ authK;  evs ∈ kerbIV ⟧
      ⟹ ¬ AKcryptSK authK' servK evs"
⟨proof⟩
```

Equivalently

```
lemma not_different_AKcryptSK:
     "⟦ AKcryptSK authK servK evs;
        authK' ≠ authK;  evs ∈ kerbIV ⟧
      ⟹ ¬ AKcryptSK authK' servK evs  ∧ servK ∈ symKeys"
⟨proof⟩
```

```
lemma AKcryptSK_not_AKcryptSK:
     "⟦ AKcryptSK authK servK evs;  evs ∈ kerbIV ⟧
      ⟹ ¬ AKcryptSK servK K evs"
⟨proof⟩
```

The only session keys that can be found with the help of session keys are those sent by Tgs in step K4.

We take some pains to express the property as a logical equivalence so that the simplifier can apply it.

```
lemma Key_analz_image_Key_lemma:
     "P ⟶ (Key K ∈ analz (Key`KK ∪ H)) ⟶ (K∈KK | Key K ∈ analz H)
      ⟹
      P ⟶ (Key K ∈ analz (Key`KK ∪ H)) = (K∈KK | Key K ∈ analz H)"
⟨proof⟩
```

```
lemma AKcryptSK_analz_insert:
     "⟦ AKcryptSK K K' evs; K ∈ symKeys; evs ∈ kerbIV ⟧
      ⟹ Key K' ∈ analz (insert (Key K) (spies evs))"
⟨proof⟩
```

```
lemma authKeys_are_not_AKcryptSK:
```

```
    "⟦ K ∈ authKeys evs ∪ range shrK;  evs ∈ kerbIV ⟧
     ⟹ ∀SK. ¬ AKcryptSK SK K evs ∧ K ∈ symKeys"
⟨proof⟩
```

lemma *not_authKeys_not_AKcryptSK:*
```
    "⟦ K ∉ authKeys evs;
       K ∉ range shrK; evs ∈ kerbIV ⟧
     ⟹ ∀SK. ¬ AKcryptSK K SK evs"
⟨proof⟩
```

6.10 Secrecy Theorems

For the Oops2 case of the next theorem

lemma *Oops2_not_AKcryptSK:*
```
    "⟦ evs ∈ kerbIV;
       Says Tgs A (Crypt authK
                  ⦃Key servK, Agent B, Number Ts, servTicket⦄)
         ∈ set evs ⟧
     ⟹ ¬ AKcryptSK servK SK evs"
⟨proof⟩
```

Big simplification law for keys SK that are not crypted by keys in KK It helps prove three, otherwise hard, facts about keys. These facts are exploited as simplification laws for analz, and also "limit the damage" in case of loss of a key to the spy. See ESORICS98. [simplified by LCP]

lemma *Key_analz_image_Key [rule_format (no_asm)]:*
```
    "evs ∈ kerbIV ⟹
     (∀SK KK. SK ∈ symKeys ∧ KK ⊆ -(range shrK) ⟶
      (∀K ∈ KK. ¬ AKcryptSK K SK evs)   ⟶
      (Key SK ∈ analz (Key`KK ∪ (spies evs))) =
      (SK ∈ KK | Key SK ∈ analz (spies evs)))"
⟨proof⟩
```

First simplification law for analz: no session keys encrypt authentication keys or shared keys.

lemma *analz_insert_freshK1:*
```
    "⟦ evs ∈ kerbIV;  K ∈ authKeys evs ∪ range shrK;
       SesKey ∉ range shrK ⟧
     ⟹ (Key K ∈ analz (insert (Key SesKey) (spies evs))) =
         (K = SesKey | Key K ∈ analz (spies evs))"
⟨proof⟩
```

Second simplification law for analz: no service keys encrypt any other keys.

lemma *analz_insert_freshK2:*
```
    "⟦ evs ∈ kerbIV;  servK ∉ (authKeys evs); servK ∉ range shrK;
       K ∈ symKeys ⟧
     ⟹ (Key K ∈ analz (insert (Key servK) (spies evs))) =
         (K = servK | Key K ∈ analz (spies evs))"
⟨proof⟩
```

Third simplification law for analz: only one authentication key encrypts a certain service key.

lemma *analz_insert_freshK3:*
```
"⟦ AKcryptSK authK servK evs;
   authK' ≠ authK; authK' ∉ range shrK; evs ∈ kerbIV ⟧
      ⟹ (Key servK ∈ analz (insert (Key authK') (spies evs))) =
              (servK = authK' | Key servK ∈ analz (spies evs))"
```
⟨*proof*⟩

lemma *analz_insert_freshK3_bis:*
```
"⟦ Says Tgs A
         (Crypt authK {|Key servK, Agent B, Number Ts, servTicket|})
      ∈ set evs;
    authK ≠ authK'; authK' ∉ range shrK; evs ∈ kerbIV ⟧
      ⟹ (Key servK ∈ analz (insert (Key authK') (spies evs))) =
              (servK = authK' | Key servK ∈ analz (spies evs))"
```
⟨*proof*⟩

a weakness of the protocol

lemma *authK_compromises_servK:*
```
"⟦ Says Tgs A
         (Crypt authK {|Key servK, Agent B, Number Ts, servTicket|})
      ∈ set evs;  authK ∈ symKeys;
    Key authK ∈ analz (spies evs); evs ∈ kerbIV ⟧
  ⟹ Key servK ∈ analz (spies evs)"
```
⟨*proof*⟩

lemma *servK_notin_authKeysD:*
```
"⟦ Crypt authK {|Key servK, Agent B, Ts,
                Crypt (shrK B) {|Agent A, Agent B, Key servK, Ts|}|}
      ∈ parts (spies evs);
    Key servK ∉ analz (spies evs);
    B ≠ Tgs; evs ∈ kerbIV ⟧
  ⟹ servK ∉ authKeys evs"
```
⟨*proof*⟩

If Spy sees the Authentication Key sent in msg K2, then the Key has expired.

lemma *Confidentiality_Kas_lemma [rule_format]:*
```
"⟦ authK ∈ symKeys; A ∉ bad;  evs ∈ kerbIV ⟧
  ⟹ Says Kas A
          (Crypt (shrK A)
             {|Key authK, Agent Tgs, Number Ta,
     Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Number Ta|}|})
       ∈ set evs ⟶
     Key authK ∈ analz (spies evs) ⟶
     expiredAK Ta evs"
```
⟨*proof*⟩

lemma *Confidentiality_Kas:*
```
"⟦ Says Kas A
         (Crypt Ka {|Key authK, Agent Tgs, Number Ta, authTicket|})
      ∈ set evs;
    ¬ expiredAK Ta evs;
    A ∉ bad;  evs ∈ kerbIV ⟧
  ⟹ Key authK ∉ analz (spies evs)"
```
⟨*proof*⟩

If Spy sees the Service Key sent in msg K4, then the Key has expired.

```
lemma Confidentiality_lemma [rule_format]:
     "⟦ Says Tgs A
            (Crypt authK
               ⦃Key servK, Agent B, Number Ts,
                 Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄⦄)
          ∈ set evs;
        Key authK ∉ analz (spies evs);
        servK ∈ symKeys;
        A ∉ bad;  B ∉ bad; evs ∈ kerbIV ⟧
      ⟹ Key servK ∈ analz (spies evs) ⟶
         expiredSK Ts evs"
⟨proof⟩
```

In the real world Tgs can't check wheter authK is secure!

```
lemma Confidentiality_Tgs:
     "⟦ Says Tgs A
              (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄)
          ∈ set evs;
         Key authK ∉ analz (spies evs);
         ¬ expiredSK Ts evs;
         A ∉ bad;  B ∉ bad; evs ∈ kerbIV ⟧
      ⟹ Key servK ∉ analz (spies evs)"
⟨proof⟩
```

In the real world Tgs CAN check what Kas sends!

```
lemma Confidentiality_Tgs_bis:
     "⟦ Says Kas A
               (Crypt Ka ⦃Key authK, Agent Tgs, Number Ta, authTicket⦄)
          ∈ set evs;
         Says Tgs A
              (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄)
          ∈ set evs;
         ¬ expiredAK Ta evs; ¬ expiredSK Ts evs;
         A ∉ bad;  B ∉ bad; evs ∈ kerbIV ⟧
      ⟹ Key servK ∉ analz (spies evs)"
⟨proof⟩
```

Most general form

```
lemmas Confidentiality_Tgs_ter = authTicket_authentic [THEN Confidentiality_Tgs_bis]

lemmas Confidentiality_Auth_A = authK_authentic [THEN Confidentiality_Kas]
```

Needs a confidentiality guarantee, hence moved here. Authenticity of servK for A

```
lemma servK_authentic_bis_r:
     "⟦ Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta, authTicket⦄
          ∈ parts (spies evs);
         Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄
          ∈ parts (spies evs);
         ¬ expiredAK Ta evs; A ∉ bad; evs ∈ kerbIV ⟧
 ⟹Says Tgs A (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄)
```

```
       ∈ set evs"
⟨proof⟩

lemma Confidentiality_Serv_A:
     "⟦ Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta, authTicket|}
           ∈ parts (spies evs);
         Crypt authK {|Key servK, Agent B, Number Ts, servTicket|}
           ∈ parts (spies evs);
         ¬ expiredAK Ta evs; ¬ expiredSK Ts evs;
         A ∉ bad;  B ∉ bad; evs ∈ kerbIV ⟧
      ⟹ Key servK ∉ analz (spies evs)"
⟨proof⟩

lemma Confidentiality_B:
     "⟦ Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|}
           ∈ parts (spies evs);
         Crypt authK {|Key servK, Agent B, Number Ts, servTicket|}
           ∈ parts (spies evs);
         Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta, authTicket|}
           ∈ parts (spies evs);
         ¬ expiredSK Ts evs; ¬ expiredAK Ta evs;
         A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerbIV ⟧
      ⟹ Key servK ∉ analz (spies evs)"
⟨proof⟩


lemma u_Confidentiality_B:
     "⟦ Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|}
           ∈ parts (spies evs);
         ¬ expiredSK Ts evs;
         A ∉ bad;  B ∉ bad;  B ≠ Tgs; evs ∈ kerbIV ⟧
      ⟹ Key servK ∉ analz (spies evs)"
⟨proof⟩
```

6.11 Parties authentication: each party verifies "the identity of another party who generated some data" (quoted from Neuman and Ts'o).

These guarantees don't assess whether two parties agree on the same session key: sending a message containing a key doesn't a priori state knowledge of the key.

Tgs_authenticates_A can be found above

```
lemma A_authenticates_Tgs:
 "⟦ Says Kas A
    (Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta, authTicket|}) ∈ set evs;
     Crypt authK {|Key servK, Agent B, Number Ts, servTicket|}
       ∈ parts (spies evs);
     Key authK ∉ analz (spies evs);
     evs ∈ kerbIV ⟧
 ⟹ Says Tgs A (Crypt authK {|Key servK, Agent B, Number Ts, servTicket|})
       ∈ set evs"
⟨proof⟩
```

lemma *B_authenticates_A:*
```
"⟦ Crypt servK ⦃Agent A, Number T3⦄ ∈ parts (spies evs);
   Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
      ∈ parts (spies evs);
   Key servK ∉ analz (spies evs);
   A ∉ bad; B ∉ bad; B ≠ Tgs; evs ∈ kerbIV ⟧
⟹ Says A B ⦃Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄,
             Crypt servK ⦃Agent A, Number T3⦄⦄ ∈ set evs"
```
⟨*proof*⟩

The second assumption tells B what kind of key servK is.

lemma *B_authenticates_A_r:*
```
"⟦ Crypt servK ⦃Agent A, Number T3⦄ ∈ parts (spies evs);
    Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
      ∈ parts (spies evs);
    Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄
      ∈ parts (spies evs);
    Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta, authTicket⦄
      ∈ parts (spies evs);
    ¬ expiredSK Ts evs; ¬ expiredAK Ta evs;
    B ≠ Tgs; A ∉ bad;  B ∉ bad;  evs ∈ kerbIV ⟧
  ⟹ Says A B ⦃Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄,
              Crypt servK ⦃Agent A, Number T3⦄ ⦄ ∈ set evs"
```
⟨*proof*⟩

u_B_authenticates_A would be the same as *B_authenticates_A* because the servK confidentiality assumption is yet unrelaxed

lemma *u_B_authenticates_A_r:*
```
"⟦ Crypt servK ⦃Agent A, Number T3⦄ ∈ parts (spies evs);
    Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
      ∈ parts (spies evs);
    ¬ expiredSK Ts evs;
    B ≠ Tgs; A ∉ bad;  B ∉ bad;  evs ∈ kerbIV ⟧
  ⟹ Says A B ⦃Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄,
              Crypt servK ⦃Agent A, Number T3⦄ ⦄ ∈ set evs"
```
⟨*proof*⟩

lemma *A_authenticates_B:*
```
"⟦ Crypt servK (Number T3) ∈ parts (spies evs);
    Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄
      ∈ parts (spies evs);
    Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta, authTicket⦄
      ∈ parts (spies evs);
    Key authK ∉ analz (spies evs); Key servK ∉ analz (spies evs);
    A ∉ bad;  B ∉ bad; evs ∈ kerbIV ⟧
   ⟹ Says B A (Crypt servK (Number T3)) ∈ set evs"
```
⟨*proof*⟩

lemma *A_authenticates_B_r:*
```
"⟦ Crypt servK (Number T3) ∈ parts (spies evs);
    Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄
      ∈ parts (spies evs);
```

```
        Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta, authTicket⦄
          ∈ parts (spies evs);
        ¬ expiredAK Ta evs; ¬ expiredSK Ts evs;
        A ∉ bad;  B ∉ bad; evs ∈ kerbIV ⟧
     ⟹ Says B A (Crypt servK (Number T3)) ∈ set evs"
```
⟨proof⟩

6.12 Key distribution guarantees An agent knows a session key if he used it to issue a cipher. These guarantees also convey a stronger form of authentication - non-injective agreement on the session key

lemma *Kas_Issues_A:*
```
   "⟦ Says Kas A (Crypt (shrK A) ⦃Key authK, Peer, Ta, authTicket⦄) ∈ set
evs;
      evs ∈ kerbIV ⟧
  ⟹ Kas Issues A with (Crypt (shrK A) ⦃Key authK, Peer, Ta, authTicket⦄)

          on evs"
```
⟨proof⟩

lemma *A_authenticates_and_keydist_to_Kas:*
```
  "⟦ Crypt (shrK A) ⦃Key authK, Peer, Ta, authTicket⦄ ∈ parts (spies evs);
     A ∉ bad; evs ∈ kerbIV ⟧
 ⟹ Kas Issues A with (Crypt (shrK A) ⦃Key authK, Peer, Ta, authTicket⦄)

          on evs"
```
⟨proof⟩

lemma *honest_never_says_newer_timestamp_in_auth:*
```
     "⟦ (CT evs) ≤ T; A ∉ bad; Number T ∈ parts {X}; evs ∈ kerbIV ⟧
     ⟹ ∀ B Y.  Says A B ⦃Y, X⦄ ∉ set evs"
```
⟨proof⟩

lemma *honest_never_says_current_timestamp_in_auth:*
```
     "⟦ (CT evs) = T; Number T ∈ parts {X}; evs ∈ kerbIV ⟧
     ⟹ ∀ A B Y. A ∉ bad ⟶ Says A B ⦃Y, X⦄ ∉ set evs"
```
⟨proof⟩

lemma *A_trusts_secure_authenticator:*
```
    "⟦ Crypt K ⦃Agent A, Number T⦄ ∈ parts (spies evs);
       Key K ∉ analz (spies evs); evs ∈ kerbIV ⟧
⟹ ∃ B X. Says A Tgs ⦃X, Crypt K ⦃Agent A, Number T⦄, Agent B⦄ ∈ set evs
∨
           Says A B ⦃X, Crypt K ⦃Agent A, Number T⦄⦄ ∈ set evs"
```
⟨proof⟩

lemma *A_Issues_Tgs:*
```
  "⟦ Says A Tgs ⦃authTicket, Crypt authK ⦃Agent A, Number T2⦄, Agent B⦄
       ∈ set evs;
     Key authK ∉ analz (spies evs);
     A ∉ bad; evs ∈ kerbIV ⟧
 ⟹ A Issues Tgs with (Crypt authK ⦃Agent A, Number T2⦄) on evs"
```

⟨proof⟩

```
lemma Tgs_authenticates_and_keydist_to_A:
  "⟦  Crypt authK ⦃Agent A, Number T2⦄ ∈ parts (spies evs);
      Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number Ta⦄
           ∈ parts (spies evs);
     Key authK ∉ analz (spies evs);
     A ∉ bad; evs ∈ kerbIV ⟧
 ⟹ A Issues Tgs with (Crypt authK ⦃Agent A, Number T2⦄) on evs"
```

⟨proof⟩

```
lemma Tgs_Issues_A:
    "⟦ Says Tgs A (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket
⦄)
         ∈ set evs;
       Key authK ∉ analz (spies evs);  evs ∈ kerbIV ⟧
  ⟹ Tgs Issues A with
         (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket ⦄) on evs"
```

⟨proof⟩

```
lemma A_authenticates_and_keydist_to_Tgs:
"⟦Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄ ∈ parts (spies evs);
  Key authK ∉ analz (spies evs); B ≠ Tgs; evs ∈ kerbIV ⟧
 ⟹ ∃A. Tgs Issues A with
         (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket ⦄) on evs"
```

⟨proof⟩

```
lemma B_Issues_A:
     "⟦ Says B A (Crypt servK (Number T3)) ∈ set evs;
         Key servK ∉ analz (spies evs);
         A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerbIV ⟧
      ⟹ B Issues A with (Crypt servK (Number T3)) on evs"
```

⟨proof⟩

```
lemma B_Issues_A_r:
     "⟦ Says B A (Crypt servK (Number T3)) ∈ set evs;
         Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
            ∈ parts (spies evs);
         Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄
            ∈ parts (spies evs);
         Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta, authTicket⦄
           ∈ parts (spies evs);
         ¬ expiredSK Ts evs; ¬ expiredAK Ta evs;
         A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerbIV ⟧
      ⟹ B Issues A with (Crypt servK (Number T3)) on evs"
```

⟨proof⟩

```
lemma u_B_Issues_A_r:
     "⟦ Says B A (Crypt servK (Number T3)) ∈ set evs;
         Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
            ∈ parts (spies evs);
         ¬ expiredSK Ts evs;
```

```
        A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerbIV ⟧
     ⟹ B Issues A with (Crypt servK (Number T3)) on evs"
```
⟨proof⟩

lemma *A_authenticates_and_keydist_to_B:*
```
    "⟦ Crypt servK (Number T3) ∈ parts (spies evs);
        Crypt authK {|Key servK, Agent B, Number Ts, servTicket|}
          ∈ parts (spies evs);
        Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta, authTicket|}
          ∈ parts (spies evs);
        Key authK ∉ analz (spies evs); Key servK ∉ analz (spies evs);
        A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerbIV ⟧
     ⟹ B Issues A with (Crypt servK (Number T3)) on evs"
```
⟨proof⟩

lemma *A_authenticates_and_keydist_to_B_r:*
```
    "⟦ Crypt servK (Number T3) ∈ parts (spies evs);
        Crypt authK {|Key servK, Agent B, Number Ts, servTicket|}
          ∈ parts (spies evs);
        Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta, authTicket|}
          ∈ parts (spies evs);
        ¬ expiredAK Ta evs; ¬ expiredSK Ts evs;
        A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerbIV ⟧
     ⟹ B Issues A with (Crypt servK (Number T3)) on evs"
```
⟨proof⟩

lemma *A_Issues_B:*
```
    "⟦ Says A B {|servTicket, Crypt servK {|Agent A, Number T3|}|}
          ∈ set evs;
        Key servK ∉ analz (spies evs);
        B ≠ Tgs; A ∉ bad;  B ∉ bad;  evs ∈ kerbIV ⟧
  ⟹ A Issues B with (Crypt servK {|Agent A, Number T3|}) on evs"
```
⟨proof⟩

lemma *A_Issues_B_r:*
```
    "⟦ Says A B {|servTicket, Crypt servK {|Agent A, Number T3|}|}
          ∈ set evs;
        Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta, authTicket|}
          ∈ parts (spies evs);
        Crypt authK {|Key servK, Agent B, Number Ts, servTicket|}
          ∈ parts (spies evs);
        ¬ expiredAK Ta evs; ¬ expiredSK Ts evs;
        B ≠ Tgs; A ∉ bad;  B ∉ bad;  evs ∈ kerbIV ⟧
  ⟹ A Issues B with (Crypt servK {|Agent A, Number T3|}) on evs"
```
⟨proof⟩

lemma *B_authenticates_and_keydist_to_A:*
```
    "⟦ Crypt servK {|Agent A, Number T3|} ∈ parts (spies evs);
        Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|}
          ∈ parts (spies evs);
        Key servK ∉ analz (spies evs);
        B ≠ Tgs; A ∉ bad;  B ∉ bad;  evs ∈ kerbIV ⟧
  ⟹ A Issues B with (Crypt servK {|Agent A, Number T3|}) on evs"
```

⟨proof⟩

```
lemma B_authenticates_and_keydist_to_A_r:
     "⟦ Crypt servK {|Agent A, Number T3|} ∈ parts (spies evs);
         Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|}
           ∈ parts (spies evs);
         Crypt authK {|Key servK, Agent B, Number Ts, servTicket|}
           ∈ parts (spies evs);
         Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta, authTicket|}
           ∈ parts (spies evs);
         ¬ expiredSK Ts evs; ¬ expiredAK Ta evs;
         B ≠ Tgs; A ∉ bad;  B ∉ bad;  evs ∈ kerbIV ⟧
   ⟹ A Issues B with (Crypt servK {|Agent A, Number T3|}) on evs"
```

⟨proof⟩

u_B_authenticates_and_keydist_to_A would be the same as *B_authenticates_and_keydist_to_A* because the servK confidentiality assumption is yet unrelaxed

```
lemma u_B_authenticates_and_keydist_to_A_r:
     "⟦ Crypt servK {|Agent A, Number T3|} ∈ parts (spies evs);
         Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|}
           ∈ parts (spies evs);
         ¬ expiredSK Ts evs;
         B ≠ Tgs; A ∉ bad;  B ∉ bad;  evs ∈ kerbIV ⟧
   ⟹ A Issues B with (Crypt servK {|Agent A, Number T3|}) on evs"
```

⟨proof⟩

end

7 The Kerberos Protocol, Version IV

theory *KerberosIV_Gets* **imports** *Public* **begin**

The "u" prefix indicates theorems referring to an updated version of the protocol. The "r" suffix indicates theorems where the confidentiality assumptions are relaxed by the corresponding arguments.

```
abbreviation
  Kas :: agent where "Kas == Server"

abbreviation
  Tgs :: agent where "Tgs == Friend 0"


axiomatization where
  Tgs_not_bad [iff]: "Tgs ∉ bad"
```

— Tgs is secure — we already know that Kas is secure

```
definition

    authKeys :: "event list ⇒ key set" where
    "authKeys evs = {authK. ∃A Peer Ta. Says Kas A
                        (Crypt (shrK A) {|Key authK, Agent Peer, Number Ta,
               (Crypt (shrK Peer) {|Agent A, Agent Peer, Key authK, Number
Ta|})
```

```
                 |}) ∈ set evs}"
```

definition

```
Unique :: "[event, event list] ⇒ bool" (‹Unique _ on _› [0, 50] 50)
where "(Unique ev on evs) = (ev ∉ set (tl (dropWhile (λz. z ≠ ev) evs)))"
```

consts

```
  authKlife   :: nat

  servKlife   :: nat

  authlife   :: nat

  replylife   :: nat
```

specification *(authKlife)*

```
authKlife_LB [iff]: "2 ≤ authKlife"
  ⟨proof⟩
```

specification *(servKlife)*

```
servKlife_LB [iff]: "2 + authKlife ≤ servKlife"
  ⟨proof⟩
```

specification *(authlife)*

```
authlife_LB [iff]: "Suc 0 ≤ authlife"
  ⟨proof⟩
```

specification *(replylife)*

```
replylife_LB [iff]: "Suc 0 ≤ replylife"
  ⟨proof⟩
```

abbreviation

```
CT :: "event list ⇒ nat" where
"CT == length"
```

abbreviation

```
expiredAK :: "[nat, event list] ⇒ bool" where
"expiredAK Ta evs == authKlife + Ta < CT evs"
```

abbreviation

```
expiredSK :: "[nat, event list] ⇒ bool" where
"expiredSK Ts evs == servKlife + Ts < CT evs"
```

abbreviation

```
expiredA :: "[nat, event list] ⇒ bool" where
"expiredA T evs == authlife + T < CT evs"
```

```
abbreviation
  valid :: "[nat, nat] ⇒ bool" (‹valid _ wrt _› [0, 50] 50) where
  "valid T1 wrt T2 == T1 ≤ replylife + T2"
```

```
definition AKcryptSK :: "[key, key, event list] ⇒ bool" where
  "AKcryptSK authK servK evs ==
     ∃A B Ts.
       Says Tgs A (Crypt authK
                     ⦃Key servK, Agent B, Number Ts,
                       Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number
Ts⦄ ⦄)
         ∈ set evs"
```

```
inductive_set "kerbIV_gets" :: "event list set"
  where

   Nil:  "[] ∈ kerbIV_gets"

 | Fake: "⟦ evsf ∈ kerbIV_gets;  X ∈ synth (analz (spies evsf)) ⟧
         ⟹ Says Spy B X  # evsf ∈ kerbIV_gets"

 | Reception: "⟦ evsr ∈ kerbIV_gets;  Says A B X ∈ set evsr ⟧
                ⟹ Gets B X # evsr ∈ kerbIV_gets"


 | K1:   "⟦ evs1 ∈ kerbIV_gets ⟧
         ⟹ Says A Kas ⦃Agent A, Agent Tgs, Number (CT evs1)⦄ # evs1
         ∈ kerbIV_gets"


 | K2:  "⟦ evs2 ∈ kerbIV_gets; Key authK ∉ used evs2; authK ∈ symKeys;
            Gets Kas ⦃Agent A, Agent Tgs, Number T1⦄ ∈ set evs2 ⟧
         ⟹ Says Kas A
                (Crypt (shrK A) ⦃Key authK, Agent Tgs, Number (CT evs2),
                       (Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK,
                          Number (CT evs2)⦄)⦄) # evs2 ∈ kerbIV_gets"


 | K3:  "⟦ evs3 ∈ kerbIV_gets;
            Says A Kas ⦃Agent A, Agent Tgs, Number T1⦄ ∈ set evs3;
            Gets A (Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta,
              authTicket⦄) ∈ set evs3;
            valid Ta wrt T1
```

```
       ⟧
        ⟹ Says A Tgs ⦃authTicket,
                          (Crypt authK ⦃Agent A, Number (CT evs3)⦄),
                          Agent B⦄ # evs3 ∈ kerbIV_gets"


 | K4:  "⟦ evs4 ∈ kerbIV_gets; Key servK ∉ used evs4; servK ∈ symKeys;
          B ≠ Tgs;  authK ∈ symKeys;
          Gets Tgs ⦃
           (Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK,
                               Number Ta⦄),
           (Crypt authK ⦃Agent A, Number T2⦄), Agent B⦄
              ∈ set evs4;
          ¬ expiredAK Ta evs4;
          ¬ expiredA T2 evs4;
          servKlife + (CT evs4) ≤ authKlife + Ta
       ⟧
        ⟹ Says Tgs A
              (Crypt authK ⦃Key servK, Agent B, Number (CT evs4),
                             Crypt (shrK B) ⦃Agent A, Agent B, Key servK,
                                              Number (CT evs4)⦄ ⦄)
              # evs4 ∈ kerbIV_gets"


 | K5:  "⟦ evs5 ∈ kerbIV_gets; authK ∈ symKeys; servK ∈ symKeys;
          Says A Tgs
              ⦃authTicket, Crypt authK ⦃Agent A, Number T2⦄,
                Agent B⦄
            ∈ set evs5;
          Gets A
           (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄)
              ∈ set evs5;
          valid Ts wrt T2 ⟧
        ⟹ Says A B ⦃servTicket,
                       Crypt servK ⦃Agent A, Number (CT evs5)⦄ ⦄
             # evs5 ∈ kerbIV_gets"


  | K6:  "⟦ evs6 ∈ kerbIV_gets;
          Gets B ⦃
            (Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄),
            (Crypt servK ⦃Agent A, Number T3⦄)⦄
          ∈ set evs6;
          ¬ expiredSK Ts evs6;
```

```
      ¬ expiredA T3 evs6
    ⟧
    ⟹ Says B A (Crypt servK (Number T3))
         # evs6 ∈ kerbIV_gets"
```

```
| Oops1: "⟦ evsO1 ∈ kerbIV_gets;  A ≠ Spy;
             Says Kas A
               (Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta,
                                  authTicket⦄)  ∈ set evsO1;
             expiredAK Ta evsO1 ⟧
          ⟹ Says A Spy ⦃Agent A, Agent Tgs, Number Ta, Key authK⦄
              # evsO1 ∈ kerbIV_gets"
```

```
| Oops2: "⟦ evsO2 ∈ kerbIV_gets;  A ≠ Spy;
             Says Tgs A
               (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄)
                  ∈ set evsO2;
             expiredSK Ts evsO2 ⟧
          ⟹ Says A Spy ⦃Agent A, Agent B, Number Ts, Key servK⦄
              # evsO2 ∈ kerbIV_gets"
```

declare *Says_imp_knows_Spy [THEN parts.Inj, dest]*
declare *parts.Body [dest]*
declare *analz_into_parts [dest]*
declare *Fake_parts_insert_in_Un [dest]*

7.1 Lemmas about reception event

lemma *Gets_imp_Says :*
"⟦ Gets B X ∈ set evs; evs ∈ kerbIV_gets ⟧ ⟹ ∃A. Says A B X ∈ set evs"
⟨*proof*⟩

lemma *Gets_imp_knows_Spy:*
"⟦ Gets B X ∈ set evs; evs ∈ kerbIV_gets ⟧ ⟹ X ∈ knows Spy evs"
⟨*proof*⟩

declare *Gets_imp_knows_Spy [THEN parts.Inj, dest]*

lemma *Gets_imp_knows:*
"⟦ Gets B X ∈ set evs; evs ∈ kerbIV_gets ⟧ ⟹ X ∈ knows B evs"
⟨*proof*⟩

7.2 Lemmas about *authKeys*

```
lemma authKeys_empty: "authKeys [] = {}"
⟨proof⟩

lemma authKeys_not_insert:
 "(∀A Ta akey Peer.
   ev ≠ Says Kas A (Crypt (shrK A) {|akey, Agent Peer, Ta,
              (Crypt (shrK Peer) {|Agent A, Agent Peer, akey, Ta|})|}))
       ⟹ authKeys (ev # evs) = authKeys evs"
  ⟨proof⟩

lemma authKeys_insert:
  "authKeys
     (Says Kas A (Crypt (shrK A) {|Key K, Agent Peer, Number Ta,
      (Crypt (shrK Peer) {|Agent A, Agent Peer, Key K, Number Ta|})|}) # evs)
       = insert K (authKeys evs)"
  ⟨proof⟩

lemma authKeys_simp:
   "K ∈ authKeys
    (Says Kas A (Crypt (shrK A) {|Key K', Agent Peer, Number Ta,
     (Crypt (shrK Peer) {|Agent A, Agent Peer, Key K', Number Ta|})|}) # evs)
        ⟹ K = K' | K ∈ authKeys evs"
  ⟨proof⟩

lemma authKeysI:
   "Says Kas A (Crypt (shrK A) {|Key K, Agent Tgs, Number Ta,
     (Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key K, Number Ta|})|}) ∈ set evs
        ⟹ K ∈ authKeys evs"
  ⟨proof⟩

lemma authKeys_used: "K ∈ authKeys evs ⟹ Key K ∈ used evs"
⟨proof⟩
```

7.3 Forwarding Lemmas

```
lemma Says_ticket_parts:
     "Says S A (Crypt K {|SesKey, B, TimeStamp, Ticket|}) ∈ set evs
      ⟹ Ticket ∈ parts (spies evs)"
⟨proof⟩

lemma Gets_ticket_parts:
     "[|Gets A (Crypt K {|SesKey, Peer, Ta, Ticket|}) ∈ set evs; evs ∈ kerbIV_gets
|]
      ⟹ Ticket ∈ parts (spies evs)"
⟨proof⟩

lemma Oops_range_spies1:
     "[| Says Kas A (Crypt KeyA {|Key authK, Peer, Ta, authTicket|})
           ∈ set evs ;
         evs ∈ kerbIV_gets |] ⟹ authK ∉ range shrK ∧ authK ∈ symKeys"
⟨proof⟩

lemma Oops_range_spies2:
```

```
"⟦ Says Tgs A (Crypt authK {|Key servK, Agent B, Ts, servTicket|})
      ∈ set evs ;
    evs ∈ kerbIV_gets ⟧ ⟹ servK ∉ range shrK ∧ servK ∈ symKeys"
```
⟨proof⟩

lemma *Spy_see_shrK [simp]:*
```
"evs ∈ kerbIV_gets ⟹ (Key (shrK A) ∈ parts (spies evs)) = (A ∈ bad)"
```
⟨proof⟩

lemma *Spy_analz_shrK [simp]:*
```
"evs ∈ kerbIV_gets ⟹ (Key (shrK A) ∈ analz (spies evs)) = (A ∈ bad)"
```
⟨proof⟩

lemma *Spy_see_shrK_D [dest!]:*
```
"⟦ Key (shrK A) ∈ parts (spies evs);  evs ∈ kerbIV_gets ⟧ ⟹ A∈bad"
```
⟨proof⟩
lemmas *Spy_analz_shrK_D = analz_subset_parts [THEN subsetD, THEN Spy_see_shrK_D, dest!]*

Nobody can have used non-existent keys!

lemma *new_keys_not_used [simp]:*
```
"⟦Key K ∉ used evs; K ∈ symKeys; evs ∈ kerbIV_gets⟧
 ⟹ K ∉ keysFor (parts (spies evs))"
```
⟨proof⟩

lemma *new_keys_not_analzd:*
```
"⟦evs ∈ kerbIV_gets; K ∈ symKeys; Key K ∉ used evs⟧
 ⟹ K ∉ keysFor (analz (spies evs))"
```
⟨proof⟩

7.4 Regularity Lemmas

These concern the form of items passed in messages

Describes the form of all components sent by Kas

lemma *Says_Kas_message_form:*
```
"⟦ Says Kas A (Crypt K {|Key authK, Agent Peer, Number Ta, authTicket|})
      ∈ set evs;
    evs ∈ kerbIV_gets ⟧ ⟹
K = shrK A  ∧ Peer = Tgs ∧
authK ∉ range shrK ∧ authK ∈ authKeys evs ∧ authK ∈ symKeys ∧
authTicket = (Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Number Ta|})"
```
⟨proof⟩

lemma *SesKey_is_session_key:*
```
"⟦ Crypt (shrK Tgs_B) {|Agent A, Agent Tgs_B, Key SesKey, Number T|}
      ∈ parts (spies evs); Tgs_B ∉ bad;
    evs ∈ kerbIV_gets ⟧
 ⟹ SesKey ∉ range shrK"
```

⟨proof⟩

```
lemma authTicket_authentic:
     "⟦ Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Number Ta|}
          ∈ parts (spies evs);
        evs ∈ kerbIV_gets ⟧
      ⟹ Says Kas A (Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta,
                  Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Number Ta|}|})
           ∈ set evs"
```

⟨proof⟩

```
lemma authTicket_crypt_authK:
     "⟦ Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Number Ta|}
          ∈ parts (spies evs);
        evs ∈ kerbIV_gets ⟧
      ⟹ authK ∈ authKeys evs"
```

⟨proof⟩

```
lemma Says_Tgs_message_form:
     "⟦ Says Tgs A (Crypt authK {|Key servK, Agent B, Number Ts, servTicket|})
          ∈ set evs;
        evs ∈ kerbIV_gets ⟧
  ⟹ B ≠ Tgs ∧
      authK ∉ range shrK ∧ authK ∈ authKeys evs ∧ authK ∈ symKeys ∧
      servK ∉ range shrK ∧ servK ∉ authKeys evs ∧ servK ∈ symKeys ∧
      servTicket = (Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|})"
```

⟨proof⟩

```
lemma authTicket_form:
     "⟦ Crypt (shrK A) {|Key authK, Agent Tgs, Ta, authTicket|}
          ∈ parts (spies evs);
        A ∉ bad;
        evs ∈ kerbIV_gets ⟧
    ⟹ authK ∉ range shrK ∧ authK ∈ symKeys ∧
        authTicket = Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Ta|}"
```

⟨proof⟩

This form holds also over an authTicket, but is not needed below.

```
lemma servTicket_form:
     "⟦ Crypt authK {|Key servK, Agent B, Ts, servTicket|}
            ∈ parts (spies evs);
          Key authK ∉ analz (spies evs);
          evs ∈ kerbIV_gets ⟧
        ⟹ servK ∉ range shrK ∧ servK ∈ symKeys ∧
    (∃A. servTicket = Crypt (shrK B) {|Agent A, Agent B, Key servK, Ts|})"
```

⟨proof⟩

Essentially the same as `authTicket_form`

```
lemma Says_kas_message_form:
     "⟦ Gets A (Crypt (shrK A)
            {|Key authK, Agent Tgs, Ta, authTicket|}) ∈ set evs;
        evs ∈ kerbIV_gets ⟧
      ⟹ authK ∉ range shrK ∧ authK ∈ symKeys ∧
```

```
          authTicket =
                  Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Ta|}
          | authTicket ∈ analz (spies evs)"
```
⟨proof⟩

lemma *Says_tgs_message_form:*
```
 "⟦ Gets A (Crypt authK {|Key servK, Agent B, Ts, servTicket|})
       ∈ set evs;  authK ∈ symKeys;
     evs ∈ kerbIV_gets ⟧
  ⟹ servK ∉ range shrK ∧
      (∃A. servTicket =
              Crypt (shrK B) {|Agent A, Agent B, Key servK, Ts|})
       | servTicket ∈ analz (spies evs)"
```
⟨proof⟩

7.5 Authenticity theorems: confirm origin of sensitive messages

lemma *authK_authentic:*
```
     "⟦ Crypt (shrK A) {|Key authK, Peer, Ta, authTicket|}
           ∈ parts (spies evs);
         A ∉ bad;  evs ∈ kerbIV_gets ⟧
      ⟹ Says Kas A (Crypt (shrK A) {|Key authK, Peer, Ta, authTicket|})
            ∈ set evs"
```
⟨proof⟩

If a certain encrypted message appears then it originated with Tgs

lemma *servK_authentic:*
```
     "⟦ Crypt authK {|Key servK, Agent B, Number Ts, servTicket|}
           ∈ parts (spies evs);
         Key authK ∉ analz (spies evs);
         authK ∉ range shrK;
         evs ∈ kerbIV_gets ⟧
 ⟹ ∃A. Says Tgs A (Crypt authK {|Key servK, Agent B, Number Ts, servTicket|})
       ∈ set evs"
```
⟨proof⟩

lemma *servK_authentic_bis:*
```
     "⟦ Crypt authK {|Key servK, Agent B, Number Ts, servTicket|}
           ∈ parts (spies evs);
         Key authK ∉ analz (spies evs);
         B ≠ Tgs;
         evs ∈ kerbIV_gets ⟧
 ⟹ ∃A. Says Tgs A (Crypt authK {|Key servK, Agent B, Number Ts, servTicket|})
       ∈ set evs"
```
⟨proof⟩

Authenticity of servK for B

lemma *servTicket_authentic_Tgs:*
```
     "⟦ Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|}
           ∈ parts (spies evs); B ≠ Tgs;  B ∉ bad;
         evs ∈ kerbIV_gets ⟧
 ⟹ ∃authK.
```

```
      Says Tgs A (Crypt authK ⦃Key servK, Agent B, Number Ts,
                 Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄⦄)
      ∈ set evs"
```
⟨proof⟩

Anticipated here from next subsection

lemma *K4_imp_K2:*
```
"⟦ Says Tgs A (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄)
     ∈ set evs;  evs ∈ kerbIV_gets⟧
   ⟹ ∃Ta. Says Kas A
        (Crypt (shrK A)
         ⦃Key authK, Agent Tgs, Number Ta,
           Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number Ta⦄⦄)
        ∈ set evs"
```
⟨proof⟩

Anticipated here from next subsection

lemma *u_K4_imp_K2:*
```
"⟦ Says Tgs A (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄)
     ∈ set evs; evs ∈ kerbIV_gets⟧
   ⟹ ∃Ta. (Says Kas A (Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta,
           Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number Ta⦄⦄)
             ∈ set evs
          ∧ servKlife + Ts ≤ authKlife + Ta)"
```
⟨proof⟩

lemma *servTicket_authentic_Kas:*
```
     "⟦ Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
           ∈ parts (spies evs);  B ≠ Tgs;  B ∉ bad;
         evs ∈ kerbIV_gets ⟧
  ⟹ ∃authK Ta.
       Says Kas A
         (Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta,
            Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number Ta⦄⦄)
        ∈ set evs"
```
⟨proof⟩

lemma *u_servTicket_authentic_Kas:*
```
     "⟦ Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
           ∈ parts (spies evs);  B ≠ Tgs;  B ∉ bad;
         evs ∈ kerbIV_gets ⟧
  ⟹ ∃authK Ta. Says Kas A (Crypt(shrK A) ⦃Key authK, Agent Tgs, Number Ta,
           Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number Ta⦄⦄)
             ∈ set evs
           ∧ servKlife + Ts ≤ authKlife + Ta"
```
⟨proof⟩

lemma *servTicket_authentic:*
```
     "⟦ Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
           ∈ parts (spies evs);  B ≠ Tgs;  B ∉ bad;
         evs ∈ kerbIV_gets ⟧
 ⟹ ∃Ta authK.
     Says Kas A (Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta,
```

```
                  Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Number
Ta|}|})
       ∈ set evs
     ∧ Says Tgs A (Crypt authK {|Key servK, Agent B, Number Ts,
                  Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|}|})
       ∈ set evs"
```
⟨proof⟩

lemma *u_servTicket_authentic:*

```
     "⟦ Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|}
           ∈ parts (spies evs);  B ≠ Tgs;  B ∉ bad;
         evs ∈ kerbIV_gets ⟧
 ⟹ ∃ Ta authK.
     (Says Kas A (Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta,
                  Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Number
Ta|}|})
       ∈ set evs
     ∧ Says Tgs A (Crypt authK {|Key servK, Agent B, Number Ts,
                  Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|}|})
       ∈ set evs
     ∧ servKlife + Ts ≤ authKlife + Ta)"
```
⟨proof⟩

lemma *u_NotexpiredSK_NotexpiredAK:*

```
     "⟦ ¬ expiredSK Ts evs; servKlife + Ts ≤ authKlife + Ta ⟧
      ⟹ ¬ expiredAK Ta evs"
```
⟨proof⟩

7.6 Reliability: friendly agents send something if something else happened

lemma *K3_imp_K2:*

```
     "⟦ Says A Tgs
             {|authTicket, Crypt authK {|Agent A, Number T2|}, Agent B|}
           ∈ set evs;
         A ∉ bad;  evs ∈ kerbIV_gets ⟧
      ⟹ ∃Ta. Says Kas A (Crypt (shrK A)
                     {|Key authK, Agent Tgs, Number Ta, authTicket|})
                  ∈ set evs"
```
⟨proof⟩

Anticipated here from next subsection. An authK is encrypted by one and only one Shared key. A servK is encrypted by one and only one authK.

lemma *Key_unique_SesKey:*

```
     "⟦ Crypt K  {|Key SesKey,  Agent B, T, Ticket|}
           ∈ parts (spies evs);
         Crypt K' {|Key SesKey,  Agent B', T', Ticket'|}
           ∈ parts (spies evs);  Key SesKey ∉ analz (spies evs);
         evs ∈ kerbIV_gets ⟧
      ⟹ K=K' ∧ B=B' ∧ T=T' ∧ Ticket=Ticket'"
```
⟨proof⟩

lemma *Tgs_authenticates_A:*

```
  "⟦  Crypt authK {|Agent A, Number T2|} ∈ parts (spies evs);
```

```
      Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Number Ta|}
           ∈ parts (spies evs);
      Key authK ∉ analz (spies evs); A ∉ bad; evs ∈ kerbIV_gets ⟧
  ⟹ ∃ B. Says A Tgs {|
          Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Number Ta|},
          Crypt authK {|Agent A, Number T2|}, Agent B |} ∈ set evs"
```

⟨proof⟩

lemma *Says_K5:*

```
     "⟦ Crypt servK {|Agent A, Number T3|} ∈ parts (spies evs);
         Says Tgs A (Crypt authK {|Key servK, Agent B, Number Ts,
                                   servTicket|}) ∈ set evs;
         Key servK ∉ analz (spies evs);
         A ∉ bad; B ∉ bad; evs ∈ kerbIV_gets ⟧
  ⟹ Says A B {|servTicket, Crypt servK {|Agent A, Number T3|}|} ∈ set evs"
```

⟨proof⟩

Anticipated here from next subsection

lemma *unique_CryptKey:*

```
     "⟦ Crypt (shrK B)  {|Agent A,  Agent B,  Key SesKey, T|}
           ∈ parts (spies evs);
         Crypt (shrK B') {|Agent A', Agent B', Key SesKey, T'|}
           ∈ parts (spies evs);  Key SesKey ∉ analz (spies evs);
         evs ∈ kerbIV_gets ⟧
      ⟹ A=A' ∧ B=B' ∧ T=T'"
```

⟨proof⟩

lemma *Says_K6:*

```
     "⟦ Crypt servK (Number T3) ∈ parts (spies evs);
         Says Tgs A (Crypt authK {|Key servK, Agent B, Number Ts,
                                   servTicket|}) ∈ set evs;
         Key servK ∉ analz (spies evs);
         A ∉ bad; B ∉ bad; evs ∈ kerbIV_gets ⟧
      ⟹ Says B A (Crypt servK (Number T3)) ∈ set evs"
```

⟨proof⟩

Needs a unicity theorem, hence moved here

lemma *servK_authentic_ter:*

```
  "⟦ Says Kas A
     (Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta, authTicket|}) ∈ set evs;
      Crypt authK {|Key servK, Agent B, Number Ts, servTicket|}
        ∈ parts (spies evs);
      Key authK ∉ analz (spies evs);
      evs ∈ kerbIV_gets ⟧
  ⟹ Says Tgs A (Crypt authK {|Key servK, Agent B, Number Ts, servTicket|})
        ∈ set evs"
```

⟨proof⟩

7.7 Unicity Theorems

The session key, if secure, uniquely identifies the Ticket whether authTicket or servTicket. As a matter of fact, one can read also Tgs in the place of B.

lemma *unique_authKeys:*

```
  "⟦ Says Kas A
         (Crypt Ka {|Key authK, Agent Tgs, Ta, X|}) ∈ set evs;
     Says Kas A'
         (Crypt Ka' {|Key authK, Agent Tgs, Ta', X'|}) ∈ set evs;
     evs ∈ kerbIV_gets ⟧ ⟹ A=A' ∧ Ka=Ka' ∧ Ta=Ta' ∧ X=X'"
```
⟨proof⟩

servK uniquely identifies the message from Tgs

lemma *unique_servKeys:*
```
  "⟦ Says Tgs A
         (Crypt K {|Key servK, Agent B, Ts, X|}) ∈ set evs;
     Says Tgs A'
         (Crypt K' {|Key servK, Agent B', Ts', X'|}) ∈ set evs;
     evs ∈ kerbIV_gets ⟧ ⟹ A=A' ∧ B=B' ∧ K=K' ∧ Ts=Ts' ∧ X=X'"
```
⟨proof⟩

Revised unicity theorems

lemma *Kas_Unique:*
```
  "⟦ Says Kas A
         (Crypt Ka {|Key authK, Agent Tgs, Ta, authTicket|}) ∈ set evs;
    evs ∈ kerbIV_gets ⟧ ⟹
 Unique (Says Kas A (Crypt Ka {|Key authK, Agent Tgs, Ta, authTicket|}))
 on evs"
```
⟨proof⟩

lemma *Tgs_Unique:*
```
  "⟦ Says Tgs A
         (Crypt authK {|Key servK, Agent B, Ts, servTicket|}) ∈ set evs;
    evs ∈ kerbIV_gets ⟧ ⟹
 Unique (Says Tgs A (Crypt authK {|Key servK, Agent B, Ts, servTicket|}))
 on evs"
```
⟨proof⟩

7.8 Lemmas About the Predicate *AKcryptSK*

lemma *not_AKcryptSK_Nil [iff]: "¬ AKcryptSK authK servK []"*
⟨proof⟩

lemma *AKcryptSKI:*
```
 "⟦ Says Tgs A (Crypt authK {|Key servK, Agent B, Number Ts, X |}) ∈ set evs;
    evs ∈ kerbIV_gets ⟧ ⟹ AKcryptSK authK servK evs"
```
⟨proof⟩

lemma *AKcryptSK_Says [simp]:*
```
  "AKcryptSK authK servK (Says S A X # evs) =
    (Tgs = S ∧
     (∃B Ts. X = Crypt authK
               {|Key servK, Agent B, Number Ts,
                 Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|}
|})
    | AKcryptSK authK servK evs)"
```
⟨proof⟩

```
lemma Auth_fresh_not_AKcryptSK:
     "⟦ Key authK ∉ used evs; evs ∈ kerbIV_gets ⟧
      ⟹ ¬ AKcryptSK authK servK evs"
⟨proof⟩
```

```
lemma Serv_fresh_not_AKcryptSK:
 "Key servK ∉ used evs ⟹ ¬ AKcryptSK authK servK evs"
  ⟨proof⟩
```

```
lemma authK_not_AKcryptSK:
     "⟦ Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, tk⦄
           ∈ parts (spies evs);  evs ∈ kerbIV_gets ⟧
      ⟹ ¬ AKcryptSK K authK evs"
⟨proof⟩
```

A secure serverkey cannot have been used to encrypt others

```
lemma servK_not_AKcryptSK:
 "⟦ Crypt (shrK B) ⦃Agent A, Agent B, Key SK, Number Ts⦄ ∈ parts (spies evs);
     Key SK ∉ analz (spies evs);  SK ∈ symKeys;
     B ≠ Tgs;  evs ∈ kerbIV_gets ⟧
  ⟹ ¬ AKcryptSK SK K evs"
⟨proof⟩
```

Long term keys are not issued as servKeys

```
lemma shrK_not_AKcryptSK:
     "evs ∈ kerbIV_gets ⟹ ¬ AKcryptSK K (shrK A) evs"
⟨proof⟩
```

The Tgs message associates servK with authK and therefore not with any other key authK.

```
lemma Says_Tgs_AKcryptSK:
     "⟦ Says Tgs A (Crypt authK ⦃Key servK, Agent B, Number Ts, X ⦄)
           ∈ set evs;
         authK' ≠ authK;  evs ∈ kerbIV_gets ⟧
      ⟹ ¬ AKcryptSK authK' servK evs"
⟨proof⟩
```

Equivalently

```
lemma not_different_AKcryptSK:
     "⟦ AKcryptSK authK servK evs;
        authK' ≠ authK;  evs ∈ kerbIV_gets ⟧
      ⟹ ¬ AKcryptSK authK' servK evs  ∧ servK ∈ symKeys"
⟨proof⟩
```

```
lemma AKcryptSK_not_AKcryptSK:
     "⟦ AKcryptSK authK servK evs;  evs ∈ kerbIV_gets ⟧
      ⟹ ¬ AKcryptSK servK K evs"
⟨proof⟩
```

The only session keys that can be found with the help of session keys are those sent by Tgs in step K4.

We take some pains to express the property as a logical equivalence so that the simplifier can apply it.

```
lemma Key_analz_image_Key_lemma:
     "P ⟶ (Key K ∈ analz (Key`KK ∪ H)) ⟶ (K ∈ KK | Key K ∈ analz H)
      ⟹
      P ⟶ (Key K ∈ analz (Key`KK ∪ H)) = (K ∈ KK | Key K ∈ analz H)"
⟨proof⟩
```

```
lemma AKcryptSK_analz_insert:
     "⟦ AKcryptSK K K' evs; K ∈ symKeys; evs ∈ kerbIV_gets ⟧
      ⟹ Key K' ∈ analz (insert (Key K) (spies evs))"
⟨proof⟩
```

```
lemma authKeys_are_not_AKcryptSK:
     "⟦ K ∈ authKeys evs ∪ range shrK;  evs ∈ kerbIV_gets ⟧
      ⟹ ∀SK. ¬ AKcryptSK SK K evs ∧ K ∈ symKeys"
⟨proof⟩
```

```
lemma not_authKeys_not_AKcryptSK:
     "⟦ K ∉ authKeys evs;
         K ∉ range shrK; evs ∈ kerbIV_gets ⟧
      ⟹ ∀SK. ¬ AKcryptSK K SK evs"
⟨proof⟩
```

7.9 Secrecy Theorems

For the Oops2 case of the next theorem

```
lemma Oops2_not_AKcryptSK:
     "⟦ evs ∈ kerbIV_gets;
         Says Tgs A (Crypt authK
                     {|Key servK, Agent B, Number Ts, servTicket|})
           ∈ set evs ⟧
      ⟹ ¬ AKcryptSK servK SK evs"
⟨proof⟩
```

Big simplification law for keys SK that are not crypted by keys in KK It helps prove three, otherwise hard, facts about keys. These facts are exploited as simplification laws for analz, and also "limit the damage" in case of loss of a key to the spy. See ESORICS98.

```
lemma Key_analz_image_Key [rule_format (no_asm)]:
     "evs ∈ kerbIV_gets ⟹
      (∀SK KK. SK ∈ symKeys ∧ KK ⊆ -(range shrK) ⟶
       (∀K ∈ KK. ¬ AKcryptSK K SK evs)   ⟶
       (Key SK ∈ analz (Key`KK ∪ (spies evs))) =
       (SK ∈ KK | Key SK ∈ analz (spies evs)))"
⟨proof⟩
```

First simplification law for analz: no session keys encrypt authentication keys or shared keys.

```
lemma analz_insert_freshK1:
     "⟦ evs ∈ kerbIV_gets;  K ∈ authKeys evs ∪ range shrK;
```

```
        SesKey ∉ range shrK ⟧
      ⟹ (Key K ∈ analz (insert (Key SesKey) (spies evs))) =
          (K = SesKey | Key K ∈ analz (spies evs))"
```
⟨*proof*⟩

Second simplification law for analz: no service keys encrypt any other keys.

lemma *analz_insert_freshK2:*
```
     "⟦ evs ∈ kerbIV_gets;  servK ∉ (authKeys evs); servK ∉ range shrK;
        K ∈ symKeys ⟧
      ⟹ (Key K ∈ analz (insert (Key servK) (spies evs))) =
          (K = servK | Key K ∈ analz (spies evs))"
```
⟨*proof*⟩

Third simplification law for analz: only one authentication key encrypts a certain service key.

lemma *analz_insert_freshK3:*
```
 "⟦ AKcryptSK authK servK evs;
    authK' ≠ authK; authK' ∉ range shrK; evs ∈ kerbIV_gets ⟧
        ⟹ (Key servK ∈ analz (insert (Key authK') (spies evs))) =
                (servK = authK' | Key servK ∈ analz (spies evs))"
```
⟨*proof*⟩

lemma *analz_insert_freshK3_bis:*
```
 "⟦ Says Tgs A
            (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄)
        ∈ set evs;
     authK ≠ authK'; authK' ∉ range shrK; evs ∈ kerbIV_gets ⟧
        ⟹ (Key servK ∈ analz (insert (Key authK') (spies evs))) =
                (servK = authK' | Key servK ∈ analz (spies evs))"
```
⟨*proof*⟩

a weakness of the protocol

lemma *authK_compromises_servK:*
```
     "⟦ Says Tgs A
              (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄)
           ∈ set evs;  authK ∈ symKeys;
         Key authK ∈ analz (spies evs); evs ∈ kerbIV_gets ⟧
      ⟹ Key servK ∈ analz (spies evs)"
```
⟨*proof*⟩

lemma *servK_notin_authKeysD:*
```
     "⟦ Crypt authK ⦃Key servK, Agent B, Ts,
                    Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Ts⦄⦄
           ∈ parts (spies evs);
         Key servK ∉ analz (spies evs);
         B ≠ Tgs; evs ∈ kerbIV_gets ⟧
      ⟹ servK ∉ authKeys evs"
```
⟨*proof*⟩

If Spy sees the Authentication Key sent in msg K2, then the Key has expired.

lemma *Confidentiality_Kas_lemma [rule_format]:*
```
     "⟦ authK ∈ symKeys; A ∉ bad;  evs ∈ kerbIV_gets ⟧
      ⟹ Says Kas A
```

```
            (Crypt (shrK A)
               {|Key authK, Agent Tgs, Number Ta,
         Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Number Ta|}|})
           ∈ set evs ⟶
         Key authK ∈ analz (spies evs) ⟶
         expiredAK Ta evs"
```

⟨proof⟩

```
lemma Confidentiality_Kas:
     "⟦ Says Kas A
            (Crypt Ka {|Key authK, Agent Tgs, Number Ta, authTicket|})
          ∈ set evs;
        ¬ expiredAK Ta evs;
        A ∉ bad;  evs ∈ kerbIV_gets ⟧
      ⟹ Key authK ∉ analz (spies evs)"
```

⟨proof⟩

If Spy sees the Service Key sent in msg K4, then the Key has expired.

```
lemma Confidentiality_lemma [rule_format]:
     "⟦ Says Tgs A
          (Crypt authK
             {|Key servK, Agent B, Number Ts,
               Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|}|})
          ∈ set evs;
       Key authK ∉ analz (spies evs);
       servK ∈ symKeys;
       A ∉ bad;  B ∉ bad; evs ∈ kerbIV_gets ⟧
      ⟹ Key servK ∈ analz (spies evs) ⟶
         expiredSK Ts evs"
```

⟨proof⟩

In the real world Tgs can't check wheter authK is secure!

```
lemma Confidentiality_Tgs:
     "⟦ Says Tgs A
            (Crypt authK {|Key servK, Agent B, Number Ts, servTicket|})
          ∈ set evs;
        Key authK ∉ analz (spies evs);
        ¬ expiredSK Ts evs;
        A ∉ bad;  B ∉ bad; evs ∈ kerbIV_gets ⟧
      ⟹ Key servK ∉ analz (spies evs)"
```

⟨proof⟩

In the real world Tgs CAN check what Kas sends!

```
lemma Confidentiality_Tgs_bis:
     "⟦ Says Kas A
             (Crypt Ka {|Key authK, Agent Tgs, Number Ta, authTicket|})
          ∈ set evs;
        Says Tgs A
            (Crypt authK {|Key servK, Agent B, Number Ts, servTicket|})
          ∈ set evs;
        ¬ expiredAK Ta evs; ¬ expiredSK Ts evs;
        A ∉ bad;  B ∉ bad; evs ∈ kerbIV_gets ⟧
      ⟹ Key servK ∉ analz (spies evs)"
```

⟨*proof*⟩

Most general form

```
lemmas Confidentiality_Tgs_ter = authTicket_authentic [THEN Confidentiality_Tgs_bis]
```

```
lemmas Confidentiality_Auth_A = authK_authentic [THEN Confidentiality_Kas]
```

Needs a confidentiality guarantee, hence moved here. Authenticity of servK for A

```
lemma servK_authentic_bis_r:
     "⟦ Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta, authTicket⦄
          ∈ parts (spies evs);
        Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄
          ∈ parts (spies evs);
        ¬ expiredAK Ta evs; A ∉ bad; evs ∈ kerbIV_gets ⟧
 ⟹Says Tgs A (Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄)
       ∈ set evs"
```
⟨*proof*⟩

```
lemma Confidentiality_Serv_A:
     "⟦ Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta, authTicket⦄
          ∈ parts (spies evs);
        Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄
          ∈ parts (spies evs);
        ¬ expiredAK Ta evs; ¬ expiredSK Ts evs;
        A ∉ bad;  B ∉ bad; evs ∈ kerbIV_gets ⟧
      ⟹ Key servK ∉ analz (spies evs)"
```
⟨*proof*⟩

```
lemma u_Confidentiality_B:
     "⟦ Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
          ∈ parts (spies evs);
        ¬ expiredSK Ts evs;
        A ∉ bad;  B ∉ bad;  B ≠ Tgs; evs ∈ kerbIV_gets ⟧
      ⟹ Key servK ∉ analz (spies evs)"
```
⟨*proof*⟩

7.10 2. Parties' strong authentication: non-injective agreement on the session key. The same guarantees also express key distribution, hence their names

Authentication here still is weak agreement - of B with A

```
lemma A_authenticates_B:
     "⟦ Crypt servK (Number T3) ∈ parts (spies evs);
        Crypt authK ⦃Key servK, Agent B, Number Ts, servTicket⦄
          ∈ parts (spies evs);
        Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta, authTicket⦄
          ∈ parts (spies evs);
        Key authK ∉ analz (spies evs); Key servK ∉ analz (spies evs);
        A ∉ bad;  B ∉ bad; evs ∈ kerbIV_gets ⟧
```

```
      ⟹ Says B A (Crypt servK (Number T3)) ∈ set evs"
```
⟨proof⟩

lemma *shrK_in_initState_Server[iff]: "Key (shrK A) ∈ initState Kas"*
⟨proof⟩

lemma *shrK_in_knows_Server [iff]: "Key (shrK A) ∈ knows Kas evs"*
⟨proof⟩

lemma *A_authenticates_and_keydist_to_Kas:*
```
   "⟦ Gets A (Crypt (shrK A) {|Key authK, Peer, Ta, authTicket|}) ∈ set evs;
      A ∉ bad;  evs ∈ kerbIV_gets ⟧
  ⟹ Says Kas A (Crypt (shrK A) {|Key authK, Peer, Ta, authTicket|}) ∈ set
evs
  ∧ Key authK ∈ analz(knows Kas evs)"
```
⟨proof⟩

lemma *K3_imp_Gets_evs:*
```
  "⟦ Says A Tgs {|Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Number Ta|},
                 Crypt authK {|Agent A, Number T2|}, Agent B|}
      ∈ set evs;  A ∉ bad; evs ∈ kerbIV_gets ⟧
 ⟹  Gets A (Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta,
                 Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Number Ta|}|})
       ∈ set evs"
```
⟨proof⟩

lemma *Tgs_authenticates_and_keydist_to_A:*
```
  "⟦  Gets Tgs {|
          Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Number Ta|},
          Crypt authK {|Agent A, Number T2|}, Agent B |} ∈ set evs;
      Key authK ∉ analz (spies evs); A ∉ bad; evs ∈ kerbIV_gets ⟧
 ⟹ ∃ B. Says A Tgs {|
          Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Number Ta|},
          Crypt authK {|Agent A, Number T2|}, Agent B |} ∈ set evs
 ∧  Key authK ∈ analz (knows A evs)"
```
⟨proof⟩

lemma *K4_imp_Gets:*
```
  "⟦ Says Tgs A (Crypt authK {|Key servK, Agent B, Number Ts, servTicket|})
       ∈ set evs; evs ∈ kerbIV_gets ⟧
 ⟹ ∃ Ta X.
     Gets Tgs {|Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Number Ta|},
X|}
       ∈ set evs"
```
⟨proof⟩

lemma *A_authenticates_and_keydist_to_Tgs:*
```
 "⟦  Gets A (Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta, authTicket|})
       ∈ set evs;
     Gets A (Crypt authK {|Key servK, Agent B, Number Ts, servTicket|})
```

```
      ∈ set evs;
     Key authK ∉ analz (spies evs); A ∉ bad;
     evs ∈ kerbIV_gets ⟧
 ⟹ Says Tgs A (Crypt authK {|Key servK, Agent B, Number Ts, servTicket|})
      ∈ set evs
  ∧ Key authK ∈ analz (knows Tgs evs)
  ∧ Key servK ∈ analz (knows Tgs evs)"
```
⟨proof⟩

lemma *K5_imp_Gets:*
```
  "⟦ Says A B {|servTicket, Crypt servK {|Agent A, Number T3|}|} ∈ set evs;
    A ∉ bad; evs ∈ kerbIV_gets ⟧
 ⟹ ∃ authK Ts authTicket T2.
    Gets A (Crypt authK {|Key servK, Agent B, Number Ts, servTicket|}) ∈ set
evs
 ∧ Says A Tgs {|authTicket, Crypt authK {|Agent A, Number T2|}, Agent B|}  ∈
set evs"
```
⟨proof⟩

lemma *K3_imp_Gets:*
```
  "⟦ Says A Tgs {|authTicket, Crypt authK {|Agent A, Number T2|}, Agent B|}
      ∈ set evs;
    A ∉ bad; evs ∈ kerbIV_gets ⟧
 ⟹ ∃ Ta. Gets A (Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta, authTicket|})
∈ set evs"
```
⟨proof⟩

lemma *B_authenticates_and_keydist_to_A:*
```
     "⟦ Gets B {|Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|},
                Crypt servK {|Agent A, Number T3|}|} ∈ set evs;
        Key servK ∉ analz (spies evs);
        A ∉ bad; B ∉ bad; B ≠ Tgs; evs ∈ kerbIV_gets ⟧
 ⟹ Says A B {|Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|},
               Crypt servK {|Agent A, Number T3|}|} ∈ set evs
  ∧ Key servK ∈ analz (knows A evs)"
```
⟨proof⟩

lemma *K6_imp_Gets:*
```
  "⟦ Says B A (Crypt servK (Number T3)) ∈ set evs;
     B ∉ bad; evs ∈ kerbIV_gets ⟧
⟹ ∃ Ts X. Gets B {|Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|},X|}
      ∈ set evs"
```
⟨proof⟩

lemma *A_authenticates_and_keydist_to_B:*
```
  "⟦ Gets A {|Crypt authK {|Key servK, Agent B, Number Ts, servTicket|},
            Crypt servK (Number T3)|} ∈ set evs;
     Gets A (Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta, authTicket|})
          ∈ set evs;
     Key authK ∉ analz (spies evs); Key servK ∉ analz (spies evs);
     A ∉ bad;  B ∉ bad; evs ∈ kerbIV_gets ⟧
 ⟹ Says B A (Crypt servK (Number T3)) ∈ set evs
```

```
  ∧ Key servK ∈ analz (knows B evs)"
```
⟨*proof*⟩

end

8 The Kerberos Protocol, Version V

theory *KerberosV* **imports** *Public* **begin**

The "u" prefix indicates theorems referring to an updated version of the protocol. The "r" suffix indicates theorems where the confidentiality assumptions are relaxed by the corresponding arguments.

abbreviation
```
  Kas :: agent where
  "Kas == Server"
```

abbreviation
```
  Tgs :: agent where
  "Tgs == Friend 0"
```

axiomatization where
```
  Tgs_not_bad [iff]: "Tgs ∉ bad"
```
— Tgs is secure — we already know that Kas is secure

definition

```
    authKeys :: "event list ⇒ key set" where
    "authKeys evs = {authK. ∃A Peer Ta.
        Says Kas A ⦃Crypt (shrK A) ⦃Key authK, Agent Peer, Ta⦄,
                     Crypt (shrK Peer) ⦃Agent A, Agent Peer, Key authK, Ta⦄
                   ⦄ ∈ set evs}"
```

definition

```
  Issues :: "[agent, agent, msg, event list] ⇒ bool"
            (‹_ Issues _ with _ on _›) where
   "A Issues B with X on evs =
      (∃Y. Says A B Y ∈ set evs ∧ X ∈ parts {Y} ∧
        X ∉ parts (spies (takeWhile (λz. z  ≠ Says A B Y) (rev evs))))"
```

consts

```
    authKlife   :: nat

    servKlife   :: nat

    authlife   :: nat
```

```
  replylife   :: nat

specification (authKlife)
  authKlife_LB [iff]: "2 ≤ authKlife"
    ⟨proof⟩

specification (servKlife)
  servKlife_LB [iff]: "2 + authKlife ≤ servKlife"
    ⟨proof⟩

specification (authlife)
  authlife_LB [iff]: "Suc 0 ≤ authlife"
    ⟨proof⟩

specification (replylife)
  replylife_LB [iff]: "Suc 0 ≤ replylife"
    ⟨proof⟩

abbreviation

  CT :: "event list ⇒ nat" where
  "CT == length"

abbreviation
  expiredAK :: "[nat, event list] ⇒ bool" where
  "expiredAK T evs == authKlife + T < CT evs"

abbreviation
  expiredSK :: "[nat, event list] ⇒ bool" where
  "expiredSK T evs == servKlife + T < CT evs"

abbreviation
  expiredA :: "[nat, event list] ⇒ bool" where
  "expiredA T evs == authlife + T < CT evs"

abbreviation
  valid :: "[nat, nat] ⇒ bool"  (‹valid _ wrt _›) where
  "valid T1 wrt T2 == T1 ≤ replylife + T2"

definition AKcryptSK :: "[key, key, event list] ⇒ bool" where
  "AKcryptSK authK servK evs ==
     ∃A B tt.
       Says Tgs A {|Crypt authK {|Key servK, Agent B, tt|},
                   Crypt (shrK B) {|Agent A, Agent B, Key servK, tt|} |}
         ∈ set evs"

inductive_set kerbV :: "event list set"
  where

   Nil:  "[] ∈ kerbV"
```

```
| Fake: "⟦ evsf ∈ kerbV;  X ∈ synth (analz (spies evsf)) ⟧
        ⟹ Says Spy B X  # evsf ∈ kerbV"



| KV1:   "⟦ evs1 ∈ kerbV ⟧
        ⟹ Says A Kas {|Agent A, Agent Tgs, Number (CT evs1)|} # evs1
        ∈ kerbV"

| KV2:  "⟦ evs2 ∈ kerbV; Key authK ∉ used evs2; authK ∈ symKeys;
          Says A' Kas {|Agent A, Agent Tgs, Number T1|} ∈ set evs2 ⟧
        ⟹ Says Kas A {|
        Crypt (shrK A) {|Key authK, Agent Tgs, Number (CT evs2)|},
      Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Number (CT evs2)|}

                      |} # evs2 ∈ kerbV"



| KV3:  "⟦ evs3 ∈ kerbV; A ≠ Kas; A ≠ Tgs;
          Says A Kas {|Agent A, Agent Tgs, Number T1|} ∈ set evs3;
          Says Kas' A {|Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta|},
                        authTicket|} ∈ set evs3;
          valid Ta wrt T1
       ⟧
        ⟹ Says A Tgs {|authTicket,
                        (Crypt authK {|Agent A, Number (CT evs3)|}),
                        Agent B|} # evs3 ∈ kerbV"

| KV4:  "⟦ evs4 ∈ kerbV; Key servK ∉ used evs4; servK ∈ symKeys;
          B ≠ Tgs;  authK ∈ symKeys;
          Says A' Tgs {|
           (Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK,
                               Number Ta|}),
           (Crypt authK {|Agent A, Number T2|}), Agent B|}
              ∈ set evs4;
          ¬ expiredAK Ta evs4;
          ¬ expiredA T2 evs4;
          servKlife + (CT evs4) ≤ authKlife + Ta
       ⟧
        ⟹ Says Tgs A {|
           Crypt authK {|Key servK, Agent B, Number (CT evs4)|},
  Crypt (shrK B) {|Agent A, Agent B, Key servK, Number (CT evs4)|}
                        |} # evs4 ∈ kerbV"



| KV5:  "⟦ evs5 ∈ kerbV; authK ∈ symKeys; servK ∈ symKeys;
          A ≠ Kas; A ≠ Tgs;
          Says A Tgs
              {|authTicket, Crypt authK {|Agent A, Number T2|},
                Agent B|}
            ∈ set evs5;
```

```
          Says Tgs' A {|Crypt authK {|Key servK, Agent B, Number Ts|},
                        servTicket|}
              ∈ set evs5;
          valid Ts wrt T2 ]]
        ==> Says A B {|servTicket,
                       Crypt servK {|Agent A, Number (CT evs5)|} |}
             # evs5 ∈ kerbV"

 | KV6:  "[[ evs6 ∈ kerbV; B ≠ Kas; B ≠ Tgs;
          Says A' B {|
            (Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|}),
            (Crypt servK {|Agent A, Number T3|})|}
          ∈ set evs6;
          ¬ expiredSK Ts evs6;
          ¬ expiredA T3 evs6
        ]]
        ==> Says B A (Crypt servK (Number Ta2))
             # evs6 ∈ kerbV"



| Oops1:"[[ evsO1 ∈ kerbV;  A ≠ Spy;
           Says Kas A {|Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta|},
                        authTicket|}  ∈ set evsO1;
            expiredAK Ta evsO1 ]]
        ==> Notes Spy {|Agent A, Agent Tgs, Number Ta, Key authK|}
             # evsO1 ∈ kerbV"


| Oops2: "[[ evsO2 ∈ kerbV;  A ≠ Spy;
            Says Tgs A {|Crypt authK {|Key servK, Agent B, Number Ts|},
                         servTicket|}  ∈ set evsO2;
            expiredSK Ts evsO2 ]]
        ==> Notes Spy {|Agent A, Agent B, Number Ts, Key servK|}
             # evsO2 ∈ kerbV"
```

declare *Says_imp_knows_Spy [THEN parts.Inj, dest]*
declare *parts.Body [dest]*
declare *analz_into_parts [dest]*
declare *Fake_parts_insert_in_Un [dest]*

8.1 Lemmas about lists, for reasoning about Issues

lemma *spies_Says_rev: "spies (evs @ [Says A B X]) = insert X (spies evs)"*
⟨*proof*⟩

lemma *spies_Gets_rev: "spies (evs @ [Gets A X]) = spies evs"*
⟨*proof*⟩

lemma *spies_Notes_rev: "spies (evs @ [Notes A X]) =*
(if A∈bad then insert X (spies evs) else spies evs)"

⟨proof⟩

lemma *spies_evs_rev: "spies evs = spies (rev evs)"*
⟨proof⟩

lemmas *parts_spies_evs_revD2 = spies_evs_rev [THEN equalityD2, THEN parts_mono]*

lemma *spies_takeWhile: "spies (takeWhile P evs) ⊆ spies evs"*
⟨proof⟩

lemmas *parts_spies_takeWhile_mono = spies_takeWhile [THEN parts_mono]*

8.2 Lemmas about *authKeys*

lemma *authKeys_empty: "authKeys [] = {}"*
⟨proof⟩

```
lemma authKeys_not_insert:
 "(∀A Ta akey Peer.
   ev ≠ Says Kas A {|Crypt (shrK A) {|akey, Agent Peer, Ta|},
                     Crypt (shrK Peer) {|Agent A, Agent Peer, akey, Ta|} |})
       ⟹ authKeys (ev # evs) = authKeys evs"
```
⟨proof⟩

```
lemma authKeys_insert:
  "authKeys
     (Says Kas A {|Crypt (shrK A) {|Key K, Agent Peer, Number Ta|},
         Crypt (shrK Peer) {|Agent A, Agent Peer, Key K, Number Ta|} |} # evs)
       = insert K (authKeys evs)"
```
⟨proof⟩

```
lemma authKeys_simp:
   "K ∈ authKeys
    (Says Kas A {|Crypt (shrK A) {|Key K', Agent Peer, Number Ta|},
        Crypt (shrK Peer) {|Agent A, Agent Peer, Key K', Number Ta|} |} # evs)
        ⟹ K = K' | K ∈ authKeys evs"
```
⟨proof⟩

```
lemma authKeysI:
   "Says Kas A {|Crypt (shrK A) {|Key K, Agent Tgs, Number Ta|},
         Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key K, Number Ta|} |} ∈ set evs
        ⟹ K ∈ authKeys evs"
```
⟨proof⟩

lemma *authKeys_used: "K ∈ authKeys evs ⟹ Key K ∈ used evs"*
⟨proof⟩

8.3 Forwarding Lemmas

```
lemma Says_ticket_parts:
     "Says S A {|Crypt K {|SesKey, B, TimeStamp|}, Ticket|}
               ∈ set evs ⟹ Ticket ∈ parts (spies evs)"
```
⟨proof⟩

```
lemma Says_ticket_analz:
     "Says S A {|Crypt K {|SesKey, B, TimeStamp|}, Ticket|}
               ∈ set evs ⟹ Ticket ∈ analz (spies evs)"
⟨proof⟩

lemma Oops_range_spies1:
     "⟦ Says Kas A {|Crypt KeyA {|Key authK, Peer, Ta|}, authTicket|}
           ∈ set evs ;
         evs ∈ kerbV ⟧ ⟹ authK ∉ range shrK ∧ authK ∈ symKeys"
⟨proof⟩

lemma Oops_range_spies2:
     "⟦ Says Tgs A {|Crypt authK {|Key servK, Agent B, Ts|}, servTicket|}
           ∈ set evs ;
         evs ∈ kerbV ⟧ ⟹ servK ∉ range shrK ∧ servK ∈ symKeys"
⟨proof⟩


lemma Spy_see_shrK [simp]:
     "evs ∈ kerbV ⟹ (Key (shrK A) ∈ parts (spies evs)) = (A ∈ bad)"
⟨proof⟩

lemma Spy_analz_shrK [simp]:
     "evs ∈ kerbV ⟹ (Key (shrK A) ∈ analz (spies evs)) = (A ∈ bad)"
⟨proof⟩

lemma Spy_see_shrK_D [dest!]:
     "⟦ Key (shrK A) ∈ parts (spies evs);  evs ∈ kerbV ⟧ ⟹ A∈bad"
⟨proof⟩

lemmas Spy_analz_shrK_D = analz_subset_parts [THEN subsetD, THEN Spy_see_shrK_D,
dest!]
```

Nobody can have used non-existent keys!

```
lemma new_keys_not_used [simp]:
    "⟦Key K ∉ used evs; K ∈ symKeys; evs ∈ kerbV⟧
     ⟹ K ∉ keysFor (parts (spies evs))"
⟨proof⟩


lemma new_keys_not_analzd:
 "⟦evs ∈ kerbV; K ∈ symKeys; Key K ∉ used evs⟧
  ⟹ K ∉ keysFor (analz (spies evs))"
⟨proof⟩
```

8.4 Regularity Lemmas

These concern the form of items passed in messages

Describes the form of all components sent by Kas

```
lemma Says_Kas_message_form:
     "⟦ Says Kas A {|Crypt K {|Key authK, Agent Peer, Ta|}, authTicket|}
```

```
         ∈ set evs;
       evs ∈ kerbV ⟧
    ⟹ authK ∉ range shrK ∧ authK ∈ authKeys evs ∧ authK ∈ symKeys ∧

  authTicket = (Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Ta|}) ∧
             K = shrK A  ∧ Peer = Tgs"
```
⟨proof⟩

lemma *SesKey_is_session_key:*
```
     "⟦ Crypt (shrK Tgs_B) {|Agent A, Agent Tgs_B, Key SesKey, Number T|}
          ∈ parts (spies evs); Tgs_B ∉ bad;
       evs ∈ kerbV ⟧
    ⟹ SesKey ∉ range shrK"
```
⟨proof⟩

lemma *authTicket_authentic:*
```
     "⟦ Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Ta|}
          ∈ parts (spies evs);
       evs ∈ kerbV ⟧
    ⟹ Says Kas A {|Crypt (shrK A) {|Key authK, Agent Tgs, Ta|},
              Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Ta|}|}
          ∈ set evs"
```
⟨proof⟩

lemma *authTicket_crypt_authK:*
```
     "⟦ Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key authK, Number Ta|}
          ∈ parts (spies evs);
       evs ∈ kerbV ⟧
    ⟹ authK ∈ authKeys evs"
```
⟨proof⟩

Describes the form of servK, servTicket and authK sent by Tgs

lemma *Says_Tgs_message_form:*
```
     "⟦ Says Tgs A {|Crypt authK {|Key servK, Agent B, Ts|}, servTicket|}
          ∈ set evs;
       evs ∈ kerbV ⟧
  ⟹ B ≠ Tgs ∧
      servK ∉ range shrK ∧ servK ∉ authKeys evs ∧ servK ∈ symKeys ∧
      servTicket = (Crypt (shrK B) {|Agent A, Agent B, Key servK, Ts|}) ∧
      authK ∉ range shrK ∧ authK ∈ authKeys evs ∧ authK ∈ symKeys"
```
⟨proof⟩

8.5 Authenticity theorems: confirm origin of sensitive messages

lemma *authK_authentic:*
```
     "⟦ Crypt (shrK A) {|Key authK, Peer, Ta|}
          ∈ parts (spies evs);
       A ∉ bad;  evs ∈ kerbV ⟧
    ⟹ ∃ AT. Says Kas A {|Crypt (shrK A) {|Key authK, Peer, Ta|}, AT|}
          ∈ set evs"
```

⟨*proof*⟩

If a certain encrypted message appears then it originated with Tgs

lemma *servK_authentic:*
```
     "⟦ Crypt authK ⦃Key servK, Agent B, Ts⦄
           ∈ parts (spies evs);
         Key authK ∉ analz (spies evs);
         authK ∉ range shrK;
         evs ∈ kerbV ⟧
 ⟹ ∃A ST. Says Tgs A ⦃Crypt authK ⦃Key servK, Agent B, Ts⦄, ST⦄
       ∈ set evs"
```
⟨*proof*⟩

lemma *servK_authentic_bis:*
```
     "⟦ Crypt authK ⦃Key servK, Agent B, Ts⦄
           ∈ parts (spies evs);
         Key authK ∉ analz (spies evs);
         B ≠ Tgs;
         evs ∈ kerbV ⟧
 ⟹ ∃A ST. Says Tgs A ⦃Crypt authK ⦃Key servK, Agent B, Ts⦄, ST⦄
       ∈ set evs"
```
⟨*proof*⟩

Authenticity of servK for B

lemma *servTicket_authentic_Tgs:*
```
     "⟦ Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Ts⦄
           ∈ parts (spies evs);  B ≠ Tgs;  B ∉ bad;
         evs ∈ kerbV ⟧
 ⟹ ∃authK.
       Says Tgs A ⦃Crypt authK ⦃Key servK, Agent B, Ts⦄,
                     Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Ts⦄⦄
       ∈ set evs"
```
⟨*proof*⟩

Anticipated here from next subsection

lemma *K4_imp_K2:*
```
"⟦ Says Tgs A ⦃Crypt authK ⦃Key servK, Agent B, Number Ts⦄, servTicket⦄
      ∈ set evs;  evs ∈ kerbV⟧
   ⟹ ∃Ta. Says Kas A
        ⦃Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta⦄,
           Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number Ta⦄ ⦄
        ∈ set evs"
```
⟨*proof*⟩

Anticipated here from next subsection

lemma *u_K4_imp_K2:*
```
"⟦ Says Tgs A ⦃Crypt authK ⦃Key servK, Agent B, Number Ts⦄, servTicket⦄  ∈
set evs; evs ∈ kerbV⟧
   ⟹ ∃Ta. Says Kas A ⦃Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta⦄,
             Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number Ta⦄ ⦄
             ∈ set evs
          ∧ servKlife + Ts ≤ authKlife + Ta"
```
⟨*proof*⟩

lemma *servTicket_authentic_Kas:*
```
    "⟦ Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
          ∈ parts (spies evs);  B ≠ Tgs;  B ∉ bad;
        evs ∈ kerbV ⟧
  ⟹ ∃authK Ta.
       Says Kas A
         ⦃Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta⦄,
           Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number Ta⦄ ⦄
        ∈ set evs"
```
⟨*proof*⟩

lemma *u_servTicket_authentic_Kas:*
```
    "⟦ Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
          ∈ parts (spies evs);  B ≠ Tgs;  B ∉ bad;
        evs ∈ kerbV ⟧
  ⟹ ∃authK Ta.
       Says Kas A
         ⦃Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta⦄,
           Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number Ta⦄ ⦄
        ∈ set evs ∧
      servKlife + Ts ≤ authKlife + Ta"
```
⟨*proof*⟩

lemma *servTicket_authentic:*
```
    "⟦ Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
          ∈ parts (spies evs);  B ≠ Tgs;  B ∉ bad;
        evs ∈ kerbV ⟧
 ⟹ ∃Ta authK.
    Says Kas A ⦃Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta⦄,
                 Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number
Ta⦄ ⦄  ∈ set evs
    ∧ Says Tgs A ⦃Crypt authK ⦃Key servK, Agent B, Number Ts⦄,
                 Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄⦄
      ∈ set evs"
```
⟨*proof*⟩

lemma *u_servTicket_authentic:*
```
    "⟦ Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
          ∈ parts (spies evs);  B ≠ Tgs;  B ∉ bad;
        evs ∈ kerbV ⟧
 ⟹ ∃Ta authK.
    Says Kas A ⦃Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta⦄,
                 Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number
Ta⦄⦄ ∈ set evs
    ∧ Says Tgs A ⦃Crypt authK ⦃Key servK, Agent B, Number Ts⦄,
               Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄⦄
      ∈ set evs
    ∧ servKlife + Ts ≤ authKlife + Ta"
```
⟨*proof*⟩

lemma *u_NotexpiredSK_NotexpiredAK:*
```
    "⟦ ¬ expiredSK Ts evs; servKlife + Ts ≤ authKlife + Ta ⟧
     ⟹ ¬ expiredAK Ta evs"
```

⟨proof⟩

8.6 Reliability: friendly agents send something if something else happened

```
lemma K3_imp_K2:
     "⟦ Says A Tgs
             ⦃authTicket, Crypt authK ⦃Agent A, Number T2⦄, Agent B⦄
           ∈ set evs;
         A ∉ bad;  evs ∈ kerbV ⟧
      ⟹ ∃Ta AT. Says Kas A ⦃Crypt (shrK A) ⦃Key authK, Agent Tgs, Ta⦄,

                                   AT⦄ ∈ set evs"
```
⟨proof⟩

Anticipated here from next subsection. An authK is encrypted by one and only one Shared key. A servK is encrypted by one and only one authK.

```
lemma Key_unique_SesKey:
     "⟦ Crypt K  ⦃Key SesKey,  Agent B, T⦄
           ∈ parts (spies evs);
         Crypt K' ⦃Key SesKey,  Agent B', T'⦄
           ∈ parts (spies evs);  Key SesKey ∉ analz (spies evs);
         evs ∈ kerbV ⟧
      ⟹ K=K' ∧ B=B' ∧ T=T'"
```
⟨proof⟩

This inevitably has an existential form in version V

```
lemma Says_K5:
     "⟦ Crypt servK ⦃Agent A, Number T3⦄ ∈ parts (spies evs);
         Says Tgs A ⦃Crypt authK ⦃Key servK, Agent B, Number Ts⦄,
                                        servTicket⦄ ∈ set evs;
         Key servK ∉ analz (spies evs);
         A ∉ bad; B ∉ bad; evs ∈ kerbV ⟧
 ⟹ ∃ ST. Says A B ⦃ST, Crypt servK ⦃Agent A, Number T3⦄⦄ ∈ set evs"
```
⟨proof⟩

Anticipated here from next subsection

```
lemma unique_CryptKey:
     "⟦ Crypt (shrK B)  ⦃Agent A,  Agent B,  Key SesKey, T⦄
           ∈ parts (spies evs);
         Crypt (shrK B') ⦃Agent A', Agent B', Key SesKey, T'⦄
           ∈ parts (spies evs);  Key SesKey ∉ analz (spies evs);
         evs ∈ kerbV ⟧
      ⟹ A=A' ∧ B=B' ∧ T=T'"
```
⟨proof⟩

```
lemma Says_K6:
     "⟦ Crypt servK (Number T3) ∈ parts (spies evs);
         Says Tgs A ⦃Crypt authK ⦃Key servK, Agent B, Number Ts⦄,
                      servTicket⦄ ∈ set evs;
         Key servK ∉ analz (spies evs);
         A ∉ bad; B ∉ bad; evs ∈ kerbV ⟧
      ⟹ Says B A (Crypt servK (Number T3)) ∈ set evs"
```

⟨proof⟩

Needs a unicity theorem, hence moved here

```
lemma servK_authentic_ter:
 "⟦ Says Kas A
       ⦃Crypt (shrK A) ⦃Key authK, Agent Tgs, Ta⦄, authTicket⦄ ∈ set evs;
     Crypt authK ⦃Key servK, Agent B, Ts⦄
       ∈ parts (spies evs);
     Key authK ∉ analz (spies evs);
     evs ∈ kerbV ⟧
 ⟹ Says Tgs A ⦃Crypt authK ⦃Key servK, Agent B, Ts⦄,
                 Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Ts⦄ ⦄
       ∈ set evs"
```
⟨proof⟩

8.7 Unicity Theorems

The session key, if secure, uniquely identifies the Ticket whether authTicket or servTicket. As a matter of fact, one can read also Tgs in the place of B.

```
lemma unique_authKeys:
     "⟦ Says Kas A
              ⦃Crypt Ka ⦃Key authK, Agent Tgs, Ta⦄, X⦄ ∈ set evs;
         Says Kas A'
              ⦃Crypt Ka' ⦃Key authK, Agent Tgs, Ta'⦄, X'⦄ ∈ set evs;
         evs ∈ kerbV ⟧ ⟹ A=A' ∧ Ka=Ka' ∧ Ta=Ta' ∧ X=X'"
```
⟨proof⟩

servK uniquely identifies the message from Tgs

```
lemma unique_servKeys:
     "⟦ Says Tgs A
              ⦃Crypt K ⦃Key servK, Agent B, Ts⦄, X⦄ ∈ set evs;
         Says Tgs A'
              ⦃Crypt K' ⦃Key servK, Agent B', Ts'⦄, X'⦄ ∈ set evs;
         evs ∈ kerbV ⟧ ⟹ A=A' ∧ B=B' ∧ K=K' ∧ Ts=Ts' ∧ X=X'"
```
⟨proof⟩

8.8 Lemmas About the Predicate *AKcryptSK*

```
lemma not_AKcryptSK_Nil [iff]: "¬ AKcryptSK authK servK []"
```
⟨proof⟩

```
lemma AKcryptSKI:
 "⟦ Says Tgs A ⦃Crypt authK ⦃Key servK, Agent B, tt⦄, X ⦄ ∈ set evs;
     evs ∈ kerbV ⟧ ⟹ AKcryptSK authK servK evs"
```
⟨proof⟩

```
lemma AKcryptSK_Says [simp]:
   "AKcryptSK authK servK (Says S A X # evs) =
     (S = Tgs ∧
      (∃B tt. X = ⦃Crypt authK ⦃Key servK, Agent B, tt⦄,
                    Crypt (shrK B) ⦃Agent A, Agent B, Key servK, tt⦄ ⦄)
     | AKcryptSK authK servK evs)"
```
⟨proof⟩

```
lemma AKcryptSK_Notes [simp]:
  "AKcryptSK authK servK (Notes A X # evs) =
    AKcryptSK authK servK evs"
⟨proof⟩
```

```
lemma Auth_fresh_not_AKcryptSK:
     "⟦ Key authK ∉ used evs; evs ∈ kerbV ⟧
      ⟹ ¬ AKcryptSK authK servK evs"
⟨proof⟩
```

```
lemma Serv_fresh_not_AKcryptSK:
 "Key servK ∉ used evs ⟹ ¬ AKcryptSK authK servK evs"
⟨proof⟩
```

```
lemma authK_not_AKcryptSK:
     "⟦ Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, tk⦄
           ∈ parts (spies evs);  evs ∈ kerbV ⟧
      ⟹ ¬ AKcryptSK K authK evs"
⟨proof⟩
```

A secure serverkey cannot have been used to encrypt others

```
lemma servK_not_AKcryptSK:
 "⟦ Crypt (shrK B) ⦃Agent A, Agent B, Key SK, tt⦄ ∈ parts (spies evs);
     Key SK ∉ analz (spies evs);  SK ∈ symKeys;
     B ≠ Tgs;  evs ∈ kerbV ⟧
  ⟹ ¬ AKcryptSK SK K evs"
⟨proof⟩
```

Long term keys are not issued as servKeys

```
lemma shrK_not_AKcryptSK:
     "evs ∈ kerbV ⟹ ¬ AKcryptSK K (shrK A) evs"
⟨proof⟩
```

The Tgs message associates servK with authK and therefore not with any other key authK.

```
lemma Says_Tgs_AKcryptSK:
     "⟦ Says Tgs A ⦃Crypt authK ⦃Key servK, Agent B, tt⦄, X ⦄
           ∈ set evs;
         authK' ≠ authK;  evs ∈ kerbV ⟧
      ⟹ ¬ AKcryptSK authK' servK evs"
⟨proof⟩
```

```
lemma AKcryptSK_not_AKcryptSK:
     "⟦ AKcryptSK authK servK evs;  evs ∈ kerbV ⟧
      ⟹ ¬ AKcryptSK servK K evs"
⟨proof⟩
```

```
lemma not_different_AKcryptSK:
     "⟦ AKcryptSK authK servK evs;
        authK' ≠ authK;  evs ∈ kerbV ⟧
```

```
  ⟹ ¬ AKcryptSK authK' servK evs  ∧ servK ∈ symKeys"
```
⟨proof⟩

The only session keys that can be found with the help of session keys are those sent by Tgs in step K4.

We take some pains to express the property as a logical equivalence so that the simplifier can apply it.

lemma *Key_analz_image_Key_lemma:*
```
"P ⟶ (Key K ∈ analz (Key`KK ∪ H)) ⟶ (K∈KK ∨ Key K ∈ analz H)
 ⟹
 P ⟶ (Key K ∈ analz (Key`KK ∪ H)) = (K∈KK ∨ Key K ∈ analz H)"
```
⟨proof⟩

lemma *AKcryptSK_analz_insert:*
```
"⟦ AKcryptSK K K' evs; K ∈ symKeys; evs ∈ kerbV ⟧
 ⟹ Key K' ∈ analz (insert (Key K) (spies evs))"
```
⟨proof⟩

lemma *authKeys_are_not_AKcryptSK:*
```
"⟦ K ∈ authKeys evs ∪ range shrK;  evs ∈ kerbV ⟧
 ⟹ ∀SK. ¬ AKcryptSK SK K evs ∧ K ∈ symKeys"
```
⟨proof⟩

lemma *not_authKeys_not_AKcryptSK:*
```
"⟦ K ∉ authKeys evs;
    K ∉ range shrK; evs ∈ kerbV ⟧
 ⟹ ∀SK. ¬ AKcryptSK K SK evs"
```
⟨proof⟩

8.9 Secrecy Theorems

For the Oops2 case of the next theorem

lemma *Oops2_not_AKcryptSK:*
```
"⟦ evs ∈ kerbV;
    Says Tgs A ⦃Crypt authK
                  ⦃Key servK, Agent B, Number Ts⦄, servTicket⦄
      ∈ set evs ⟧
 ⟹ ¬ AKcryptSK servK SK evs"
```
⟨proof⟩

Big simplification law for keys SK that are not crypted by keys in KK It helps prove three, otherwise hard, facts about keys. These facts are exploited as simplification laws for analz, and also "limit the damage" in case of loss of a key to the spy. See ESORICS98.

lemma *Key_analz_image_Key [rule_format (no_asm)]:*
```
"evs ∈ kerbV ⟹
 (∀SK KK. SK ∈ symKeys ∧ KK ⊆ -(range shrK) ⟶
  (∀K ∈ KK. ¬ AKcryptSK K SK evs)   ⟶
  (Key SK ∈ analz (Key`KK ∪ (spies evs))) =
  (SK ∈ KK | Key SK ∈ analz (spies evs)))"
```
⟨proof⟩

First simplification law for analz: no session keys encrypt authentication keys or shared keys.

```
lemma analz_insert_freshK1:
     "⟦ evs ∈ kerbV;  K ∈ authKeys evs ∪ range shrK;
        SesKey ∉ range shrK ⟧
      ⟹ (Key K ∈ analz (insert (Key SesKey) (spies evs))) =
          (K = SesKey | Key K ∈ analz (spies evs))"
⟨proof⟩
```

Second simplification law for analz: no service keys encrypt any other keys.

```
lemma analz_insert_freshK2:
     "⟦ evs ∈ kerbV;  servK ∉ (authKeys evs); servK ∉ range shrK;
        K ∈ symKeys ⟧
      ⟹ (Key K ∈ analz (insert (Key servK) (spies evs))) =
          (K = servK | Key K ∈ analz (spies evs))"
⟨proof⟩
```

Third simplification law for analz: only one authentication key encrypts a certain service key.

```
lemma analz_insert_freshK3:
 "⟦ AKcryptSK authK servK evs;
    authK' ≠ authK; authK' ∉ range shrK; evs ∈ kerbV ⟧
        ⟹ (Key servK ∈ analz (insert (Key authK') (spies evs))) =
                (servK = authK' | Key servK ∈ analz (spies evs))"
⟨proof⟩
```

```
lemma analz_insert_freshK3_bis:
 "⟦ Says Tgs A ⦃Crypt authK ⦃Key servK, Agent B, Number Ts⦄, servTicket⦄
        ∈ set evs;
     authK ≠ authK'; authK' ∉ range shrK; evs ∈ kerbV ⟧
        ⟹ (Key servK ∈ analz (insert (Key authK') (spies evs))) =
                (servK = authK' | Key servK ∈ analz (spies evs))"
⟨proof⟩
```

a weakness of the protocol

```
lemma authK_compromises_servK:
     "⟦ Says Tgs A ⦃Crypt authK ⦃Key servK, Agent B, Number Ts⦄, servTicket⦄
        ∈ set evs;  authK ∈ symKeys;
         Key authK ∈ analz (spies evs); evs ∈ kerbV ⟧
      ⟹ Key servK ∈ analz (spies evs)"
  ⟨proof⟩
```

lemma `servK_notin_authKeysD` not needed in version V

If Spy sees the Authentication Key sent in msg K2, then the Key has expired.

```
lemma Confidentiality_Kas_lemma [rule_format]:
     "⟦ authK ∈ symKeys; A ∉ bad;  evs ∈ kerbV ⟧
      ⟹ Says Kas A
               ⦃Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta⦄,
          Crypt (shrK Tgs) ⦃Agent A, Agent Tgs, Key authK, Number Ta⦄⦄
            ∈ set evs ⟶
          Key authK ∈ analz (spies evs) ⟶
```

```
          expiredAK Ta evs"
```
⟨proof⟩

lemma *Confidentiality_Kas:*
```
     "⟦ Says Kas A
              {|Crypt Ka {|Key authK, Agent Tgs, Number Ta|}, authTicket|}
           ∈ set evs;
        ¬ expiredAK Ta evs;
        A ∉ bad;  evs ∈ kerbV ⟧
      ⟹ Key authK ∉ analz (spies evs)"
```
⟨proof⟩

If Spy sees the Service Key sent in msg K4, then the Key has expired.

lemma *Confidentiality_lemma [rule_format]:*
```
     "⟦ Says Tgs A
            {|Crypt authK {|Key servK, Agent B, Number Ts|},
              Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|}|}
           ∈ set evs;
        Key authK ∉ analz (spies evs);
        servK ∈ symKeys;
        A ∉ bad;  B ∉ bad; evs ∈ kerbV ⟧
      ⟹ Key servK ∈ analz (spies evs) ⟶
          expiredSK Ts evs"
```
⟨proof⟩

In the real world Tgs can't check wheter authK is secure!

lemma *Confidentiality_Tgs:*
```
     "⟦ Says Tgs A
              {|Crypt authK {|Key servK, Agent B, Number Ts|}, servTicket|}
           ∈ set evs;
         Key authK ∉ analz (spies evs);
         ¬ expiredSK Ts evs;
         A ∉ bad;  B ∉ bad; evs ∈ kerbV ⟧
      ⟹ Key servK ∉ analz (spies evs)"
```
⟨proof⟩

In the real world Tgs CAN check what Kas sends!

lemma *Confidentiality_Tgs_bis:*
```
     "⟦ Says Kas A
               {|Crypt Ka {|Key authK, Agent Tgs, Number Ta|}, authTicket|}
           ∈ set evs;
         Says Tgs A
              {|Crypt authK {|Key servK, Agent B, Number Ts|}, servTicket|}
           ∈ set evs;
         ¬ expiredAK Ta evs; ¬ expiredSK Ts evs;
         A ∉ bad;  B ∉ bad; evs ∈ kerbV ⟧
      ⟹ Key servK ∉ analz (spies evs)"
```
⟨proof⟩

Most general form

lemmas *Confidentiality_Tgs_ter = authTicket_authentic [THEN Confidentiality_Tgs_bis]*

lemmas *Confidentiality_Auth_A = authK_authentic [THEN exE, THEN Confidentiality_Kas]*

Needs a confidentiality guarantee, hence moved here. Authenticity of servK for A

```
lemma servK_authentic_bis_r:
     "⟦ Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta⦄
          ∈ parts (spies evs);
        Crypt authK ⦃Key servK, Agent B, Number Ts⦄
          ∈ parts (spies evs);
        ¬ expiredAK Ta evs; A ∉ bad; evs ∈ kerbV ⟧
 ⟹ Says Tgs A ⦃Crypt authK ⦃Key servK, Agent B, Number Ts⦄,
                Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄ ⦄
      ∈ set evs"
⟨proof⟩

lemma Confidentiality_Serv_A:
     "⟦ Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta⦄
          ∈ parts (spies evs);
        Crypt authK ⦃Key servK, Agent B, Number Ts⦄
          ∈ parts (spies evs);
        ¬ expiredAK Ta evs; ¬ expiredSK Ts evs;
        A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerbV ⟧
      ⟹ Key servK ∉ analz (spies evs)"
⟨proof⟩

lemma Confidentiality_B:
     "⟦ Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
          ∈ parts (spies evs);
        Crypt authK ⦃Key servK, Agent B, Number Ts⦄
          ∈ parts (spies evs);
        Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta⦄
          ∈ parts (spies evs);
        ¬ expiredSK Ts evs; ¬ expiredAK Ta evs;
        A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerbV ⟧
      ⟹ Key servK ∉ analz (spies evs)"
⟨proof⟩

lemma u_Confidentiality_B:
     "⟦ Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
          ∈ parts (spies evs);
        ¬ expiredSK Ts evs;
        A ∉ bad;  B ∉ bad;  B ≠ Tgs; evs ∈ kerbV ⟧
      ⟹ Key servK ∉ analz (spies evs)"
⟨proof⟩
```

8.10 Authentication

Each party verifies "the identity of another party who generated some data" (quoted from Neuman and Ts'o).

These guarantees don't assess whether two parties agree on the same session key: sending a message containing a key doesn't a priori state knowledge of the key.

These didn't have existential form in version IV

```
lemma B_authenticates_A:
     "⟦ Crypt servK {|Agent A, Number T3|} ∈ parts (spies evs);
        Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|}
           ∈ parts (spies evs);
        Key servK ∉ analz (spies evs);
        A ∉ bad; B ∉ bad; B ≠ Tgs; evs ∈ kerbV ⟧
  ⟹ ∃ ST. Says A B {|ST, Crypt servK {|Agent A, Number T3|} |} ∈ set evs"
⟨proof⟩
```

The second assumption tells B what kind of key servK is.

```
lemma B_authenticates_A_r:
     "⟦ Crypt servK {|Agent A, Number T3|} ∈ parts (spies evs);
         Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|}
           ∈ parts (spies evs);
         Crypt authK {|Key servK, Agent B, Number Ts|}
           ∈ parts (spies evs);
         Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta|}
           ∈ parts (spies evs);
         ¬ expiredSK Ts evs; ¬ expiredAK Ta evs;
         B ≠ Tgs; A ∉ bad;  B ∉ bad;  evs ∈ kerbV ⟧
  ⟹ ∃ ST. Says A B {|ST, Crypt servK {|Agent A, Number T3|} |} ∈ set evs"
⟨proof⟩
```

`u_B_authenticates_A` would be the same as `B_authenticates_A` because the servK confidentiality assumption is yet unrelaxed

```
lemma u_B_authenticates_A_r:
     "⟦ Crypt servK {|Agent A, Number T3|} ∈ parts (spies evs);
         Crypt (shrK B) {|Agent A, Agent B, Key servK, Number Ts|}
           ∈ parts (spies evs);
         ¬ expiredSK Ts evs;
         B ≠ Tgs; A ∉ bad;  B ∉ bad;  evs ∈ kerbV ⟧
  ⟹ ∃ ST. Says A B {|ST, Crypt servK {|Agent A, Number T3|} |} ∈ set evs"
⟨proof⟩
```

```
lemma A_authenticates_B:
     "⟦ Crypt servK (Number T3) ∈ parts (spies evs);
         Crypt authK {|Key servK, Agent B, Number Ts|}
           ∈ parts (spies evs);
         Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta|}
           ∈ parts (spies evs);
         Key authK ∉ analz (spies evs); Key servK ∉ analz (spies evs);
         A ∉ bad;  B ∉ bad; evs ∈ kerbV ⟧
      ⟹ Says B A (Crypt servK (Number T3)) ∈ set evs"
  ⟨proof⟩
```

```
lemma A_authenticates_B_r:
     "⟦ Crypt servK (Number T3) ∈ parts (spies evs);
         Crypt authK {|Key servK, Agent B, Number Ts|}
           ∈ parts (spies evs);
         Crypt (shrK A) {|Key authK, Agent Tgs, Number Ta|}
           ∈ parts (spies evs);
         ¬ expiredAK Ta evs; ¬ expiredSK Ts evs;
         A ∉ bad;  B ∉ bad; evs ∈ kerbV ⟧
      ⟹ Says B A (Crypt servK (Number T3)) ∈ set evs"
```

⟨proof⟩

8.11 Parties' knowledge of session keys

An agent knows a session key if he used it to issue a cipher. These guarantees can be interpreted both in terms of key distribution and of non-injective agreement on the session key.

```
lemma Kas_Issues_A:
   "⟦ Says Kas A ⦃Crypt (shrK A) ⦃Key authK, Peer, Ta⦄, authTicket⦄ ∈ set
evs;
      evs ∈ kerbV ⟧
  ⟹ Kas Issues A with (Crypt (shrK A) ⦃Key authK, Peer, Ta⦄)
          on evs"
```

⟨proof⟩

```
lemma A_authenticates_and_keydist_to_Kas:
  "⟦ Crypt (shrK A) ⦃Key authK, Peer, Ta⦄ ∈ parts (spies evs);
     A ∉ bad; evs ∈ kerbV ⟧
 ⟹ Kas Issues A with (Crypt (shrK A) ⦃Key authK, Peer, Ta⦄)
          on evs"
```

⟨proof⟩

```
lemma Tgs_Issues_A:
    "⟦ Says Tgs A ⦃Crypt authK ⦃Key servK, Agent B, Number Ts⦄, servTicket⦄
         ∈ set evs;
       Key authK ∉ analz (spies evs);  evs ∈ kerbV ⟧
  ⟹ Tgs Issues A with
          (Crypt authK ⦃Key servK, Agent B, Number Ts⦄) on evs"
```

⟨proof⟩

```
lemma A_authenticates_and_keydist_to_Tgs:
     "⟦ Crypt authK ⦃Key servK, Agent B, Number Ts⦄
          ∈ parts (spies evs);
       Key authK ∉ analz (spies evs); B ≠ Tgs; evs ∈ kerbV ⟧
  ⟹ ∃A. Tgs Issues A with
          (Crypt authK ⦃Key servK, Agent B, Number Ts⦄) on evs"
```

⟨proof⟩

```
lemma B_Issues_A:
     "⟦ Says B A (Crypt servK (Number T3)) ∈ set evs;
         Key servK ∉ analz (spies evs);
         A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerbV ⟧
      ⟹ B Issues A with (Crypt servK (Number T3)) on evs"
```

⟨proof⟩

```
lemma A_authenticates_and_keydist_to_B:
     "⟦ Crypt servK (Number T3) ∈ parts (spies evs);
         Crypt authK ⦃Key servK, Agent B, Number Ts⦄
           ∈ parts (spies evs);
         Crypt (shrK A) ⦃Key authK, Agent Tgs, Number Ta⦄
           ∈ parts (spies evs);
         Key authK ∉ analz (spies evs); Key servK ∉ analz (spies evs);
         A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerbV ⟧
```

```
    ⟹ B Issues A with (Crypt servK (Number T3)) on evs"
```
⟨proof⟩

But can prove a less general fact conerning only authenticators!

lemma *honest_never_says_newer_timestamp_in_auth:*
```
  "⟦ (CT evs) ≤ T; Number T ∈ parts {X}; A ∉ bad; evs ∈ kerbV ⟧
  ⟹ Says A B ⦃Y, X⦄ ∉ set evs"
```
⟨proof⟩

lemma *honest_never_says_current_timestamp_in_auth:*
```
  "⟦ (CT evs) = T; Number T ∈ parts {X}; A ∉ bad; evs ∈ kerbV ⟧
  ⟹ Says A B ⦃Y, X⦄ ∉ set evs"
```
⟨proof⟩

lemma *A_Issues_B:*
```
  "⟦ Says A B ⦃ST, Crypt servK ⦃Agent A, Number T3⦄⦄ ∈ set evs;
     Key servK ∉ analz (spies evs);
     B ≠ Tgs; A ∉ bad;  B ∉ bad;  evs ∈ kerbV ⟧
 ⟹ A Issues B with (Crypt servK ⦃Agent A, Number T3⦄) on evs"
```
⟨proof⟩

lemma *B_authenticates_and_keydist_to_A:*
```
  "⟦ Crypt servK ⦃Agent A, Number T3⦄ ∈ parts (spies evs);
     Crypt (shrK B) ⦃Agent A, Agent B, Key servK, Number Ts⦄
       ∈ parts (spies evs);
     Key servK ∉ analz (spies evs);
     B ≠ Tgs; A ∉ bad;  B ∉ bad;  evs ∈ kerbV ⟧
 ⟹ A Issues B with (Crypt servK ⦃Agent A, Number T3⦄) on evs"
```
⟨proof⟩

8.12 Novel guarantees, never studied before

Because honest agents always say the right timestamp in authenticators, we can prove unicity guarantees based exactly on timestamps. Classical unicity guarantees are based on nonces. Of course assuming the agent to be different from the Spy, rather than not in bad, would suffice below. Similar guarantees must also hold of Kerberos IV.

Notice that an honest agent can send the same timestamp on two different traces of the same length, but not on the same trace!

lemma *unique_timestamp_authenticator1:*
```
  "⟦ Says A Kas ⦃Agent A, Agent Tgs, Number T1⦄ ∈ set evs;
     Says A Kas' ⦃Agent A, Agent Tgs', Number T1⦄ ∈ set evs;
     A ∉bad; evs ∈ kerbV ⟧
⟹ Kas=Kas' ∧ Tgs=Tgs'"
```
⟨proof⟩

lemma *unique_timestamp_authenticator2:*
```
  "⟦ Says A Tgs ⦃AT, Crypt AK ⦃Agent A, Number T2⦄, Agent B⦄ ∈ set evs;
  Says A Tgs' ⦃AT', Crypt AK' ⦃Agent A, Number T2⦄, Agent B'⦄ ∈ set evs;
     A ∉bad; evs ∈ kerbV ⟧
⟹ Tgs=Tgs' ∧ AT=AT' ∧ AK=AK' ∧ B=B'"
```

⟨*proof*⟩

lemma *unique_timestamp_authenticator3:*
```
"⟦ Says A B {|ST, Crypt SK {|Agent A, Number T|}|} ∈ set evs;
     Says A B' {|ST', Crypt SK' {|Agent A, Number T|}|} ∈ set evs;
     A ∉bad; evs ∈ kerbV ⟧
  ⟹ B=B' ∧ ST=ST' ∧ SK=SK'"
```
⟨*proof*⟩

The second part of the message is treated as an authenticator by the last simplification step, even if it is not an authenticator!

lemma *unique_timestamp_authticket:*
```
"⟦ Says Kas A {|X, Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key AK, T|}|} ∈
set evs;
    Says Kas A' {|X', Crypt (shrK Tgs') {|Agent A', Agent Tgs', Key AK',
T|}|} ∈ set evs;
     evs ∈ kerbV ⟧
  ⟹ A=A' ∧ X=X' ∧ Tgs=Tgs' ∧ AK=AK'"
```
⟨*proof*⟩

The second part of the message is treated as an authenticator by the last simplification step, even if it is not an authenticator!

lemma *unique_timestamp_servticket:*
```
"⟦ Says Tgs A {|X, Crypt (shrK B) {|Agent A, Agent B, Key SK, T|}|} ∈ set
evs;
    Says Tgs A' {|X', Crypt (shrK B') {|Agent A', Agent B', Key SK', T|}|}
∈ set evs;
     evs ∈ kerbV ⟧
  ⟹ A=A' ∧ X=X' ∧ B=B' ∧ SK=SK'"
```
⟨*proof*⟩

lemma *Kas_never_says_newer_timestamp:*
```
"⟦ (CT evs) ≤ T; Number T ∈ parts {X}; evs ∈ kerbV ⟧
   ⟹ ∀ A. Says Kas A X ∉ set evs"
```
⟨*proof*⟩

lemma *Kas_never_says_current_timestamp:*
```
"⟦ (CT evs) = T; Number T ∈ parts {X}; evs ∈ kerbV ⟧
   ⟹ ∀ A. Says Kas A X ∉ set evs"
```
⟨*proof*⟩

lemma *unique_timestamp_msg2:*
```
"⟦ Says Kas A {|Crypt (shrK A) {|Key AK, Agent Tgs, T|}, AT|} ∈ set evs;
   Says Kas A' {|Crypt (shrK A') {|Key AK', Agent Tgs', T|}, AT'|} ∈ set evs;
     evs ∈ kerbV ⟧
  ⟹ A=A' ∧ AK=AK' ∧ Tgs=Tgs' ∧ AT=AT'"
```
⟨*proof*⟩

lemma *Tgs_never_says_newer_timestamp:*
```
"⟦ (CT evs) ≤ T; Number T ∈ parts {X}; evs ∈ kerbV ⟧
   ⟹ ∀ A. Says Tgs A X ∉ set evs"
```
⟨*proof*⟩

lemma *Tgs_never_says_current_timestamp:*
```
    "⟦ (CT evs) = T; Number T ∈ parts {X}; evs ∈ kerbV ⟧
    ⟹ ∀ A. Says Tgs A X ∉ set evs"
```
⟨*proof*⟩

lemma *unique_timestamp_msg4:*
```
    "⟦ Says Tgs A ⦃Crypt (shrK A) ⦃Key SK, Agent B, T⦄, ST⦄ ∈ set evs;
      Says Tgs A' ⦃Crypt (shrK A') ⦃Key SK', Agent B', T⦄, ST'⦄ ∈ set evs;
        evs ∈ kerbV ⟧
  ⟹ A=A' ∧ SK=SK' ∧ B=B' ∧ ST=ST'"
```
⟨*proof*⟩

end

9 The Original Otway-Rees Protocol

theory *OtwayRees* **imports** *Public* **begin**

From page 244 of Burrows, Abadi and Needham (1989). A Logic of Authentication. Proc. Royal Soc. 426

This is the original version, which encrypts Nonce NB.

inductive_set *otway* :: *"event list set"*
  where
```
   Nil:  "[] ∈ otway"
   — Initial trace is empty
 | Fake: "⟦evsf ∈ otway;  X ∈ synth (analz (knows Spy evsf)) ⟧
          ⟹ Says Spy B X  # evsf ∈ otway"
   — The spy can say almost anything.
 | Reception: "⟦evsr ∈ otway;  Says A B X ∈set evsr⟧ ⟹ Gets B X # evsr
∈ otway"
   — A message that has been sent can be received by the intended recipient.
 | OR1:  "⟦evs1 ∈ otway;  Nonce NA ∉ used evs1⟧
          ⟹ Says A B ⦃Nonce NA, Agent A, Agent B,
                         Crypt (shrK A) ⦃Nonce NA, Agent A, Agent B⦄ ⦄
                 # evs1 ∈ otway"
  — Alice initiates a protocol run
 | OR2:  "⟦evs2 ∈ otway;  Nonce NB ∉ used evs2;
             Gets B ⦃Nonce NA, Agent A, Agent B, X⦄ ∈ set evs2⟧
          ⟹ Says B Server
                  ⦃Nonce NA, Agent A, Agent B, X,
                    Crypt (shrK B)
                      ⦃Nonce NA, Nonce NB, Agent A, Agent B⦄⦄
                 # evs2 ∈ otway"
   — Bob's response to Alice's message. Note that NB is encrypted.
 | OR3:  "⟦evs3 ∈ otway;  Key KAB ∉ used evs3;
             Gets Server
                  ⦃Nonce NA, Agent A, Agent B,
                    Crypt (shrK A) ⦃Nonce NA, Agent A, Agent B⦄,
                    Crypt (shrK B) ⦃Nonce NA, Nonce NB, Agent A, Agent B⦄⦄
               ∈ set evs3⟧
          ⟹ Says Server B
                  ⦃Nonce NA,
```

```
                     Crypt (shrK A) ⦃Nonce NA, Key KAB⦄,
                     Crypt (shrK B) ⦃Nonce NB, Key KAB⦄⦄
                  # evs3 ∈ otway"
```

— The Server receives Bob's message and checks that the three NAs match. Then he sends a new session key to Bob with a packet for forwarding to Alice

```
 | OR4:  "⟦evs4 ∈ otway;  B ≠ Server;
             Says B Server ⦃Nonce NA, Agent A, Agent B, X',
                             Crypt (shrK B)
                                   ⦃Nonce NA, Nonce NB, Agent A, Agent B⦄⦄
               ∈ set evs4;
             Gets B ⦃Nonce NA, X, Crypt (shrK B) ⦃Nonce NB, Key K⦄⦄
               ∈ set evs4⟧
          ⟹ Says B A ⦃Nonce NA, X⦄ # evs4 ∈ otway"
```

— Bob receives the Server's (?) message and compares the Nonces with those in the message he previously sent the Server. Need `B ≠ Server` because we allow messages to self.

```
 | Oops: "⟦evso ∈ otway;
             Says Server B ⦃Nonce NA, X, Crypt (shrK B) ⦃Nonce NB, Key K⦄⦄
               ∈ set evso⟧
          ⟹ Notes Spy ⦃Nonce NA, Nonce NB, Key K⦄ # evso ∈ otway"
```

— This message models possible leaks of session keys. The nonces identify the protocol run

```
declare Says_imp_analz_Spy [dest]
declare parts.Body  [dest]
declare analz_into_parts [dest]
declare Fake_parts_insert_in_Un  [dest]
```

A "possibility property": there are traces that reach the end

```
lemma "⟦B ≠ Server; Key K ∉ used []⟧
      ⟹ ∃evs ∈ otway.
             Says B A ⦃Nonce NA, Crypt (shrK A) ⦃Nonce NA, Key K⦄⦄
               ∈ set evs"
⟨proof⟩

lemma Gets_imp_Says [dest!]:
     "⟦Gets B X ∈ set evs; evs ∈ otway⟧ ⟹ ∃A. Says A B X ∈ set evs"
⟨proof⟩

lemma OR2_analz_knows_Spy:
     "⟦Gets B ⦃N, Agent A, Agent B, X⦄ ∈ set evs;  evs ∈ otway⟧
      ⟹ X ∈ analz (knows Spy evs)"
  ⟨proof⟩

lemma OR4_analz_knows_Spy:
     "⟦Gets B ⦃N, X, Crypt (shrK B) X'⦄ ∈ set evs;  evs ∈ otway⟧
      ⟹ X ∈ analz (knows Spy evs)"
  ⟨proof⟩

lemmas OR2_parts_knows_Spy =
```

```
OR2_analz_knows_Spy [THEN analz_into_parts]
```

Theorems of the form *X ∉ parts (knows Spy evs)* imply that NOBODY sends messages containing X!

Spy never sees a good agent's shared key!

```
lemma Spy_see_shrK [simp]:
     "evs ∈ otway ⟹ (Key (shrK A) ∈ parts (knows Spy evs)) = (A ∈ bad)"
⟨proof⟩
```

```
lemma Spy_analz_shrK [simp]:
     "evs ∈ otway ⟹ (Key (shrK A) ∈ analz (knows Spy evs)) = (A ∈ bad)"
⟨proof⟩
```

```
lemma Spy_see_shrK_D [dest!]:
     "⟦Key (shrK A) ∈ parts (knows Spy evs);  evs ∈ otway⟧ ⟹ A ∈ bad"
⟨proof⟩
```

9.1 Towards Secrecy: Proofs Involving analz

Describes the form of K and NA when the Server sends this message. Also for Oops case.

```
lemma Says_Server_message_form:
     "⟦Says Server B ⦃NA, X, Crypt (shrK B) ⦃NB, Key K⦄⦄ ∈ set evs;
         evs ∈ otway⟧
      ⟹ K ∉ range shrK ∧ (∃i. NA = Nonce i) ∧ (∃j. NB = Nonce j)"
⟨proof⟩
```

Session keys are not used to encrypt other session keys

The equality makes the induction hypothesis easier to apply

```
lemma analz_image_freshK [rule_format]:
 "evs ∈ otway ⟹
   ∀K KK. KK ⊆ -(range shrK) ⟶
          (Key K ∈ analz (Key`KK ∪ (knows Spy evs))) =
          (K ∈ KK | Key K ∈ analz (knows Spy evs))"
⟨proof⟩
```

```
lemma analz_insert_freshK:
  "⟦evs ∈ otway;  KAB ∉ range shrK⟧ ⟹
      (Key K ∈ analz (insert (Key KAB) (knows Spy evs))) =
      (K = KAB | Key K ∈ analz (knows Spy evs))"
⟨proof⟩
```

The Key K uniquely identifies the Server's message.

```
lemma unique_session_keys:
     "⟦Says Server B ⦃NA, X, Crypt (shrK B) ⦃NB, K⦄⦄   ∈ set evs;
         Says Server B' ⦃NA',X',Crypt (shrK B') ⦃NB',K⦄⦄ ∈ set evs;
         evs ∈ otway⟧ ⟹ X=X' ∧ B=B' ∧ NA=NA' ∧ NB=NB'"
⟨proof⟩
```

9.2 Authenticity properties relating to NA

Only OR1 can have caused such a part of a message to appear.

```
lemma Crypt_imp_OR1 [rule_format]:
 "⟦A ∉ bad;  evs ∈ otway⟧
  ⟹ Crypt (shrK A) ⦃NA, Agent A, Agent B⦄ ∈ parts (knows Spy evs) ⟶
      Says A B ⦃NA, Agent A, Agent B,
                 Crypt (shrK A) ⦃NA, Agent A, Agent B⦄⦄
        ∈ set evs"
```
⟨*proof*⟩

```
lemma Crypt_imp_OR1_Gets:
     "⟦Gets B ⦃NA, Agent A, Agent B,
                 Crypt (shrK A) ⦃NA, Agent A, Agent B⦄⦄ ∈ set evs;
         A ∉ bad; evs ∈ otway⟧
       ⟹ Says A B ⦃NA, Agent A, Agent B,
                       Crypt (shrK A) ⦃NA, Agent A, Agent B⦄⦄
              ∈ set evs"
```
⟨*proof*⟩

The Nonce NA uniquely identifies A's message

```
lemma unique_NA:
     "⟦Crypt (shrK A) ⦃NA, Agent A, Agent B⦄ ∈ parts (knows Spy evs);
         Crypt (shrK A) ⦃NA, Agent A, Agent C⦄ ∈ parts (knows Spy evs);
         evs ∈ otway;  A ∉ bad⟧
      ⟹ B = C"
```
⟨*proof*⟩

It is impossible to re-use a nonce in both OR1 and OR2. This holds because OR2 encrypts Nonce NB. It prevents the attack that can occur in the over-simplified version of this protocol: see *OtwayRees_Bad*.

```
lemma no_nonce_OR1_OR2:
   "⟦Crypt (shrK A) ⦃NA, Agent A, Agent B⦄ ∈ parts (knows Spy evs);
       A ∉ bad;  evs ∈ otway⟧
    ⟹ Crypt (shrK A) ⦃NA', NA, Agent A', Agent A⦄ ∉ parts (knows Spy evs)"
```
⟨*proof*⟩

Crucial property: If the encrypted message appears, and A has used NA to start a run, then it originated with the Server!

```
lemma NA_Crypt_imp_Server_msg [rule_format]:
     "⟦A ∉ bad;  evs ∈ otway⟧
      ⟹ Says A B ⦃NA, Agent A, Agent B,
                     Crypt (shrK A) ⦃NA, Agent A, Agent B⦄⦄ ∈ set evs ⟶
          Crypt (shrK A) ⦃NA, Key K⦄ ∈ parts (knows Spy evs)
          ⟶ (∃NB. Says Server B
                         ⦃NA,
                           Crypt (shrK A) ⦃NA, Key K⦄,
                           Crypt (shrK B) ⦃NB, Key K⦄⦄ ∈ set evs)"
```
⟨*proof*⟩

Corollary: if A receives B's OR4 message and the nonce NA agrees then the key really did come from the Server! CANNOT prove this of the bad form of this protocol, even though we can prove *Spy_not_see_encrypted_key*

```
lemma A_trusts_OR4:
    "⟦Says A  B {|NA, Agent A, Agent B,
                    Crypt (shrK A) {|NA, Agent A, Agent B|}|} ∈ set evs;
       Says B' A {|NA, Crypt (shrK A) {|NA, Key K|}|} ∈ set evs;
    A ∉ bad;  evs ∈ otway⟧
  ⟹ ∃NB. Says Server B
              {|NA,
                Crypt (shrK A) {|NA, Key K|},
                Crypt (shrK B) {|NB, Key K|}|}
                ∈ set evs"
⟨proof⟩
```

Crucial secrecy property: Spy does not see the keys sent in msg OR3 Does not in itself guarantee security: an attack could violate the premises, e.g. by having `A = Spy`

```
lemma secrecy_lemma:
 "⟦A ∉ bad;  B ∉ bad;  evs ∈ otway⟧
  ⟹ Says Server B
        {|NA, Crypt (shrK A) {|NA, Key K|},
          Crypt (shrK B) {|NB, Key K|}|} ∈ set evs ⟶
      Notes Spy {|NA, NB, Key K|} ∉ set evs ⟶
      Key K ∉ analz (knows Spy evs)"
  ⟨proof⟩
```

```
theorem Spy_not_see_encrypted_key:
    "⟦Says Server B
         {|NA, Crypt (shrK A) {|NA, Key K|},
               Crypt (shrK B) {|NB, Key K|}|} ∈ set evs;
       Notes Spy {|NA, NB, Key K|} ∉ set evs;
       A ∉ bad;  B ∉ bad;  evs ∈ otway⟧
      ⟹ Key K ∉ analz (knows Spy evs)"
⟨proof⟩
```

This form is an immediate consequence of the previous result. It is similar to the assertions established by other methods. It is equivalent to the previous result in that the Spy already has `analz` and `synth` at his disposal. However, the conclusion `Key K ∉ knows Spy evs` appears not to be inductive: all the cases other than Fake are trivial, while Fake requires `Key K ∉ analz (knows Spy evs)`.

```
lemma Spy_not_know_encrypted_key:
    "⟦Says Server B
         {|NA, Crypt (shrK A) {|NA, Key K|},
               Crypt (shrK B) {|NB, Key K|}|} ∈ set evs;
       Notes Spy {|NA, NB, Key K|} ∉ set evs;
       A ∉ bad;  B ∉ bad;  evs ∈ otway⟧
      ⟹ Key K ∉ knows Spy evs"
⟨proof⟩
```

A's guarantee. The Oops premise quantifies over NB because A cannot know what it is.

```
lemma A_gets_good_key:
    "⟦Says A  B {|NA, Agent A, Agent B,
                    Crypt (shrK A) {|NA, Agent A, Agent B|}|} ∈ set evs;
       Says B' A {|NA, Crypt (shrK A) {|NA, Key K|}|} ∈ set evs;
```

```
         ∀NB. Notes Spy {|NA, NB, Key K|} ∉ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ otway⟧
      ⟹ Key K ∉ analz (knows Spy evs)"
```

⟨proof⟩

9.3 Authenticity properties relating to NB

Only OR2 can have caused such a part of a message to appear. We do not know anything about X: it does NOT have to have the right form.

lemma *Crypt_imp_OR2:*

```
      "⟦Crypt (shrK B) {|NA, NB, Agent A, Agent B|} ∈ parts (knows Spy evs);
         B ∉ bad;  evs ∈ otway⟧
       ⟹ ∃X. Says B Server
                 {|NA, Agent A, Agent B, X,
                   Crypt (shrK B) {|NA, NB, Agent A, Agent B|}|}
                 ∈ set evs"
```

⟨proof⟩

The Nonce NB uniquely identifies B's message

lemma *unique_NB:*

```
      "⟦Crypt (shrK B) {|NA, NB, Agent A, Agent B|} ∈ parts(knows Spy evs);
         Crypt (shrK B) {|NC, NB, Agent C, Agent B|} ∈ parts(knows Spy evs);
           evs ∈ otway;  B ∉ bad⟧
         ⟹ NC = NA ∧ C = A"
```

⟨proof⟩

If the encrypted message appears, and B has used Nonce NB, then it originated with the Server! Quite messy proof.

lemma *NB_Crypt_imp_Server_msg [rule_format]:*

```
  "⟦B ∉ bad;  evs ∈ otway⟧
  ⟹ Crypt (shrK B) {|NB, Key K|} ∈ parts (knows Spy evs)
      ⟶ (∀X'. Says B Server
                     {|NA, Agent A, Agent B, X',
                       Crypt (shrK B) {|NA, NB, Agent A, Agent B|}|}
           ∈ set evs
           ⟶ Says Server B
                {|NA, Crypt (shrK A) {|NA, Key K|},
                      Crypt (shrK B) {|NB, Key K|}|}
                    ∈ set evs)"
```

⟨proof⟩

Guarantee for B: if it gets a message with matching NB then the Server has sent the correct message.

theorem *B_trusts_OR3:*

```
      "⟦Says B Server {|NA, Agent A, Agent B, X',
                           Crypt (shrK B) {|NA, NB, Agent A, Agent B|}|}
           ∈ set evs;
         Gets B {|NA, X, Crypt (shrK B) {|NB, Key K|}|} ∈ set evs;
         B ∉ bad;  evs ∈ otway⟧
      ⟹ Says Server B
              {|NA,
                Crypt (shrK A) {|NA, Key K|},
```

```
                  Crypt (shrK B) {|NB, Key K|}|}
                  ∈ set evs"
```
⟨proof⟩

The obvious combination of *B_trusts_OR3* with *Spy_not_see_encrypted_key*

lemma *B_gets_good_key:*
```
     "[[Says B Server {|NA, Agent A, Agent B, X',
                         Crypt (shrK B) {|NA, NB, Agent A, Agent B|}|}
           ∈ set evs;
         Gets B {|NA, X, Crypt (shrK B) {|NB, Key K|}|} ∈ set evs;
         Notes Spy {|NA, NB, Key K|} ∉ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ otway]]
      ⟹ Key K ∉ analz (knows Spy evs)"
```
⟨proof⟩

lemma *OR3_imp_OR2:*
```
     "[[Says Server B
              {|NA, Crypt (shrK A) {|NA, Key K|},
                Crypt (shrK B) {|NB, Key K|}|} ∈ set evs;
         B ∉ bad;  evs ∈ otway]]
  ⟹ ∃X. Says B Server {|NA, Agent A, Agent B, X,
                            Crypt (shrK B) {|NA, NB, Agent A, Agent B|}|}
              ∈ set evs"
```
⟨proof⟩

After getting and checking OR4, agent A can trust that B has been active. We could probably prove that X has the expected form, but that is not strictly necessary for authentication.

theorem *A_auths_B:*
```
     "[[Says B' A {|NA, Crypt (shrK A) {|NA, Key K|}|} ∈ set evs;
         Says A  B {|NA, Agent A, Agent B,
                     Crypt (shrK A) {|NA, Agent A, Agent B|}|} ∈ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ otway]]
  ⟹ ∃NB X. Says B Server {|NA, Agent A, Agent B, X,
                               Crypt (shrK B)  {|NA, NB, Agent A, Agent B|}|}
                  ∈ set evs"
```
⟨proof⟩

end

10 The Otway-Rees Protocol as Modified by Abadi and Needham

theory *OtwayRees_AN* **imports** *Public* **begin**

This simplified version has minimal encryption and explicit messages.

Note that the formalization does not even assume that nonces are fresh. This is because the protocol does not rely on uniqueness of nonces for security, only for freshness, and the proof script does not prove freshness properties.

From page 11 of Abadi and Needham (1996). Prudent Engineering Practice for Cryptographic Protocols. IEEE Trans. SE 22 (1)

```
inductive_set otway :: "event list set"
  where
   Nil: — The empty trace
        "[] ∈ otway"

 | Fake: — The Spy may say anything he can say. The sender field is correct, but
agents don't use that information.
         "⟦evsf ∈ otway;  X ∈ synth (analz (knows Spy evsf))⟧
          ⟹ Says Spy B X  # evsf ∈ otway"


 | Reception: — A message that has been sent can be received by the intended
recipient.
              "⟦evsr ∈ otway;  Says A B X ∈set evsr⟧
               ⟹ Gets B X # evsr ∈ otway"

 | OR1:  — Alice initiates a protocol run
         "evs1 ∈ otway
          ⟹ Says A B ⦃Agent A, Agent B, Nonce NA⦄ # evs1 ∈ otway"

 | OR2:  — Bob's response to Alice's message.
         "⟦evs2 ∈ otway;
             Gets B ⦃Agent A, Agent B, Nonce NA⦄ ∈set evs2⟧
          ⟹ Says B Server ⦃Agent A, Agent B, Nonce NA, Nonce NB⦄
                 # evs2 ∈ otway"

 | OR3:  — The Server receives Bob's message. Then he sends a new session key to
Bob with a packet for forwarding to Alice.
         "⟦evs3 ∈ otway;  Key KAB ∉ used evs3;
             Gets Server ⦃Agent A, Agent B, Nonce NA, Nonce NB⦄
               ∈set evs3⟧
          ⟹ Says Server B
               ⦃Crypt (shrK A) ⦃Nonce NA, Agent A, Agent B, Key KAB⦄,
                 Crypt (shrK B) ⦃Nonce NB, Agent A, Agent B, Key KAB⦄⦄
              # evs3 ∈ otway"

 | OR4:  — Bob receives the Server's (?) message and compares the Nonces with
those in the message he previously sent the Server. Need B ≠ Server because we
allow messages to self.
         "⟦evs4 ∈ otway;  B ≠ Server;
             Says B Server ⦃Agent A, Agent B, Nonce NA, Nonce NB⦄ ∈set evs4;
             Gets B ⦃X, Crypt(shrK B)⦃Nonce NB,Agent A,Agent B,Key K⦄⦄
               ∈set evs4⟧
          ⟹ Says B A X # evs4 ∈ otway"

 | Oops: — This message models possible leaks of session keys. The nonces identify
the protocol run.
         "⟦evso ∈ otway;
             Says Server B
                    ⦃Crypt (shrK A) ⦃Nonce NA, Agent A, Agent B, Key K⦄,
                      Crypt (shrK B) ⦃Nonce NB, Agent A, Agent B, Key K⦄⦄
               ∈set evso⟧
          ⟹ Notes Spy ⦃Nonce NA, Nonce NB, Key K⦄ # evso ∈ otway"
```

```
declare Says_imp_knows_Spy [THEN analz.Inj, dest]
declare parts.Body  [dest]
declare analz_into_parts [dest]
declare Fake_parts_insert_in_Un  [dest]
```

A "possibility property": there are traces that reach the end

```
lemma "⟦B ≠ Server; Key K ∉ used []⟧
      ⟹ ∃evs ∈ otway.
            Says B A (Crypt (shrK A) {|Nonce NA, Agent A, Agent B, Key K|})
              ∈ set evs"
⟨proof⟩

lemma Gets_imp_Says [dest!]:
     "⟦Gets B X ∈ set evs; evs ∈ otway⟧ ⟹ ∃A. Says A B X ∈ set evs"
⟨proof⟩
```

For reasoning about the encrypted portion of messages

```
lemma OR4_analz_knows_Spy:
     "⟦Gets B {|X, Crypt(shrK B) X'|} ∈ set evs;  evs ∈ otway⟧
      ⟹ X ∈ analz (knows Spy evs)"
⟨proof⟩
```

Theorems of the form `X ∉ parts (knows Spy evs)` imply that NOBODY sends messages containing X!

Spy never sees a good agent's shared key!

```
lemma Spy_see_shrK [simp]:
     "evs ∈ otway ⟹ (Key (shrK A) ∈ parts (knows Spy evs)) = (A ∈ bad)"
⟨proof⟩

lemma Spy_analz_shrK [simp]:
     "evs ∈ otway ⟹ (Key (shrK A) ∈ analz (knows Spy evs)) = (A ∈ bad)"
⟨proof⟩

lemma Spy_see_shrK_D [dest!]:
     "⟦Key (shrK A) ∈ parts (knows Spy evs);  evs ∈ otway⟧ ⟹ A ∈ bad"
⟨proof⟩
```

10.1 Proofs involving analz

Describes the form of K and NA when the Server sends this message.

```
lemma Says_Server_message_form:
     "⟦Says Server B
            {|Crypt (shrK A) {|NA, Agent A, Agent B, Key K|},
              Crypt (shrK B) {|NB, Agent A, Agent B, Key K|}|}
           ∈ set evs;
         evs ∈ otway⟧
      ⟹ K ∉ range shrK ∧ (∃i. NA = Nonce i) ∧ (∃j. NB = Nonce j)"
⟨proof⟩
```

Session keys are not used to encrypt other session keys

The equality makes the induction hypothesis easier to apply

lemma analz_image_freshK [rule_format]:
```
 "evs ∈ otway ⟹
   ∀K KK. KK ⊆ -(range shrK) ⟶
          (Key K ∈ analz (Key`KK ∪ (knows Spy evs))) =
          (K ∈ KK | Key K ∈ analz (knows Spy evs))"
```
⟨proof⟩

lemma analz_insert_freshK:
```
  "⟦evs ∈ otway;  KAB ∉ range shrK⟧ ⟹
      (Key K ∈ analz (insert (Key KAB) (knows Spy evs))) =
      (K = KAB | Key K ∈ analz (knows Spy evs))"
```
⟨proof⟩

The Key K uniquely identifies the Server's message.

lemma unique_session_keys:
```
     "⟦Says Server B
          ⦃Crypt (shrK A) ⦃NA, Agent A, Agent B, K⦄,
            Crypt (shrK B) ⦃NB, Agent A, Agent B, K⦄⦄
         ∈ set evs;
        Says Server B'
          ⦃Crypt (shrK A') ⦃NA', Agent A', Agent B', K⦄,
            Crypt (shrK B') ⦃NB', Agent A', Agent B', K⦄⦄
         ∈ set evs;
        evs ∈ otway⟧
     ⟹ A=A' ∧ B=B' ∧ NA=NA' ∧ NB=NB'"
```
⟨proof⟩

10.2 Authenticity properties relating to NA

If the encrypted message appears then it originated with the Server!

lemma NA_Crypt_imp_Server_msg [rule_format]:
```
    "⟦A ∉ bad;  A ≠ B;  evs ∈ otway⟧
     ⟹ Crypt (shrK A) ⦃NA, Agent A, Agent B, Key K⦄ ∈ parts (knows Spy evs)
       ⟶ (∃NB. Says Server B
                    ⦃Crypt (shrK A) ⦃NA, Agent A, Agent B, Key K⦄,
                      Crypt (shrK B) ⦃NB, Agent A, Agent B, Key K⦄⦄
                    ∈ set evs)"
```
⟨proof⟩

Corollary: if A receives B's OR4 message then it originated with the Server. Freshness may be inferred from nonce NA.

lemma A_trusts_OR4:
```
     "⟦Says B' A (Crypt (shrK A) ⦃NA, Agent A, Agent B, Key K⦄) ∈ set evs;
         A ∉ bad;  A ≠ B;  evs ∈ otway⟧
      ⟹ ∃NB. Says Server B
                  ⦃Crypt (shrK A) ⦃NA, Agent A, Agent B, Key K⦄,
                    Crypt (shrK B) ⦃NB, Agent A, Agent B, Key K⦄⦄
                 ∈ set evs"
```
⟨proof⟩

Crucial secrecy property: Spy does not see the keys sent in msg OR3 Does not in itself guarantee security: an attack could violate the premises, e.g. by having `A = Spy`

lemma *secrecy_lemma:*
```
"⟦A ∉ bad;  B ∉ bad;  evs ∈ otway⟧
 ⟹ Says Server B
      ⦃Crypt (shrK A) ⦃NA, Agent A, Agent B, Key K⦄,
        Crypt (shrK B) ⦃NB, Agent A, Agent B, Key K⦄⦄
     ∈ set evs ⟶
     Notes Spy ⦃NA, NB, Key K⦄ ∉ set evs ⟶
     Key K ∉ analz (knows Spy evs)"
```
⟨*proof*⟩

lemma *Spy_not_see_encrypted_key:*
```
"⟦Says Server B
      ⦃Crypt (shrK A) ⦃NA, Agent A, Agent B, Key K⦄,
        Crypt (shrK B) ⦃NB, Agent A, Agent B, Key K⦄⦄
     ∈ set evs;
   Notes Spy ⦃NA, NB, Key K⦄ ∉ set evs;
   A ∉ bad;  B ∉ bad;  evs ∈ otway⟧
 ⟹ Key K ∉ analz (knows Spy evs)"
```
⟨*proof*⟩

A's guarantee. The Oops premise quantifies over NB because A cannot know what it is.

lemma *A_gets_good_key:*
```
"⟦Says B' A (Crypt (shrK A) ⦃NA, Agent A, Agent B, Key K⦄) ∈ set evs;
   ∀NB. Notes Spy ⦃NA, NB, Key K⦄ ∉ set evs;
   A ∉ bad;  B ∉ bad;  A ≠ B;  evs ∈ otway⟧
 ⟹ Key K ∉ analz (knows Spy evs)"
```
⟨*proof*⟩

10.3 Authenticity properties relating to NB

If the encrypted message appears then it originated with the Server!

lemma *NB_Crypt_imp_Server_msg [rule_format]:*
```
"⟦B ∉ bad;  A ≠ B;  evs ∈ otway⟧
 ⟹ Crypt (shrK B) ⦃NB, Agent A, Agent B, Key K⦄ ∈ parts (knows Spy evs)
     ⟶ (∃NA. Says Server B
                 ⦃Crypt (shrK A) ⦃NA, Agent A, Agent B, Key K⦄,
                   Crypt (shrK B) ⦃NB, Agent A, Agent B, Key K⦄⦄
                 ∈ set evs)"
```
⟨*proof*⟩

Guarantee for B: if it gets a well-formed certificate then the Server has sent the correct message in round 3.

lemma *B_trusts_OR3:*
```
"⟦Says S B ⦃X, Crypt (shrK B) ⦃NB, Agent A, Agent B, Key K⦄⦄
      ∈ set evs;
   B ∉ bad;  A ≠ B;  evs ∈ otway⟧
 ⟹ ∃NA. Says Server B
            ⦃Crypt (shrK A) ⦃NA, Agent A, Agent B, Key K⦄,
```

```
                    Crypt (shrK B) {|NB, Agent A, Agent B, Key K|}|}
                  ∈ set evs"
```
⟨proof⟩

The obvious combination of *B_trusts_OR3* with *Spy_not_see_encrypted_key*

lemma *B_gets_good_key:*
```
     "⟦Gets B {|X, Crypt (shrK B) {|NB, Agent A, Agent B, Key K|}|}
          ∈ set evs;
         ∀NA. Notes Spy {|NA, NB, Key K|} ∉ set evs;
         A ∉ bad;  B ∉ bad;  A ≠ B;  evs ∈ otway⟧
      ⟹ Key K ∉ analz (knows Spy evs)"
```
⟨proof⟩

end

11 The Otway-Rees Protocol: The Faulty BAN Version

theory *OtwayRees_Bad* **imports** *Public* **begin**

The FAULTY version omitting encryption of Nonce NB, as suggested on page 247 of Burrows, Abadi and Needham (1988). A Logic of Authentication. Proc. Royal Soc. 426

This file illustrates the consequences of such errors. We can still prove impressive-looking properties such as *Spy_not_see_encrypted_key*, yet the protocol is open to a middleperson attack. Attempting to prove some key lemmas indicates the possibility of this attack.

inductive_set *otway :: "event list set"*
  where
   Nil: — The empty trace
```
        "[] ∈ otway"
```

 | Fake: — The Spy may say anything he can say. The sender field is correct, but agents don't use that information.
```
         "⟦evsf ∈ otway;  X ∈ synth (analz (knows Spy evsf))⟧
          ⟹ Says Spy B X  # evsf ∈ otway"
```

 | Reception: — A message that has been sent can be received by the intended recipient.
```
              "⟦evsr ∈ otway;  Says A B X ∈set evsr⟧
               ⟹ Gets B X # evsr ∈ otway"
```

 | OR1: — Alice initiates a protocol run
```
         "⟦evs1 ∈ otway;  Nonce NA ∉ used evs1⟧
          ⟹ Says A B {|Nonce NA, Agent A, Agent B,
                         Crypt (shrK A) {|Nonce NA, Agent A, Agent B|}|}
                 # evs1 ∈ otway"
```

 | OR2: — Bob's response to Alice's message. This variant of the protocol does NOT encrypt NB.
```
         "⟦evs2 ∈ otway;  Nonce NB ∉ used evs2;
```

```
          Gets B {|Nonce NA, Agent A, Agent B, X|} ∈ set evs2⟧
        ⟹ Says B Server
               {|Nonce NA, Agent A, Agent B, X, Nonce NB,
                 Crypt (shrK B) {|Nonce NA, Agent A, Agent B|}|}
             # evs2 ∈ otway"
```

| OR3: — The Server receives Bob's message and checks that the three NAs match. Then he sends a new session key to Bob with a packet for forwarding to Alice.

```
        "⟦evs3 ∈ otway;  Key KAB ∉ used evs3;
          Gets Server
               {|Nonce NA, Agent A, Agent B,
                 Crypt (shrK A) {|Nonce NA, Agent A, Agent B|},
                 Nonce NB,
                 Crypt (shrK B) {|Nonce NA, Agent A, Agent B|}|}
            ∈ set evs3⟧
        ⟹ Says Server B
               {|Nonce NA,
                 Crypt (shrK A) {|Nonce NA, Key KAB|},
                 Crypt (shrK B) {|Nonce NB, Key KAB|}|}
             # evs3 ∈ otway"
```

| OR4: — Bob receives the Server's (?) message and compares the Nonces with those in the message he previously sent the Server. Need *B ≠ Server* because we allow messages to self.

```
        "⟦evs4 ∈ otway;  B ≠ Server;
          Says B Server {|Nonce NA, Agent A, Agent B, X', Nonce NB,
                          Crypt (shrK B) {|Nonce NA, Agent A, Agent B|}|}
            ∈ set evs4;
          Gets B {|Nonce NA, X, Crypt (shrK B) {|Nonce NB, Key K|}|}
            ∈ set evs4⟧
        ⟹ Says B A {|Nonce NA, X|} # evs4 ∈ otway"
```

| Oops: — This message models possible leaks of session keys. The nonces identify the protocol run.

```
        "⟦evso ∈ otway;
          Says Server B {|Nonce NA, X, Crypt (shrK B) {|Nonce NB, Key K|}|}
            ∈ set evso⟧
        ⟹ Notes Spy {|Nonce NA, Nonce NB, Key K|} # evso ∈ otway"
```

declare *Says_imp_knows_Spy [THEN analz.Inj, dest]*
declare *parts.Body [dest]*
declare *analz_into_parts [dest]*
declare *Fake_parts_insert_in_Un [dest]*

A "possibility property": there are traces that reach the end

lemma *"⟦B ≠ Server; Key K ∉ used []⟧
⟹ ∃NA. ∃evs ∈ otway.
Says B A {|Nonce NA, Crypt (shrK A) {|Nonce NA, Key K|}|}
∈ set evs"*
⟨*proof*⟩

lemma *Gets_imp_Says [dest!]:*
"⟦Gets B X ∈ set evs; evs ∈ otway⟧ ⟹ ∃A. Says A B X ∈ set evs"

⟨proof⟩

11.1 For reasoning about the encrypted portion of messages

lemma *OR2_analz_knows_Spy:*
```
"⟦Gets B {|N, Agent A, Agent B, X|} ∈ set evs;  evs ∈ otway⟧
 ⟹ X ∈ analz (knows Spy evs)"
```
⟨proof⟩

lemma *OR4_analz_knows_Spy:*
```
"⟦Gets B {|N, X, Crypt (shrK B) X'|} ∈ set evs;  evs ∈ otway⟧
 ⟹ X ∈ analz (knows Spy evs)"
```
⟨proof⟩

lemma *Oops_parts_knows_Spy:*
```
"Says Server B {|NA, X, Crypt K' {|NB,K|}|} ∈ set evs
 ⟹ K ∈ parts (knows Spy evs)"
```
⟨proof⟩

Forwarding lemma: see comments in OtwayRees.thy

lemmas *OR2_parts_knows_Spy =*
```
OR2_analz_knows_Spy [THEN analz_into_parts]
```

Theorems of the form *X ∉ parts (knows Spy evs)* imply that NOBODY sends messages containing X!

Spy never sees a good agent's shared key!

lemma *Spy_see_shrK [simp]:*
```
"evs ∈ otway ⟹ (Key (shrK A) ∈ parts (knows Spy evs)) = (A ∈ bad)"
```
⟨proof⟩

lemma *Spy_analz_shrK [simp]:*
```
"evs ∈ otway ⟹ (Key (shrK A) ∈ analz (knows Spy evs)) = (A ∈ bad)"
```
⟨proof⟩

lemma *Spy_see_shrK_D [dest!]:*
```
"⟦Key (shrK A) ∈ parts (knows Spy evs);  evs ∈ otway⟧ ⟹ A ∈ bad"
```
⟨proof⟩

11.2 Proofs involving analz

Describes the form of K and NA when the Server sends this message. Also for Oops case.

lemma *Says_Server_message_form:*
```
"⟦Says Server B {|NA, X, Crypt (shrK B) {|NB, Key K|}|} ∈ set evs;
   evs ∈ otway⟧
 ⟹ K ∉ range shrK ∧ (∃i. NA = Nonce i) ∧ (∃j. NB = Nonce j)"
```
⟨proof⟩

Session keys are not used to encrypt other session keys

The equality makes the induction hypothesis easier to apply

```
lemma analz_image_freshK [rule_format]:
 "evs ∈ otway ⟹
   ∀K KK. KK ⊆ -(range shrK) ⟶
          (Key K ∈ analz (Key`KK ∪ (knows Spy evs))) =
          (K ∈ KK | Key K ∈ analz (knows Spy evs))"
⟨proof⟩
```

```
lemma analz_insert_freshK:
  "⟦evs ∈ otway;  KAB ∉ range shrK⟧ ⟹
      (Key K ∈ analz (insert (Key KAB) (knows Spy evs))) =
      (K = KAB | Key K ∈ analz (knows Spy evs))"
⟨proof⟩
```

The Key K uniquely identifies the Server's message.

```
lemma unique_session_keys:
     "⟦Says Server B ⦃NA, X, Crypt (shrK B) ⦃NB, K⦄⦄   ∈ set evs;
         Says Server B' ⦃NA',X',Crypt (shrK B') ⦃NB',K⦄⦄ ∈ set evs;
         evs ∈ otway⟧ ⟹ X=X' ∧ B=B' ∧ NA=NA' ∧ NB=NB'"
⟨proof⟩
```

Crucial secrecy property: Spy does not see the keys sent in msg OR3 Does not in itself guarantee security: an attack could violate the premises, e.g. by having `A = Spy`

```
lemma secrecy_lemma:
 "⟦A ∉ bad;  B ∉ bad;  evs ∈ otway⟧
  ⟹ Says Server B
        ⦃NA, Crypt (shrK A) ⦃NA, Key K⦄,
          Crypt (shrK B) ⦃NB, Key K⦄⦄ ∈ set evs ⟶
      Notes Spy ⦃NA, NB, Key K⦄ ∉ set evs ⟶
      Key K ∉ analz (knows Spy evs)"
⟨proof⟩
```

```
lemma Spy_not_see_encrypted_key:
     "⟦Says Server B
          ⦃NA, Crypt (shrK A) ⦃NA, Key K⦄,
                Crypt (shrK B) ⦃NB, Key K⦄⦄ ∈ set evs;
         Notes Spy ⦃NA, NB, Key K⦄ ∉ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ otway⟧
      ⟹ Key K ∉ analz (knows Spy evs)"
⟨proof⟩
```

11.3 Attempting to prove stronger properties

Only OR1 can have caused such a part of a message to appear. The premise `A ≠ B` prevents OR2's similar-looking cryptogram from being picked up. Original Otway-Rees doesn't need it.

```
lemma Crypt_imp_OR1 [rule_format]:
     "⟦A ∉ bad;  A ≠ B;  evs ∈ otway⟧
      ⟹ Crypt (shrK A) ⦃NA, Agent A, Agent B⦄ ∈ parts (knows Spy evs) ⟶
         Says A B ⦃NA, Agent A, Agent B,
```

```
                    Crypt (shrK A) {|NA, Agent A, Agent B|}  ∈ set evs"
```
⟨proof⟩

Crucial property: If the encrypted message appears, and A has used NA to start a run, then it originated with the Server! The premise *A ≠ B* allows use of *Crypt_imp_OR1*

Only it is FALSE. Somebody could make a fake message to Server substituting some other nonce NA' for NB.

```
lemma "[A ∉ bad;  A ≠ B;  evs ∈ otway]
       ⟹ Crypt (shrK A) {|NA, Key K|} ∈ parts (knows Spy evs) ⟶
          Says A B {|NA, Agent A, Agent B,
                     Crypt (shrK A) {|NA, Agent A, Agent B|}|}
           ∈ set evs  ⟶
          (∃B NB. Says Server B
               {|NA,
                 Crypt (shrK A) {|NA, Key K|},
                 Crypt (shrK B) {|NB, Key K|}|} ∈ set evs)"
```
⟨proof⟩

end

12 Bella's version of the Otway-Rees protocol

theory *OtwayReesBella* **imports** *Public* **begin**

Bella's modifications to a version of the Otway-Rees protocol taken from the BAN paper only concern message 7. The updated protocol makes the goal of key distribution of the session key available to A. Investigating the principle of Goal Availability undermines the BAN claim about the original protocol, that "this protocol does not make use of Kab as an encryption key, so neither principal can know whether the key is known to the other". The updated protocol makes no use of the session key to encrypt but informs A that B knows it.

```
inductive_set orb :: "event list set"
 where

  Nil:  "[]∈ orb"

| Fake: "[evsa∈ orb;  X∈ synth (analz (knows Spy evsa))]
         ⟹ Says Spy B X  # evsa ∈ orb"

| Reception: "[evsr∈ orb;  Says A B X ∈ set evsr]
              ⟹ Gets B X # evsr ∈ orb"

| OR1:  "[evs1∈ orb;  Nonce NA ∉ used evs1]
         ⟹ Says A B {|Nonce M, Agent A, Agent B,
                 Crypt (shrK A) {|Nonce NA, Nonce M, Agent A, Agent B|}|}
               # evs1 ∈ orb"

| OR2:  "[evs2∈ orb;  Nonce NB ∉ used evs2;
           Gets B {|Nonce M, Agent A, Agent B, X|} ∈ set evs2]
        ⟹ Says B Server
```

```
                ⦃Nonce M, Agent A, Agent B, X,
           Crypt (shrK B) ⦃Nonce NB, Nonce M, Nonce M, Agent A, Agent B⦄⦄
               # evs2 ∈ orb"

| OR3:  "⟦evs3∈ orb;  Key KAB ∉ used evs3;
         Gets Server
            ⦃Nonce M, Agent A, Agent B,
              Crypt (shrK A) ⦃Nonce NA, Nonce M, Agent A, Agent B⦄,
              Crypt (shrK B) ⦃Nonce NB, Nonce M, Nonce M, Agent A, Agent
B⦄⦄
          ∈ set evs3⟧
        ⟹ Says Server B ⦃Nonce M,
                Crypt (shrK B) ⦃Crypt (shrK A) ⦃Nonce NA, Key KAB⦄,
                                  Nonce NB, Key KAB⦄⦄
              # evs3 ∈ orb"



| OR4:  "⟦evs4∈ orb; B ≠ Server; ∀ p q. X ≠ ⦃p, q⦄;
         Says B Server ⦃Nonce M, Agent A, Agent B, X',
               Crypt (shrK B) ⦃Nonce NB, Nonce M, Nonce M, Agent A, Agent
B⦄⦄
           ∈ set evs4;
         Gets B ⦃Nonce M, Crypt (shrK B) ⦃X, Nonce NB, Key KAB⦄⦄
           ∈ set evs4⟧
        ⟹ Says B A ⦃Nonce M, X⦄ # evs4 ∈ orb"


| Oops: "⟦evso∈ orb;
          Says Server B ⦃Nonce M,
                Crypt (shrK B) ⦃Crypt (shrK A) ⦃Nonce NA, Key KAB⦄,
                                  Nonce NB, Key KAB⦄⦄
            ∈ set evso⟧
 ⟹ Notes Spy ⦃Agent A, Agent B, Nonce NA, Nonce NB, Key KAB⦄ # evso
    ∈ orb"
```

declare *knows_Spy_partsEs [elim]*
declare *analz_into_parts [dest]*
declare *Fake_parts_insert_in_Un [dest]*

Fragile proof, with backtracking in the possibility call.

lemma *possibility_thm: "⟦A ≠ Server; B ≠ Server; Key K ∉ used[]⟧*
⟹ ∃ evs ∈ orb.
Says B A ⦃Nonce M, Crypt (shrK A) ⦃Nonce Na, Key K⦄⦄ ∈ set evs"
⟨*proof*⟩

lemma *Gets_imp_Says :*
"⟦Gets B X ∈ set evs; evs ∈ orb⟧ ⟹ ∃A. Says A B X ∈ set evs"
⟨*proof*⟩

lemma *Gets_imp_knows_Spy:*

```
    "⟦Gets B X ∈ set evs; evs ∈ orb⟧  ⟹ X ∈ knows Spy evs"
```
⟨proof⟩

declare *Gets_imp_knows_Spy [THEN parts.Inj, dest]*

lemma *Gets_imp_knows:*
```
    "⟦Gets B X ∈ set evs; evs ∈ orb⟧  ⟹ X ∈ knows B evs"
```
⟨proof⟩

lemma *OR2_analz_knows_Spy:*
```
  "⟦Gets B {|Nonce M, Agent A, Agent B, X|} ∈ set evs; evs ∈ orb⟧
   ⟹ X ∈ analz (knows Spy evs)"
```
⟨proof⟩

lemma *OR4_parts_knows_Spy:*
```
  "⟦Gets B {|Nonce M, Crypt (shrK B) {|X, Nonce Nb, Key Kab|}|}  ∈ set evs;
     evs ∈ orb⟧   ⟹ X ∈ parts (knows Spy evs)"
```
⟨proof⟩

lemma *Oops_parts_knows_Spy:*
```
   "Says Server B {|Nonce M, Crypt K' {|X, Nonce Nb, K|}|} ∈ set evs
    ⟹ K ∈ parts (knows Spy evs)"
```
⟨proof⟩

lemmas *OR2_parts_knows_Spy =*
```
   OR2_analz_knows_Spy [THEN analz_into_parts]
```

⟨ML⟩

lemma *Spy_see_shrK [simp]:*
```
   "evs ∈ orb ⟹ (Key (shrK A) ∈ parts (knows Spy evs)) = (A ∈ bad)"
```
⟨proof⟩

lemma *Spy_analz_shrK [simp]:*
```
"evs ∈ orb ⟹ (Key (shrK A) ∈ analz (knows Spy evs)) = (A ∈ bad)"
```
⟨proof⟩

lemma *Spy_see_shrK_D [dest!]:*
```
    "⟦Key (shrK A) ∈ parts (knows Spy evs);  evs ∈ orb⟧ ⟹ A ∈ bad"
```
⟨proof⟩

lemma *new_keys_not_used [simp]:*
```
  "⟦Key K ∉ used evs; K ∈ symKeys; evs ∈ orb⟧  ⟹ K ∉ keysFor (parts (knows
Spy evs))"
```
⟨proof⟩

12.1 Proofs involving analz

Describes the form of K and NA when the Server sends this message. Also for Oops case.

lemma *Says_Server_message_form:*
```
"⟦Says Server B  {|Nonce M, Crypt (shrK B) {|X, Nonce Nb, Key K|}|} ∈ set evs;
```

```
     evs ∈ orb⟧
 ⟹ K ∉ range shrK ∧ (∃ A Na. X=(Crypt (shrK A) ⦃Nonce Na, Key K⦄))"
```
⟨proof⟩

lemma *Says_Server_imp_Gets:*
```
 "⟦Says Server B ⦃Nonce M, Crypt (shrK B) ⦃Crypt (shrK A) ⦃Nonce Na, Key K⦄,
                                              Nonce Nb, Key K⦄⦄ ∈ set evs;
    evs ∈ orb⟧
  ⟹  Gets Server ⦃Nonce M, Agent A, Agent B,
                   Crypt (shrK A) ⦃Nonce Na, Nonce M, Agent A, Agent B⦄,
               Crypt (shrK B) ⦃Nonce Nb, Nonce M, Nonce M, Agent A, Agent
B⦄⦄
         ∈ set evs"
```
⟨proof⟩

lemma *A_trusts_OR1:*
```
"⟦Crypt (shrK A) ⦃Nonce Na, Nonce M, Agent A, Agent B⦄ ∈ parts (knows Spy
evs);
    A ∉ bad; evs ∈ orb⟧
 ⟹ Says A B ⦃Nonce M, Agent A, Agent B, Crypt (shrK A) ⦃Nonce Na, Nonce
M, Agent A, Agent B⦄⦄ ∈ set evs"
```
⟨proof⟩

lemma *B_trusts_OR2:*
```
 "⟦Crypt (shrK B) ⦃Nonce Nb, Nonce M, Nonce M, Agent A, Agent B⦄
      ∈ parts (knows Spy evs);  B ∉ bad; evs ∈ orb⟧
  ⟹ (∃ X. Says B Server ⦃Nonce M, Agent A, Agent B, X,
              Crypt (shrK B) ⦃Nonce Nb, Nonce M, Nonce M, Agent A, Agent B⦄⦄

          ∈ set evs)"
```
⟨proof⟩

lemma *B_trusts_OR3:*
```
"⟦Crypt (shrK B) ⦃X, Nonce Nb, Key K⦄ ∈ parts (knows Spy evs);
   B ∉ bad; evs ∈ orb⟧
⟹ ∃ M. Says Server B ⦃Nonce M, Crypt (shrK B) ⦃X, Nonce Nb, Key K⦄⦄
         ∈ set evs"
```
⟨proof⟩

lemma *Gets_Server_message_form:*
```
"⟦Gets B ⦃Nonce M, Crypt (shrK B) ⦃X, Nonce Nb, Key K⦄⦄ ∈ set evs;
    evs ∈ orb⟧
 ⟹ (K ∉ range shrK ∧ (∃ A Na. X = (Crypt (shrK A) ⦃Nonce Na, Key K⦄)))

             | X ∈ analz (knows Spy evs)"
```
⟨proof⟩

lemma *unique_Na:* `"⟦Says A B  ⦃Nonce M, Agent A, Agent B, Crypt (shrK A) ⦃Nonce`
```
Na, Nonce M, Agent A, Agent B⦄⦄ ∈ set evs;
         Says A B' ⦃Nonce M', Agent A, Agent B', Crypt (shrK A) ⦃Nonce Na,
```

```
Nonce M', Agent A, Agent B'⦄⦄ ∈ set evs;
    A ∉ bad; evs ∈ orb⟧ ⟹ B=B' ∧ M=M'"
```
⟨proof⟩

lemma unique_Nb:
```
"⟦Says B Server ⦃Nonce M, Agent A, Agent B, X, Crypt (shrK
B) ⦃Nonce Nb, Nonce M, Nonce M, Agent A, Agent B⦄⦄ ∈ set evs;
         Says B Server ⦃Nonce M', Agent A', Agent B, X', Crypt (shrK B) ⦃Nonce
Nb,Nonce M', Nonce M', Agent A', Agent B⦄⦄ ∈ set evs;
    B ∉ bad; evs ∈ orb⟧ ⟹   M=M' ∧ A=A' ∧ X=X'"
```
⟨proof⟩

lemma analz_image_freshCryptK_lemma:
```
"(Crypt K X ∈ analz (Key`nE ∪ H)) ⟶ (Crypt K X ∈ analz H) ⟹
        (Crypt K X ∈ analz (Key`nE ∪ H)) = (Crypt K X ∈ analz H)"
```
⟨proof⟩

⟨ML⟩

lemma analz_image_freshCryptK [rule_format]:
```
"evs ∈ orb ⟹
     Key K ∉ analz (knows Spy evs) ⟶
       (∀ KK. KK ⊆ - (range shrK) ⟶
             (Crypt K X ∈ analz (Key`KK ∪ (knows Spy evs))) =
             (Crypt K X ∈ analz (knows Spy evs)))"
```
⟨proof⟩

lemma analz_insert_freshCryptK:
```
"⟦evs ∈ orb;  Key K ∉ analz (knows Spy evs);
         Seskey ∉ range shrK⟧ ⟹
         (Crypt K X ∈ analz (insert (Key Seskey) (knows Spy evs))) =
         (Crypt K X ∈ analz (knows Spy evs))"
```
⟨proof⟩

lemma analz_hard:
```
"⟦Says A B ⦃Nonce M, Agent A, Agent B,
             Crypt (shrK A) ⦃Nonce Na, Nonce M, Agent A, Agent B⦄⦄ ∈set evs;

   Crypt (shrK A) ⦃Nonce Na, Key K⦄ ∈ analz (knows Spy evs);
   A ∉ bad; B ∉ bad; evs ∈ orb⟧
 ⟹  Says B A ⦃Nonce M, Crypt (shrK A) ⦃Nonce Na, Key K⦄⦄ ∈ set evs"
```
⟨proof⟩

lemma Gets_Server_message_form':
```
"⟦Gets B ⦃Nonce M, Crypt (shrK B) ⦃X, Nonce Nb, Key K⦄⦄  ∈ set evs;
   B ∉ bad; evs ∈ orb⟧
  ⟹ K ∉ range shrK ∧ (∃ A Na. X = (Crypt (shrK A) ⦃Nonce Na, Key K⦄))"
```
⟨proof⟩

lemma *OR4_imp_Gets:*

```
"⟦Says B A {|Nonce M, Crypt (shrK A) {|Nonce Na, Key K|}|} ∈ set evs;
   B ∉ bad; evs ∈ orb⟧
 ⟹ (∃ Nb. Gets B {|Nonce M, Crypt (shrK B) {|Crypt (shrK A) {|Nonce Na, Key
K|},
                                              Nonce Nb, Key K|}|} ∈ set evs)"
```

⟨*proof*⟩

lemma *A_keydist_to_B:*

```
"⟦Says A B {|Nonce M, Agent A, Agent B,
            Crypt (shrK A) {|Nonce Na, Nonce M, Agent A, Agent B|}|} ∈set evs;

   Gets A {|Nonce M, Crypt (shrK A) {|Nonce Na, Key K|}|} ∈ set evs;
   A ∉ bad; B ∉ bad; evs ∈ orb⟧
  ⟹ Key K ∈ analz (knows B evs)"
```

⟨*proof*⟩

Other properties as for the original protocol

end

13 The Woo-Lam Protocol

theory *WooLam* **imports** *Public* **begin**

Simplified version from page 11 of Abadi and Needham (1996). Prudent Engineering Practice for Cryptographic Protocols. IEEE Trans. S.E. 22(1), pages 6-15.

Note: this differs from the Woo-Lam protocol discussed by Lowe (1996): Some New Attacks upon Security Protocols. Computer Security Foundations Workshop

inductive_set *woolam :: "event list set"*
  where

```
   Nil:  "[] ∈ woolam"


 | Fake: "⟦evsf ∈ woolam;  X ∈ synth (analz (spies evsf))⟧
          ⟹ Says Spy B X  # evsf ∈ woolam"


 | WL1:  "evs1 ∈ woolam ⟹ Says A B (Agent A) # evs1 ∈ woolam"


 | WL2:  "⟦evs2 ∈ woolam;  Says A' B (Agent A) ∈ set evs2⟧
          ⟹ Says B A (Nonce NB) # evs2 ∈ woolam"


 | WL3:  "⟦evs3 ∈ woolam;
```

```
            Says A  B (Agent A)  ∈ set evs3;
            Says B' A (Nonce NB) ∈ set evs3⟧
          ⟹ Says A B (Crypt (shrK A) (Nonce NB)) # evs3 ∈ woolam"

 | WL4:  "⟦evs4 ∈ woolam;
            Says A'  B X         ∈ set evs4;
            Says A'' B (Agent A) ∈ set evs4⟧
          ⟹ Says B Server {|Agent A, Agent B, X|} # evs4 ∈ woolam"

 | WL5:  "⟦evs5 ∈ woolam;
            Says B' Server {|Agent A, Agent B, Crypt (shrK A) (Nonce NB)|}
              ∈ set evs5⟧
          ⟹ Says Server B (Crypt (shrK B) {|Agent A, Nonce NB|})
                # evs5 ∈ woolam"
```

declare *Says_imp_knows_Spy [THEN analz.Inj, dest]*
declare *parts.Body [dest]*
declare *analz_into_parts [dest]*
declare *Fake_parts_insert_in_Un [dest]*

lemma *"∃NB. ∃evs ∈ woolam.*
Says Server B (Crypt (shrK B) {|Agent A, Nonce NB|}) ∈ set evs"
⟨*proof*⟩

lemma *Spy_see_shrK [simp]:*
"evs ∈ woolam ⟹ (Key (shrK A) ∈ parts (spies evs)) = (A ∈ bad)"
⟨*proof*⟩

lemma *Spy_analz_shrK [simp]:*
"evs ∈ woolam ⟹ (Key (shrK A) ∈ analz (spies evs)) = (A ∈ bad)"
⟨*proof*⟩

lemma *Spy_see_shrK_D [dest!]:*
"⟦Key (shrK A) ∈ parts (knows Spy evs); evs ∈ woolam⟧ ⟹ A ∈ bad"
⟨*proof*⟩

lemma *NB_Crypt_imp_Alice_msg:*
```
"⟦Crypt (shrK A) (Nonce NB) ∈ parts (spies evs);
    A ∉ bad;  evs ∈ woolam⟧
 ⟹ ∃B. Says A B (Crypt (shrK A) (Nonce NB)) ∈ set evs"
```
⟨*proof*⟩

lemma *Server_trusts_WL4 [dest]:*
```
"⟦Says B' Server {|Agent A, Agent B, Crypt (shrK A) (Nonce NB)|}
      ∈ set evs;
    A ∉ bad;  evs ∈ woolam⟧
 ⟹ ∃B. Says A B (Crypt (shrK A) (Nonce NB)) ∈ set evs"
```
⟨*proof*⟩

lemma *Server_sent_WL5 [dest]:*
```
"⟦Says Server B (Crypt (shrK B) {|Agent A, NB|}) ∈ set evs;
    evs ∈ woolam⟧
 ⟹ ∃B'. Says B' Server {|Agent A, Agent B, Crypt (shrK A) NB|}
        ∈ set evs"
```
⟨*proof*⟩

lemma *NB_Crypt_imp_Server_msg [rule_format]:*
```
"⟦Crypt (shrK B) {|Agent A, NB|} ∈ parts (spies evs);
    B ∉ bad;  evs ∈ woolam⟧
 ⟹ Says Server B (Crypt (shrK B) {|Agent A, NB|}) ∈ set evs"
```
⟨*proof*⟩

lemma *B_trusts_WL5:*
```
"⟦Says S B (Crypt (shrK B) {|Agent A, Nonce NB|}) ∈ set evs;
    A ∉ bad;  B ∉ bad;  evs ∈ woolam⟧
 ⟹ ∃B. Says A B (Crypt (shrK A) (Nonce NB)) ∈ set evs"
```
⟨*proof*⟩

lemma *B_said_WL2:*
```
"⟦Says B A (Nonce NB) ∈ set evs;  B ≠ Spy;  evs ∈ woolam⟧
 ⟹ ∃A'. Says A' B (Agent A) ∈ set evs"
```
⟨*proof*⟩

lemma *"⟦A ∉ bad; B ≠ Spy; evs ∈ woolam⟧*
```
⟹ Crypt (shrK A) (Nonce NB) ∈ parts (spies evs) ∧
    Says B A (Nonce NB) ∈ set evs
    ⟶ Says A B (Crypt (shrK A) (Nonce NB)) ∈ set evs"
```
⟨*proof*⟩

end

14 The Otway-Bull Recursive Authentication Protocol

theory *Recur* **imports** *Public* **begin**

End marker for message bundles

```
abbreviation
  END :: "msg" where
  "END == Number 0"


inductive_set
  respond :: "event list ⇒ (msg*msg*key)set"
  for evs :: "event list"
  where
   One:  "Key KAB ∉ used evs
          ⟹ (Hash[Key(shrK A)] ⦃Agent A, Agent B, Nonce NA, END⦄,
               ⦃Crypt (shrK A) ⦃Key KAB, Agent B, Nonce NA⦄, END⦄,
               KAB)   ∈ respond evs"


 | Cons: "⟦(PA, RA, KAB) ∈ respond evs;
             Key KBC ∉ used evs;  Key KBC ∉ parts {RA};
             PA = Hash[Key(shrK A)] ⦃Agent A, Agent B, Nonce NA, P⦄⟧
          ⟹ (Hash[Key(shrK B)] ⦃Agent B, Agent C, Nonce NB, PA⦄,
               ⦃Crypt (shrK B) ⦃Key KBC, Agent C, Nonce NB⦄,
                 Crypt (shrK B) ⦃Key KAB, Agent A, Nonce NB⦄,
                 RA⦄,
               KBC)
              ∈ respond evs"



inductive_set
  responses :: "event list => msg set"
  for evs :: "event list"
  where

   Nil:  "END ∈ responses evs"

 | Cons: "⟦RA ∈ responses evs;  Key KAB ∉ used evs⟧
          ⟹ ⦃Crypt (shrK B) ⦃Key KAB, Agent A, Nonce NB⦄,
                RA⦄  ∈ responses evs"


inductive_set recur :: "event list set"
  where

   Nil:  "[] ∈ recur"
```

```
| Fake: "⟦evsf ∈ recur;  X ∈ synth (analz (knows Spy evsf))⟧
        ⟹ Says Spy B X  # evsf ∈ recur"


| RA1:  "⟦evs1 ∈ recur;  Nonce NA ∉ used evs1⟧
        ⟹ Says A B (Hash[Key(shrK A)] ⦃Agent A, Agent B, Nonce NA, END⦄)
            # evs1 ∈ recur"


| RA2:  "⟦evs2 ∈ recur;  Nonce NB ∉ used evs2;
           Says A' B PA ∈ set evs2⟧
        ⟹ Says B C (Hash[Key(shrK B)] ⦃Agent B, Agent C, Nonce NB, PA⦄)
            # evs2 ∈ recur"


| RA3:  "⟦evs3 ∈ recur;  Says B' Server PB ∈ set evs3;
          (PB,RB,K) ∈ respond evs3⟧
        ⟹ Says Server B RB # evs3 ∈ recur"


| RA4:  "⟦evs4 ∈ recur;
           Says B  C ⦃XH, Agent B, Agent C, Nonce NB,
                       XA, Agent A, Agent B, Nonce NA, P⦄ ∈ set evs4;
           Says C' B ⦃Crypt (shrK B) ⦃Key KBC, Agent C, Nonce NB⦄,
                       Crypt (shrK B) ⦃Key KAB, Agent A, Nonce NB⦄,
                       RA⦄ ∈ set evs4⟧
        ⟹ Says B A RA # evs4 ∈ recur"
```

declare *Says_imp_knows_Spy [THEN analz.Inj, dest]*
declare *parts.Body [dest]*
declare *analz_into_parts [dest]*
declare *Fake_parts_insert_in_Un [dest]*

Simplest case: Alice goes directly to the server

```
lemma "Key K ∉ used []
       ⟹ ∃NA. ∃evs ∈ recur.
              Says Server A ⦃Crypt (shrK A) ⦃Key K, Agent Server, Nonce NA⦄,
                    END⦄  ∈ set evs"
```
⟨*proof*⟩

Case two: Alice, Bob and the server

```
lemma "⟦Key K ∉ used []; Key K' ∉ used []; K ≠ K';
         Nonce NA ∉ used []; Nonce NB ∉ used []; NA < NB⟧
       ⟹ ∃NA. ∃evs ∈ recur.
        Says B A ⦃Crypt (shrK A) ⦃Key K, Agent B, Nonce NA⦄,
                   END⦄  ∈ set evs"
```
⟨*proof*⟩

```
lemma "⟦Key K ∉ used []; Key K' ∉ used [];
         Key K'' ∉ used []; K ≠ K'; K' ≠ K''; K ≠ K'';
```

```
        Nonce NA ∉ used []; Nonce NB ∉ used []; Nonce NC ∉ used [];
        NA < NB; NB < NC⟧
     ⟹ ∃K. ∃NA. ∃evs ∈ recur.
           Says B A {|Crypt (shrK A) {|Key K, Agent B, Nonce NA|},
                      END|}  ∈ set evs"
```
⟨proof⟩

lemma *respond_imp_not_used: "(PA,RB,KAB) ∈ respond evs ⟹ Key KAB ∉ used evs"*
⟨proof⟩

lemma *Key_in_parts_respond [rule_format]:*
"⟦Key K ∈ parts {RB}; (PB,RB,K') ∈ respond evs⟧ ⟹ Key K ∉ used evs"
⟨proof⟩

Simple inductive reasoning about responses

lemma *respond_imp_responses:*
"(PA,RB,KAB) ∈ respond evs ⟹ RB ∈ responses evs"
⟨proof⟩

lemmas *RA2_analz_spies = Says_imp_spies [THEN analz.Inj]*

lemma *RA4_analz_spies:*
"Says C' B {|Crypt K X, X', RA|} ∈ set evs ⟹ RA ∈ analz (spies evs)"
⟨proof⟩

lemmas *RA2_parts_spies = RA2_analz_spies [THEN analz_into_parts]*
lemmas *RA4_parts_spies = RA4_analz_spies [THEN analz_into_parts]*

lemma *Spy_see_shrK [simp]:*
"evs ∈ recur ⟹ (Key (shrK A) ∈ parts (spies evs)) = (A ∈ bad)"
⟨proof⟩

lemma *Spy_analz_shrK [simp]:*
"evs ∈ recur ⟹ (Key (shrK A) ∈ analz (spies evs)) = (A ∈ bad)"
⟨proof⟩

lemma *Spy_see_shrK_D [dest!]:*
"⟦Key (shrK A) ∈ parts (knows Spy evs); evs ∈ recur⟧ ⟹ A ∈ bad"
⟨proof⟩

lemma *resp_analz_image_freshK_lemma:*
```
     "⟦RB ∈ responses evs;
         ∀K KK. KK ⊆ - (range shrK) ⟶
                   (Key K ∈ analz (Key‘KK ∪ H)) =
                   (K ∈ KK | Key K ∈ analz H)⟧
     ⟹ ∀K KK. KK ⊆ - (range shrK) ⟶
                   (Key K ∈ analz (insert RB (Key‘KK ∪ H))) =
                   (K ∈ KK | Key K ∈ analz (insert RB H))"
```
⟨*proof*⟩

Version for the protocol. Proof is easy, thanks to the lemma.

lemma *raw_analz_image_freshK:*
```
 "evs ∈ recur ⟹
   ∀K KK. KK ⊆ - (range shrK) ⟶
          (Key K ∈ analz (Key‘KK ∪ (spies evs))) =
          (K ∈ KK | Key K ∈ analz (spies evs))"
```
⟨*proof*⟩

lemmas *resp_analz_image_freshK =*
```
       resp_analz_image_freshK_lemma [OF _ raw_analz_image_freshK]
```

lemma *analz_insert_freshK:*
```
     "⟦evs ∈ recur;  KAB ∉ range shrK⟧
      ⟹ (Key K ∈ analz (insert (Key KAB) (spies evs))) =
          (K = KAB | Key K ∈ analz (spies evs))"
```
⟨*proof*⟩

Everything that's hashed is already in past traffic.

lemma *Hash_imp_body:*
```
     "⟦Hash {|Key(shrK A), X|} ∈ parts (spies evs);
         evs ∈ recur;  A ∉ bad⟧ ⟹ X ∈ parts (spies evs)"
```
⟨*proof*⟩

lemma *unique_NA:*
```
  "⟦Hash {|Key(shrK A), Agent A, B, NA, P|} ∈ parts (spies evs);
      Hash {|Key(shrK A), Agent A, B',NA, P'|} ∈ parts (spies evs);
      evs ∈ recur;  A ∉ bad⟧
    ⟹ B=B' ∧ P=P'"
```
⟨*proof*⟩

lemma *shrK_in_analz_respond [simp]:*

```
"⟦RB ∈ responses evs;  evs ∈ recur⟧
 ⟹ (Key (shrK B) ∈ analz (insert RB (spies evs))) = (B∈bad)"
```
⟨*proof*⟩

lemma *resp_analz_insert_lemma:*
```
"⟦Key K ∈ analz (insert RB H);
    ∀K KK. KK ⊆ - (range shrK) ⟶
              (Key K ∈ analz (Key`KK ∪ H)) =
              (K ∈ KK | Key K ∈ analz H);
    RB ∈ responses evs⟧
 ⟹ (Key K ∈ parts{RB} | Key K ∈ analz H)"
```
⟨*proof*⟩

lemmas *resp_analz_insert =*
```
  resp_analz_insert_lemma [OF _ raw_analz_image_freshK]
```

The last key returned by respond indeed appears in a certificate

lemma *respond_certificate:*
```
"(Hash[Key(shrK A)] ⦃Agent A, B, NA, P⦄, RA, K) ∈ respond evs
 ⟹ Crypt (shrK A) ⦃Key K, B, NA⦄ ∈ parts {RA}"
```
⟨*proof*⟩

lemma *unique_lemma [rule_format]:*
```
"(PB,RB,KXY) ∈ respond evs ⟹
 ∀A B N. Crypt (shrK A) ⦃Key K, Agent B, N⦄ ∈ parts {RB} ⟶
 (∀A' B' N'. Crypt (shrK A') ⦃Key K, Agent B', N'⦄ ∈ parts {RB} ⟶
 (A'=A ∧ B'=B) | (A'=B ∧ B'=A))"
```
⟨*proof*⟩

lemma *unique_session_keys:*
```
"⟦Crypt (shrK A) ⦃Key K, Agent B, N⦄ ∈ parts {RB};
    Crypt (shrK A') ⦃Key K, Agent B', N'⦄ ∈ parts {RB};
    (PB,RB,KXY) ∈ respond evs⟧
 ⟹ (A'=A ∧ B'=B) | (A'=B ∧ B'=A)"
```
⟨*proof*⟩

lemma *respond_Spy_not_see_session_key [rule_format]:*
```
"⟦(PB,RB,KAB) ∈ respond evs;  evs ∈ recur⟧
 ⟹ ∀A A' N. A ∉ bad ∧ A' ∉ bad ⟶
     Crypt (shrK A) ⦃Key K, Agent A', N⦄ ∈ parts{RB} ⟶
     Key K ∉ analz (insert RB (spies evs))"
```
⟨*proof*⟩

lemma *Spy_not_see_session_key:*
```
"⟦Crypt (shrK A) ⦃Key K, Agent A', N⦄ ∈ parts (spies evs);
    A ∉ bad;  A' ∉ bad;  evs ∈ recur⟧
 ⟹ Key K ∉ analz (spies evs)"
```
⟨*proof*⟩

The response never contains Hashes

lemma *Hash_in_parts_respond:*
```
     "⟦Hash ⦃Key (shrK B), M⦄ ∈ parts (insert RB H);
         (PB,RB,K) ∈ respond evs⟧
      ⟹ Hash ⦃Key (shrK B), M⦄ ∈ parts H"
```
⟨*proof*⟩

Only RA1 or RA2 can have caused such a part of a message to appear. This result is of no use to B, who cannot verify the Hash. Moreover, it can say nothing about how recent A's message is. It might later be used to prove B's presence to A at the run's conclusion.

lemma *Hash_auth_sender [rule_format]:*
```
     "⟦Hash ⦃Key(shrK A), Agent A, Agent B, NA, P⦄ ∈ parts(spies evs);
         A ∉ bad;  evs ∈ recur⟧
      ⟹ Says A B (Hash[Key(shrK A)] ⦃Agent A, Agent B, NA, P⦄) ∈ set evs"
```
⟨*proof*⟩

Certificates can only originate with the Server.

lemma *Cert_imp_Server_msg:*
```
     "⟦Crypt (shrK A) Y ∈ parts (spies evs);
         A ∉ bad;  evs ∈ recur⟧
      ⟹ ∃C RC. Says Server C RC ∈ set evs  ∧
                   Crypt (shrK A) Y ∈ parts {RC}"
```
⟨*proof*⟩

end

15 The Yahalom Protocol

theory *Yahalom* **imports** *Public* **begin**

From page 257 of Burrows, Abadi and Needham (1989). A Logic of Authentication. Proc. Royal Soc. 426

This theory has the prototypical example of a secrecy relation, KeyCryptNonce.

inductive_set *yahalom :: "event list set"*
  where

```
   Nil:  "[] ∈ yahalom"


 | Fake: "⟦evsf ∈ yahalom;  X ∈ synth (analz (knows Spy evsf))⟧
          ⟹ Says Spy B X  # evsf ∈ yahalom"


 | Reception: "⟦evsr ∈ yahalom;  Says A B X ∈ set evsr⟧
               ⟹ Gets B X # evsr ∈ yahalom"


 | YM1:  "⟦evs1 ∈ yahalom;  Nonce NA ∉ used evs1⟧
          ⟹ Says A B ⦃Agent A, Nonce NA⦄ # evs1 ∈ yahalom"
```

```
| YM2:  "⟦evs2 ∈ yahalom;  Nonce NB ∉ used evs2;
            Gets B ⦃Agent A, Nonce NA⦄ ∈ set evs2⟧
         ⟹ Says B Server
                 ⦃Agent B, Crypt (shrK B) ⦃Agent A, Nonce NA, Nonce NB⦄⦄
               # evs2 ∈ yahalom"


| YM3:  "⟦evs3 ∈ yahalom;  Key KAB ∉ used evs3;  KAB ∈ symKeys;
            Gets Server
                 ⦃Agent B, Crypt (shrK B) ⦃Agent A, Nonce NA, Nonce NB⦄⦄
              ∈ set evs3⟧
         ⟹ Says Server A
                  ⦃Crypt (shrK A) ⦃Agent B, Key KAB, Nonce NA, Nonce NB⦄,
                    Crypt (shrK B) ⦃Agent A, Key KAB⦄⦄
               # evs3 ∈ yahalom"

| YM4:
```

— Alice receives the Server's (?) message, checks her Nonce, and uses the new session key to send Bob his Nonce. The premise *A ≠ Server* is needed for *Says_Server_not_range*. Alice can check that K is symmetric by its length.

```
        "⟦evs4 ∈ yahalom;  A ≠ Server;  K ∈ symKeys;
            Gets A ⦃Crypt(shrK A) ⦃Agent B, Key K, Nonce NA, Nonce NB⦄, X⦄
               ∈ set evs4;
            Says A B ⦃Agent A, Nonce NA⦄ ∈ set evs4⟧
         ⟹ Says A B ⦃X, Crypt K (Nonce NB)⦄ # evs4 ∈ yahalom"


| Oops: "⟦evso ∈ yahalom;
            Says Server A ⦃Crypt (shrK A)
                                 ⦃Agent B, Key K, Nonce NA, Nonce NB⦄,
                             X⦄  ∈ set evso⟧
         ⟹ Notes Spy ⦃Nonce NA, Nonce NB, Key K⦄ # evso ∈ yahalom"
```

definition *KeyWithNonce :: "[key, nat, event list] ⇒ bool"* **where**

```
  "KeyWithNonce K NB evs ==
     ∃A B na X.
       Says Server A ⦃Crypt (shrK A) ⦃Agent B, Key K, na, Nonce NB⦄, X⦄
         ∈ set evs"
```

declare *Says_imp_analz_Spy [dest]*
declare *parts.Body [dest]*
declare *Fake_parts_insert_in_Un [dest]*
declare *analz_into_parts [dest]*

A "possibility property": there are traces that reach the end

lemma *"⟦A ≠ Server; K ∈ symKeys; Key K ∉ used []⟧*

```
      ⟹ ∃X NB. ∃evs ∈ yahalom.
            Says A B ⦃X, Crypt K (Nonce NB)⦄ ∈ set evs"
```

⟨*proof*⟩

15.1 Regularity Lemmas for Yahalom

lemma *Gets_imp_Says:*
```
"⟦Gets B X ∈ set evs; evs ∈ yahalom⟧ ⟹ ∃A. Says A B X ∈ set evs"
```
⟨*proof*⟩

Must be proved separately for each protocol

lemma *Gets_imp_knows_Spy:*
```
"⟦Gets B X ∈ set evs; evs ∈ yahalom⟧  ⟹ X ∈ knows Spy evs"
```
⟨*proof*⟩

lemmas *Gets_imp_analz_Spy = Gets_imp_knows_Spy [THEN analz.Inj]*
declare *Gets_imp_analz_Spy [dest]*

Lets us treat YM4 using a similar argument as for the Fake case.

lemma *YM4_analz_knows_Spy:*
```
"⟦Gets A {|Crypt (shrK A) Y, X|} ∈ set evs;  evs ∈ yahalom⟧
 ⟹ X ∈ analz (knows Spy evs)"
```
⟨*proof*⟩

lemmas *YM4_parts_knows_Spy =*
YM4_analz_knows_Spy [THEN analz_into_parts]

For Oops

lemma *YM4_Key_parts_knows_Spy:*
```
"Says Server A {|Crypt (shrK A) {|B,K,NA,NB|}, X|} ∈ set evs
 ⟹ K ∈ parts (knows Spy evs)"
```
⟨*proof*⟩

Theorems of the form *X ∉ parts (knows Spy evs)* imply that NOBODY sends messages containing X!

Spy never sees a good agent's shared key!

lemma *Spy_see_shrK [simp]:*
```
"evs ∈ yahalom ⟹ (Key (shrK A) ∈ parts (knows Spy evs)) = (A ∈ bad)"
```
⟨*proof*⟩

lemma *Spy_analz_shrK [simp]:*
```
"evs ∈ yahalom ⟹ (Key (shrK A) ∈ analz (knows Spy evs)) = (A ∈ bad)"
```
⟨*proof*⟩

lemma *Spy_see_shrK_D [dest!]:*
```
"⟦Key (shrK A) ∈ parts (knows Spy evs);  evs ∈ yahalom⟧ ⟹ A ∈ bad"
```
⟨*proof*⟩

Nobody can have used non-existent keys! Needed to apply *analz_insert_Key*

lemma *new_keys_not_used [simp]:*
```
"⟦Key K ∉ used evs; K ∈ symKeys; evs ∈ yahalom⟧
⟹ K ∉ keysFor (parts (spies evs))"
```
⟨*proof*⟩

Earlier, all protocol proofs declared this theorem. But only a few proofs need it, e.g. Yahalom and Kerberos IV.

lemma *new_keys_not_analzd:*
```
 "⟦K ∈ symKeys; evs ∈ yahalom; Key K ∉ used evs⟧
  ⟹ K ∉ keysFor (analz (knows Spy evs))"
```
⟨*proof*⟩

Describes the form of K when the Server sends this message. Useful for Oops as well as main secrecy property.

lemma *Says_Server_not_range [simp]:*
```
     "⟦Says Server A ⦃Crypt (shrK A) ⦃Agent B, Key K, na, nb⦄, X⦄
          ∈ set evs;   evs ∈ yahalom⟧
      ⟹ K ∉ range shrK"
```
⟨*proof*⟩

15.2 Secrecy Theorems

Session keys are not used to encrypt other session keys

lemma *analz_image_freshK [rule_format]:*
```
 "evs ∈ yahalom ⟹
   ∀K KK. KK ⊆ - (range shrK) ⟶
          (Key K ∈ analz (Key`KK ∪ (knows Spy evs))) =
          (K ∈ KK | Key K ∈ analz (knows Spy evs))"
```
⟨*proof*⟩

lemma *analz_insert_freshK:*
```
     "⟦evs ∈ yahalom;  KAB ∉ range shrK⟧ ⟹
      (Key K ∈ analz (insert (Key KAB) (knows Spy evs))) =
      (K = KAB | Key K ∈ analz (knows Spy evs))"
```
⟨*proof*⟩

The Key K uniquely identifies the Server's message.

lemma *unique_session_keys:*
```
     "⟦Says Server A
          ⦃Crypt (shrK A) ⦃Agent B, Key K, na, nb⦄, X⦄ ∈ set evs;
        Says Server A'
          ⦃Crypt (shrK A') ⦃Agent B', Key K, na', nb'⦄, X'⦄ ∈ set evs;
        evs ∈ yahalom⟧
     ⟹ A=A' ∧ B=B' ∧ na=na' ∧ nb=nb'"
```
⟨*proof*⟩

Crucial secrecy property: Spy does not see the keys sent in msg YM3

lemma *secrecy_lemma:*
```
     "⟦A ∉ bad;  B ∉ bad;  evs ∈ yahalom⟧
      ⟹ Says Server A
            ⦃Crypt (shrK A) ⦃Agent B, Key K, na, nb⦄,
              Crypt (shrK B) ⦃Agent A, Key K⦄⦄
           ∈ set evs ⟶
          Notes Spy ⦃na, nb, Key K⦄ ∉ set evs ⟶
          Key K ∉ analz (knows Spy evs)"
```
⟨*proof*⟩

Final version

lemma *Spy_not_see_encrypted_key:*

```
    "⟦Says Server A
          ⦃Crypt (shrK A) ⦃Agent B, Key K, na, nb⦄,
            Crypt (shrK B) ⦃Agent A, Key K⦄⦄
         ∈ set evs;
       Notes Spy ⦃na, nb, Key K⦄ ∉ set evs;
       A ∉ bad;  B ∉ bad;  evs ∈ yahalom⟧
     ⟹ Key K ∉ analz (knows Spy evs)"
```
⟨*proof*⟩

15.2.1 Security Guarantee for A upon receiving YM3

If the encrypted message appears then it originated with the Server

lemma *A_trusts_YM3:*
```
    "⟦Crypt (shrK A) ⦃Agent B, Key K, na, nb⦄ ∈ parts (knows Spy evs);
       A ∉ bad;  evs ∈ yahalom⟧
      ⟹ Says Server A
          ⦃Crypt (shrK A) ⦃Agent B, Key K, na, nb⦄,
            Crypt (shrK B) ⦃Agent A, Key K⦄⦄
         ∈ set evs"
```
⟨*proof*⟩

The obvious combination of *A_trusts_YM3* with *Spy_not_see_encrypted_key*

lemma *A_gets_good_key:*
```
    "⟦Crypt (shrK A) ⦃Agent B, Key K, na, nb⦄ ∈ parts (knows Spy evs);
       Notes Spy ⦃na, nb, Key K⦄ ∉ set evs;
       A ∉ bad;  B ∉ bad;  evs ∈ yahalom⟧
     ⟹ Key K ∉ analz (knows Spy evs)"
```
⟨*proof*⟩

15.2.2 Security Guarantees for B upon receiving YM4

B knows, by the first part of A's message, that the Server distributed the key for A and B. But this part says nothing about nonces.

lemma *B_trusts_YM4_shrK:*
```
    "⟦Crypt (shrK B) ⦃Agent A, Key K⦄ ∈ parts (knows Spy evs);
       B ∉ bad;  evs ∈ yahalom⟧
     ⟹ ∃NA NB. Says Server A
                   ⦃Crypt (shrK A) ⦃Agent B, Key K,
                                     Nonce NA, Nonce NB⦄,
                     Crypt (shrK B) ⦃Agent A, Key K⦄⦄
                  ∈ set evs"
```
⟨*proof*⟩

B knows, by the second part of A's message, that the Server distributed the key quoting nonce NB. This part says nothing about agent names. Secrecy of NB is crucial. Note that *Nonce NB ∉ analz (knows Spy evs)* must be the FIRST antecedent of the induction formula.

lemma *B_trusts_YM4_newK [rule_format]:*
```
    "⟦Crypt K (Nonce NB) ∈ parts (knows Spy evs);
      Nonce NB ∉ analz (knows Spy evs);  evs ∈ yahalom⟧
     ⟹ ∃A B NA. Says Server A
                   ⦃Crypt (shrK A) ⦃Agent B, Key K, Nonce NA, Nonce NB⦄,
```

```
                        Crypt (shrK B) {|Agent A, Key K|}|}
                      ∈ set evs"
```
⟨proof⟩

15.2.3 Towards proving secrecy of Nonce NB

Lemmas about the predicate KeyWithNonce

lemma *KeyWithNonceI:*
```
 "Says Server A
          {|Crypt (shrK A) {|Agent B, Key K, na, Nonce NB|}, X|}
        ∈ set evs ⟹ KeyWithNonce K NB evs"
```
⟨proof⟩

lemma *KeyWithNonce_Says [simp]:*
```
   "KeyWithNonce K NB (Says S A X # evs) =
      (Server = S ∧
       (∃B n X'. X = {|Crypt (shrK A) {|Agent B, Key K, n, Nonce NB|}, X'|})
      | KeyWithNonce K NB evs)"
```
⟨proof⟩

lemma *KeyWithNonce_Notes [simp]:*
```
   "KeyWithNonce K NB (Notes A X # evs) = KeyWithNonce K NB evs"
```
⟨proof⟩

lemma *KeyWithNonce_Gets [simp]:*
```
   "KeyWithNonce K NB (Gets A X # evs) = KeyWithNonce K NB evs"
```
⟨proof⟩

A fresh key cannot be associated with any nonce (with respect to a given trace).

lemma *fresh_not_KeyWithNonce:*
```
     "Key K ∉ used evs ⟹ ¬ KeyWithNonce K NB evs"
```
⟨proof⟩

The Server message associates K with NB' and therefore not with any other nonce NB.

lemma *Says_Server_KeyWithNonce:*
```
 "⟦Says Server A {|Crypt (shrK A) {|Agent B, Key K, na, Nonce NB'|}, X|}
       ∈ set evs;
     NB ≠ NB';  evs ∈ yahalom⟧
  ⟹ ¬ KeyWithNonce K NB evs"
```
⟨proof⟩

The only nonces that can be found with the help of session keys are those distributed as nonce NB by the Server. The form of the theorem recalls `analz_image_freshK`, but it is much more complicated.

As with `analz_image_freshK`, we take some pains to express the property as a logical equivalence so that the simplifier can apply it.

lemma *Nonce_secrecy_lemma:*
```
     "P ⟶ (X ∈ analz (G ∪ H)) ⟶ (X ∈ analz H)  ⟹
      P ⟶ (X ∈ analz (G ∪ H)) = (X ∈ analz H)"
```

⟨proof⟩

lemma *Nonce_secrecy:*

```
"evs ∈ yahalom ⟹
 (∀KK. KK ⊆ - (range shrK) ⟶
      (∀K ∈ KK. K ∈ symKeys ⟶ ¬ KeyWithNonce K NB evs)   ⟶
      (Nonce NB ∈ analz (Key`KK ∪ (knows Spy evs))) =
      (Nonce NB ∈ analz (knows Spy evs)))"
```

⟨proof⟩

Version required below: if NB can be decrypted using a session key then it was distributed with that key. The more general form above is required for the induction to carry through.

lemma *single_Nonce_secrecy:*

```
"⟦Says Server A
    ⦃Crypt (shrK A) ⦃Agent B, Key KAB, na, Nonce NB'⦄, X⦄
   ∈ set evs;
   NB ≠ NB';  KAB ∉ range shrK;  evs ∈ yahalom⟧
 ⟹ (Nonce NB ∈ analz (insert (Key KAB) (knows Spy evs))) =
    (Nonce NB ∈ analz (knows Spy evs))"
```

⟨proof⟩

15.2.4 The Nonce NB uniquely identifies B's message.

lemma *unique_NB:*

```
"⟦Crypt (shrK B) ⦃Agent A, Nonce NA, nb⦄ ∈ parts (knows Spy evs);
   Crypt (shrK B') ⦃Agent A', Nonce NA', nb⦄ ∈ parts (knows Spy evs);
  evs ∈ yahalom;  B ∉ bad;  B' ∉ bad⟧
 ⟹ NA' = NA ∧ A' = A ∧ B' = B"
```

⟨proof⟩

Variant useful for proving secrecy of NB. Because nb is assumed to be secret, we no longer must assume B, B' not bad.

lemma *Says_unique_NB:*

```
"⟦Says C S   ⦃X,  Crypt (shrK B) ⦃Agent A, Nonce NA, nb⦄⦄
     ∈ set evs;
   Gets S' ⦃X', Crypt (shrK B') ⦃Agent A', Nonce NA', nb⦄⦄
     ∈ set evs;
   nb ∉ analz (knows Spy evs);  evs ∈ yahalom⟧
 ⟹ NA' = NA ∧ A' = A ∧ B' = B"
```

⟨proof⟩

15.2.5 A nonce value is never used both as NA and as NB

lemma *no_nonce_YM1_YM2:*

```
"⟦Crypt (shrK B') ⦃Agent A', Nonce NB, nb'⦄ ∈ parts(knows Spy evs);
  Nonce NB ∉ analz (knows Spy evs);  evs ∈ yahalom⟧
⟹ Crypt (shrK B)  ⦃Agent A, na, Nonce NB⦄ ∉ parts(knows Spy evs)"
```

⟨proof⟩

The Server sends YM3 only in response to YM2.

lemma *Says_Server_imp_YM2:*

```
"⟦Says Server A ⦃Crypt (shrK A) ⦃Agent B, k, na, nb⦄, X⦄ ∈ set evs;
```

```
         evs ∈ yahalom⟧
      ⟹ Gets Server ⦃Agent B, Crypt (shrK B) ⦃Agent A, na, nb⦄⦄
             ∈ set evs"
```
⟨proof⟩

A vital theorem for B, that nonce NB remains secure from the Spy.

theorem *Spy_not_see_NB :*
```
     "⟦Says B Server
                ⦃Agent B, Crypt (shrK B) ⦃Agent A, Nonce NA, Nonce NB⦄⦄
           ∈ set evs;
         (∀k. Notes Spy ⦃Nonce NA, Nonce NB, k⦄ ∉ set evs);
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom⟧
      ⟹ Nonce NB ∉ analz (knows Spy evs)"
```
⟨proof⟩

B's session key guarantee from YM4. The two certificates contribute to a single conclusion about the Server's message. Note that the "Notes Spy" assumption must quantify over ∀ POSSIBLE keys instead of our particular K. If this run is broken and the spy substitutes a certificate containing an old key, B has no means of telling.

lemma *B_trusts_YM4:*
```
     "⟦Gets B ⦃Crypt (shrK B) ⦃Agent A, Key K⦄,
                  Crypt K (Nonce NB)⦄ ∈ set evs;
         Says B Server
           ⦃Agent B, Crypt (shrK B) ⦃Agent A, Nonce NA, Nonce NB⦄⦄
           ∈ set evs;
         ∀k. Notes Spy ⦃Nonce NA, Nonce NB, k⦄ ∉ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom⟧
       ⟹ Says Server A
                   ⦃Crypt (shrK A) ⦃Agent B, Key K,
                             Nonce NA, Nonce NB⦄,
                     Crypt (shrK B) ⦃Agent A, Key K⦄⦄
             ∈ set evs"
```
⟨proof⟩

The obvious combination of *B_trusts_YM4* with *Spy_not_see_encrypted_key*

lemma *B_gets_good_key:*
```
     "⟦Gets B ⦃Crypt (shrK B) ⦃Agent A, Key K⦄,
                  Crypt K (Nonce NB)⦄ ∈ set evs;
         Says B Server
           ⦃Agent B, Crypt (shrK B) ⦃Agent A, Nonce NA, Nonce NB⦄⦄
           ∈ set evs;
         ∀k. Notes Spy ⦃Nonce NA, Nonce NB, k⦄ ∉ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom⟧
      ⟹ Key K ∉ analz (knows Spy evs)"
```
⟨proof⟩

15.3 Authenticating B to A

The encryption in message YM2 tells us it cannot be faked.

lemma *B_Said_YM2 [rule_format]:*
```
     "⟦Crypt (shrK B) ⦃Agent A, Nonce NA, nb⦄ ∈ parts (knows Spy evs);
```

```
        evs ∈ yahalom⟧
      ⟹ B ∉ bad ⟶
          Says B Server ⦃Agent B, Crypt (shrK B) ⦃Agent A, Nonce NA, nb⦄⦄
            ∈ set evs"
```
⟨proof⟩

If the server sends YM3 then B sent YM2

lemma *YM3_auth_B_to_A_lemma:*
```
     "⟦Says Server A ⦃Crypt (shrK A) ⦃Agent B, Key K, Nonce NA, nb⦄, X⦄
       ∈ set evs;  evs ∈ yahalom⟧
      ⟹ B ∉ bad ⟶
          Says B Server ⦃Agent B, Crypt (shrK B) ⦃Agent A, Nonce NA, nb⦄⦄
            ∈ set evs"
```
⟨proof⟩

If A receives YM3 then B has used nonce NA (and therefore is alive)

theorem *YM3_auth_B_to_A:*
```
     "⟦Gets A ⦃Crypt (shrK A) ⦃Agent B, Key K, Nonce NA, nb⦄, X⦄
           ∈ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom⟧
      ⟹ Says B Server ⦃Agent B, Crypt (shrK B) ⦃Agent A, Nonce NA, nb⦄⦄
       ∈ set evs"
```
⟨proof⟩

15.4 Authenticating A to B using the certificate Crypt K (Nonce NB)

Assuming the session key is secure, if both certificates are present then A has said NB. We can't be sure about the rest of A's message, but only NB matters for freshness.

theorem *A_Said_YM3_lemma [rule_format]:*
```
     "evs ∈ yahalom
      ⟹ Key K ∉ analz (knows Spy evs) ⟶
          Crypt K (Nonce NB) ∈ parts (knows Spy evs) ⟶
          Crypt (shrK B) ⦃Agent A, Key K⦄ ∈ parts (knows Spy evs) ⟶
          B ∉ bad ⟶
          (∃X. Says A B ⦃X, Crypt K (Nonce NB)⦄ ∈ set evs)"
```
⟨proof⟩

If B receives YM4 then A has used nonce NB (and therefore is alive). Moreover, A associates K with NB (thus is talking about the same run). Other premises guarantee secrecy of K.

theorem *YM4_imp_A_Said_YM3 [rule_format]:*
```
     "⟦Gets B ⦃Crypt (shrK B) ⦃Agent A, Key K⦄,
                  Crypt K (Nonce NB)⦄ ∈ set evs;
         Says B Server
           ⦃Agent B, Crypt (shrK B) ⦃Agent A, Nonce NA, Nonce NB⦄⦄
           ∈ set evs;
         (∀NA k. Notes Spy ⦃Nonce NA, Nonce NB, k⦄ ∉ set evs);
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom⟧
      ⟹ ∃X. Says A B ⦃X, Crypt K (Nonce NB)⦄ ∈ set evs"
```
⟨proof⟩

end

16 The Yahalom Protocol, Variant 2

theory *Yahalom2* **imports** *Public* **begin**

This version trades encryption of NB for additional explicitness in YM3. Also in YM3, care is taken to make the two certificates distinct.

From page 259 of Burrows, Abadi and Needham (1989). A Logic of Authentication. Proc. Royal Soc. 426

This theory has the prototypical example of a secrecy relation, KeyCryptNonce.

```
inductive_set yahalom :: "event list set"
  where

   Nil:  "[] ∈ yahalom"


 | Fake: "⟦evsf ∈ yahalom;  X ∈ synth (analz (knows Spy evsf))⟧
         ⟹ Says Spy B X  # evsf ∈ yahalom"


 | Reception: "⟦evsr ∈ yahalom;  Says A B X ∈ set evsr⟧
              ⟹ Gets B X # evsr ∈ yahalom"


 | YM1:  "⟦evs1 ∈ yahalom;  Nonce NA ∉ used evs1⟧
         ⟹ Says A B {|Agent A, Nonce NA|} # evs1 ∈ yahalom"


 | YM2:  "⟦evs2 ∈ yahalom;  Nonce NB ∉ used evs2;
             Gets B {|Agent A, Nonce NA|} ∈ set evs2⟧
         ⟹ Says B Server
                  {|Agent B, Nonce NB, Crypt (shrK B) {|Agent A, Nonce NA|}|}
                # evs2 ∈ yahalom"


 | YM3:  "⟦evs3 ∈ yahalom;  Key KAB ∉ used evs3;
             Gets Server {|Agent B, Nonce NB,
                           Crypt (shrK B) {|Agent A, Nonce NA|}|}
               ∈ set evs3⟧
         ⟹ Says Server A
               {|Nonce NB,
                 Crypt (shrK A) {|Agent B, Key KAB, Nonce NA|},
                 Crypt (shrK B) {|Agent A, Agent B, Key KAB, Nonce NB|}|}
                 # evs3 ∈ yahalom"


 | YM4:  "⟦evs4 ∈ yahalom;
             Gets A {|Nonce NB, Crypt (shrK A) {|Agent B, Key K, Nonce NA|},
                      X|}  ∈ set evs4;
             Says A B {|Agent A, Nonce NA|} ∈ set evs4⟧
```

```
          ⟹ Says A B {|X, Crypt K (Nonce NB)|} # evs4 ∈ yahalom"

 | Oops: "⟦evso ∈ yahalom;
             Says Server A {|Nonce NB,
                           Crypt (shrK A) {|Agent B, Key K, Nonce NA|},
                           X|}  ∈ set evso⟧
          ⟹ Notes Spy {|Nonce NA, Nonce NB, Key K|} # evso ∈ yahalom"
```

declare *Says_imp_knows_Spy [THEN analz.Inj, dest]*
declare *parts.Body [dest]*
declare *Fake_parts_insert_in_Un [dest]*
declare *analz_into_parts [dest]*

A "possibility property": there are traces that reach the end

lemma *"Key K ∉ used []*
⟹ ∃X NB. ∃evs ∈ yahalom.
Says A B {|X, Crypt K (Nonce NB)|} ∈ set evs"
⟨*proof*⟩

lemma *Gets_imp_Says:*
"⟦Gets B X ∈ set evs; evs ∈ yahalom⟧ ⟹ ∃A. Says A B X ∈ set evs"
⟨*proof*⟩

Must be proved separately for each protocol

lemma *Gets_imp_knows_Spy:*
"⟦Gets B X ∈ set evs; evs ∈ yahalom⟧ ⟹ X ∈ knows Spy evs"
⟨*proof*⟩

declare *Gets_imp_knows_Spy [THEN analz.Inj, dest]*

16.1 Inductive Proofs

Result for reasoning about the encrypted portion of messages. Lets us treat YM4 using a similar argument as for the Fake case.

lemma *YM4_analz_knows_Spy:*
"⟦Gets A {|NB, Crypt (shrK A) Y, X|} ∈ set evs; evs ∈ yahalom⟧
⟹ X ∈ analz (knows Spy evs)"
⟨*proof*⟩

lemmas *YM4_parts_knows_Spy =*
YM4_analz_knows_Spy [THEN analz_into_parts]

Spy never sees a good agent's shared key!

lemma *Spy_see_shrK [simp]:*
"evs ∈ yahalom ⟹ (Key (shrK A) ∈ parts (knows Spy evs)) = (A ∈ bad)"
⟨*proof*⟩

lemma *Spy_analz_shrK [simp]:*
"evs ∈ yahalom ⟹ (Key (shrK A) ∈ analz (knows Spy evs)) = (A ∈ bad)"
⟨*proof*⟩

```
lemma Spy_see_shrK_D [dest!]:
    "⟦Key (shrK A) ∈ parts (knows Spy evs);  evs ∈ yahalom⟧ ⟹ A ∈ bad"
⟨proof⟩
```

Nobody can have used non-existent keys! Needed to apply `analz_insert_Key`

```
lemma new_keys_not_used [simp]:
   "⟦Key K ∉ used evs; K ∈ symKeys; evs ∈ yahalom⟧
    ⟹ K ∉ keysFor (parts (spies evs))"
⟨proof⟩
```

Describes the form of K when the Server sends this message. Useful for Oops as well as main secrecy property.

```
lemma Says_Server_message_form:
    "⟦Says Server A {|nb', Crypt (shrK A) {|Agent B, Key K, na|}, X|}
        ∈ set evs;  evs ∈ yahalom⟧
     ⟹ K ∉ range shrK"
⟨proof⟩
```

```
lemma analz_image_freshK [rule_format]:
 "evs ∈ yahalom ⟹
   ∀K KK. KK ⊆ - (range shrK) ⟶
          (Key K ∈ analz (Key`KK ∪ (knows Spy evs))) =
          (K ∈ KK | Key K ∈ analz (knows Spy evs))"
⟨proof⟩
```

```
lemma analz_insert_freshK:
    "⟦evs ∈ yahalom;  KAB ∉ range shrK⟧ ⟹
     (Key K ∈ analz (insert (Key KAB) (knows Spy evs))) =
     (K = KAB | Key K ∈ analz (knows Spy evs))"
⟨proof⟩
```

The Key K uniquely identifies the Server's message

```
lemma unique_session_keys:
    "⟦Says Server A
        {|nb, Crypt (shrK A) {|Agent B, Key K, na|}, X|} ∈ set evs;
      Says Server A'
        {|nb', Crypt (shrK A') {|Agent B', Key K, na'|}, X'|} ∈ set evs;
      evs ∈ yahalom⟧
    ⟹ A=A' ∧ B=B' ∧ na=na' ∧ nb=nb'"
⟨proof⟩
```

16.2 Crucial Secrecy Property: Spy Does Not See Key *KAB*

```
lemma secrecy_lemma:
    "⟦A ∉ bad;  B ∉ bad;  evs ∈ yahalom⟧
     ⟹ Says Server A
          {|nb, Crypt (shrK A) {|Agent B, Key K, na|},
```

```
                Crypt (shrK B) {|Agent A, Agent B, Key K, nb|}|}
         ∈ set evs ⟶
        Notes Spy {|na, nb, Key K|} ∉ set evs ⟶
        Key K ∉ analz (knows Spy evs)"
```
⟨proof⟩

Final version

lemma *Spy_not_see_encrypted_key:*
```
     "⟦Says Server A
           {|nb, Crypt (shrK A) {|Agent B, Key K, na|},
                 Crypt (shrK B) {|Agent A, Agent B, Key K, nb|}|}
        ∈ set evs;
        Notes Spy {|na, nb, Key K|} ∉ set evs;
        A ∉ bad;  B ∉ bad;  evs ∈ yahalom⟧
      ⟹ Key K ∉ analz (knows Spy evs)"
```
⟨proof⟩

This form is an immediate consequence of the previous result. It is similar to the assertions established by other methods. It is equivalent to the previous result in that the Spy already has `analz` and `synth` at his disposal. However, the conclusion `Key K ∉ knows Spy evs` appears not to be inductive: all the cases other than Fake are trivial, while Fake requires `Key K ∉ analz (knows Spy evs)`.

lemma *Spy_not_know_encrypted_key:*
```
     "⟦Says Server A
           {|nb, Crypt (shrK A) {|Agent B, Key K, na|},
                 Crypt (shrK B) {|Agent A, Agent B, Key K, nb|}|}
        ∈ set evs;
        Notes Spy {|na, nb, Key K|} ∉ set evs;
        A ∉ bad;  B ∉ bad;  evs ∈ yahalom⟧
      ⟹ Key K ∉ knows Spy evs"
```
⟨proof⟩

16.3 Security Guarantee for A upon receiving YM3

If the encrypted message appears then it originated with the Server. May now apply `Spy_not_see_encrypted_key`, subject to its conditions.

lemma *A_trusts_YM3:*
```
     "⟦Crypt (shrK A) {|Agent B, Key K, na|} ∈ parts (knows Spy evs);
        A ∉ bad;  evs ∈ yahalom⟧
      ⟹ ∃nb. Says Server A
                   {|nb, Crypt (shrK A) {|Agent B, Key K, na|},
                         Crypt (shrK B) {|Agent A, Agent B, Key K, nb|}|}
                 ∈ set evs"
```
⟨proof⟩

The obvious combination of `A_trusts_YM3` with `Spy_not_see_encrypted_key`

theorem *A_gets_good_key:*
```
     "⟦Crypt (shrK A) {|Agent B, Key K, na|} ∈ parts (knows Spy evs);
        ∀nb. Notes Spy {|na, nb, Key K|} ∉ set evs;
        A ∉ bad;  B ∉ bad;  evs ∈ yahalom⟧
      ⟹ Key K ∉ analz (knows Spy evs)"
```
⟨proof⟩

16.4 Security Guarantee for B upon receiving YM4

B knows, by the first part of A's message, that the Server distributed the key for A and B, and has associated it with NB.

lemma *B_trusts_YM4_shrK:*
```
    "⟦Crypt (shrK B) ⦃Agent A, Agent B, Key K, Nonce NB⦄
          ∈ parts (knows Spy evs);
        B ∉ bad;  evs ∈ yahalom⟧
  ⟹ ∃NA. Says Server A
            ⦃Nonce NB,
              Crypt (shrK A) ⦃Agent B, Key K, Nonce NA⦄,
              Crypt (shrK B) ⦃Agent A, Agent B, Key K, Nonce NB⦄⦄
            ∈ set evs"
```
⟨*proof*⟩

With this protocol variant, we don't need the 2nd part of YM4 at all: Nonce NB is available in the first part.

What can B deduce from receipt of YM4? Stronger and simpler than Yahalom because we do not have to show that NB is secret.

lemma *B_trusts_YM4:*
```
    "⟦Gets B ⦃Crypt (shrK B) ⦃Agent A, Agent B, Key K, Nonce NB⦄,  X⦄
          ∈ set evs;
        A ∉ bad;  B ∉ bad;  evs ∈ yahalom⟧
  ⟹ ∃NA. Says Server A
            ⦃Nonce NB,
              Crypt (shrK A) ⦃Agent B, Key K, Nonce NA⦄,
              Crypt (shrK B) ⦃Agent A, Agent B, Key K, Nonce NB⦄⦄
           ∈ set evs"
```
⟨*proof*⟩

The obvious combination of *B_trusts_YM4* with *Spy_not_see_encrypted_key*

theorem *B_gets_good_key:*
```
    "⟦Gets B ⦃Crypt (shrK B) ⦃Agent A, Agent B, Key K, Nonce NB⦄, X⦄
          ∈ set evs;
        ∀na. Notes Spy ⦃na, Nonce NB, Key K⦄ ∉ set evs;
        A ∉ bad;  B ∉ bad;  evs ∈ yahalom⟧
      ⟹ Key K ∉ analz (knows Spy evs)"
```
⟨*proof*⟩

16.5 Authenticating B to A

The encryption in message YM2 tells us it cannot be faked.

lemma *B_Said_YM2:*
```
    "⟦Crypt (shrK B) ⦃Agent A, Nonce NA⦄ ∈ parts (knows Spy evs);
        B ∉ bad;  evs ∈ yahalom⟧
      ⟹ ∃NB. Says B Server ⦃Agent B, Nonce NB,
                                  Crypt (shrK B) ⦃Agent A, Nonce NA⦄⦄
                      ∈ set evs"
```
⟨*proof*⟩

If the server sends YM3 then B sent YM2, perhaps with a different NB

```
lemma YM3_auth_B_to_A_lemma:
    "⟦Says Server A ⦃nb, Crypt (shrK A) ⦃Agent B, Key K, Nonce NA⦄, X⦄
          ∈ set evs;
       B ∉ bad;  evs ∈ yahalom⟧
     ⟹ ∃nb'. Says B Server ⦃Agent B, nb',
                                  Crypt (shrK B) ⦃Agent A, Nonce NA⦄⦄
                    ∈ set evs"
⟨proof⟩
```

If A receives YM3 then B has used nonce NA (and therefore is alive)

```
theorem YM3_auth_B_to_A:
    "⟦Gets A ⦃nb, Crypt (shrK A) ⦃Agent B, Key K, Nonce NA⦄, X⦄
          ∈ set evs;
       A ∉ bad;  B ∉ bad;  evs ∈ yahalom⟧
 ⟹ ∃nb'. Says B Server
               ⦃Agent B, nb', Crypt (shrK B) ⦃Agent A, Nonce NA⦄⦄
            ∈ set evs"
⟨proof⟩
```

16.6 Authenticating A to B

using the certificate `Crypt K (Nonce NB)`

Assuming the session key is secure, if both certificates are present then A has said NB. We can't be sure about the rest of A's message, but only NB matters for freshness. Note that `Key K ∉ analz (knows Spy evs)` must be the FIRST antecedent of the induction formula.

This lemma allows a use of `unique_session_keys` in the next proof, which otherwise is extremely slow.

```
lemma secure_unique_session_keys:
    "⟦Crypt (shrK A) ⦃Agent B, Key K, na⦄ ∈ analz (spies evs);
       Crypt (shrK A') ⦃Agent B', Key K, na'⦄ ∈ analz (spies evs);
       Key K ∉ analz (knows Spy evs);  evs ∈ yahalom⟧
    ⟹ A=A' ∧ B=B'"
⟨proof⟩
```

```
lemma Auth_A_to_B_lemma [rule_format]:
    "evs ∈ yahalom
     ⟹ Key K ∉ analz (knows Spy evs) ⟶
        K ∈ symKeys ⟶
        Crypt K (Nonce NB) ∈ parts (knows Spy evs) ⟶
        Crypt (shrK B) ⦃Agent A, Agent B, Key K, Nonce NB⦄
          ∈ parts (knows Spy evs) ⟶
        B ∉ bad ⟶
        (∃X. Says A B ⦃X, Crypt K (Nonce NB)⦄ ∈ set evs)"
⟨proof⟩
```

If B receives YM4 then A has used nonce NB (and therefore is alive). Moreover, A associates K with NB (thus is talking about the same run). Other premises guarantee secrecy of K.

```
theorem YM4_imp_A_Said_YM3 [rule_format]:
```

```
     "⟦Gets B {|Crypt (shrK B) {|Agent A, Agent B, Key K, Nonce NB|},
                 Crypt K (Nonce NB)|} ∈ set evs;
         (∀NA. Notes Spy {|Nonce NA, Nonce NB, Key K|} ∉ set evs);
         K ∈ symKeys;  A ∉ bad;  B ∉ bad;  evs ∈ yahalom⟧
      ⟹ ∃X. Says A B {|X, Crypt K (Nonce NB)|} ∈ set evs"
```

⟨*proof*⟩

end

17 The Yahalom Protocol: A Flawed Version

theory *Yahalom_Bad* **imports** *Public* **begin**

Demonstrates of why Oops is necessary. This protocol can be attacked because it doesn't keep NB secret, but without Oops it can be "verified" anyway. The issues are discussed in lcp's LICS 2000 invited lecture.

inductive_set *yahalom* :: *"event list set"*
  where

```
   Nil:  "[] ∈ yahalom"


 | Fake: "⟦evsf ∈ yahalom;  X ∈ synth (analz (knows Spy evsf))⟧
         ⟹ Says Spy B X  # evsf ∈ yahalom"


 | Reception: "⟦evsr ∈ yahalom;  Says A B X ∈ set evsr⟧
              ⟹ Gets B X # evsr ∈ yahalom"


 | YM1:  "⟦evs1 ∈ yahalom;  Nonce NA ∉ used evs1⟧
         ⟹ Says A B {|Agent A, Nonce NA|} # evs1 ∈ yahalom"


 | YM2:  "⟦evs2 ∈ yahalom;  Nonce NB ∉ used evs2;
             Gets B {|Agent A, Nonce NA|} ∈ set evs2⟧
         ⟹ Says B Server
                  {|Agent B, Nonce NB, Crypt (shrK B) {|Agent A, Nonce NA|}|}
               # evs2 ∈ yahalom"


 | YM3:  "⟦evs3 ∈ yahalom;  Key KAB ∉ used evs3;  KAB ∈ symKeys;
             Gets Server
                  {|Agent B, Nonce NB, Crypt (shrK B) {|Agent A, Nonce NA|}|}
              ∈ set evs3⟧
         ⟹ Says Server A
                   {|Crypt (shrK A) {|Agent B, Key KAB, Nonce NA, Nonce NB|},
                     Crypt (shrK B) {|Agent A, Key KAB|}|}
               # evs3 ∈ yahalom"


 | YM4:  "⟦evs4 ∈ yahalom;  A ≠ Server;  K ∈ symKeys;
             Gets A {|Crypt(shrK A) {|Agent B, Key K, Nonce NA, Nonce NB|}, X|}
```

```
               ∈ set evs4;
            Says A B {|Agent A, Nonce NA|} ∈ set evs4]]
         ⟹ Says A B {|X, Crypt K (Nonce NB)|} # evs4 ∈ yahalom"
```

declare *Says_imp_knows_Spy [THEN analz.Inj, dest]*
declare *parts.Body [dest]*
declare *Fake_parts_insert_in_Un [dest]*
declare *analz_into_parts [dest]*

A "possibility property": there are traces that reach the end

lemma *"[[A ≠ Server; Key K ∉ used []; K ∈ symKeys]]*
⟹ ∃X NB. ∃evs ∈ yahalom.
Says A B {|X, Crypt K (Nonce NB)|} ∈ set evs"
⟨*proof*⟩

17.1 Regularity Lemmas for Yahalom

lemma *Gets_imp_Says:*
"[[Gets B X ∈ set evs; evs ∈ yahalom]] ⟹ ∃A. Says A B X ∈ set evs"
⟨*proof*⟩

lemma *Gets_imp_knows_Spy:*
"[[Gets B X ∈ set evs; evs ∈ yahalom]] ⟹ X ∈ knows Spy evs"
⟨*proof*⟩

declare *Gets_imp_knows_Spy [THEN analz.Inj, dest]*

17.2 For reasoning about the encrypted portion of messages

Lets us treat YM4 using a similar argument as for the Fake case.

lemma *YM4_analz_knows_Spy:*
"[[Gets A {|Crypt (shrK A) Y, X|} ∈ set evs; evs ∈ yahalom]]
⟹ X ∈ analz (knows Spy evs)"
⟨*proof*⟩

lemmas *YM4_parts_knows_Spy =*
YM4_analz_knows_Spy [THEN analz_into_parts]

Theorems of the form *X ∉ parts (knows Spy evs)* imply that NOBODY sends messages containing X!

Spy never sees a good agent's shared key!

lemma *Spy_see_shrK [simp]:*
"evs ∈ yahalom ⟹ (Key (shrK A) ∈ parts (knows Spy evs)) = (A ∈ bad)"
⟨*proof*⟩

lemma *Spy_analz_shrK [simp]:*
"evs ∈ yahalom ⟹ (Key (shrK A) ∈ analz (knows Spy evs)) = (A ∈ bad)"
⟨*proof*⟩

```
lemma Spy_see_shrK_D [dest!]:
     "⟦Key (shrK A) ∈ parts (knows Spy evs);  evs ∈ yahalom⟧ ⟹ A ∈ bad"
⟨proof⟩
```

Nobody can have used non-existent keys! Needed to apply `analz_insert_Key`

```
lemma new_keys_not_used [simp]:
    "⟦Key K ∉ used evs; K ∈ symKeys; evs ∈ yahalom⟧
     ⟹ K ∉ keysFor (parts (spies evs))"
⟨proof⟩
```

17.3 Secrecy Theorems

17.4 Session keys are not used to encrypt other session keys

```
lemma analz_image_freshK [rule_format]:
 "evs ∈ yahalom ⟹
   ∀K KK. KK ⊆ - (range shrK) ⟶
          (Key K ∈ analz (Key`KK ∪ (knows Spy evs))) =
          (K ∈ KK | Key K ∈ analz (knows Spy evs))"
⟨proof⟩
```

```
lemma analz_insert_freshK:
     "⟦evs ∈ yahalom;  KAB ∉ range shrK⟧ ⟹
      (Key K ∈ analz (insert (Key KAB) (knows Spy evs))) =
      (K = KAB | Key K ∈ analz (knows Spy evs))"
⟨proof⟩
```

The Key K uniquely identifies the Server's message.

```
lemma unique_session_keys:
     "⟦Says Server A
          ⦃Crypt (shrK A) ⦃Agent B, Key K, na, nb⦄, X⦄ ∈ set evs;
        Says Server A'
          ⦃Crypt (shrK A') ⦃Agent B', Key K, na', nb'⦄, X'⦄ ∈ set evs;
        evs ∈ yahalom⟧
     ⟹ A=A' ∧ B=B' ∧ na=na' ∧ nb=nb'"
⟨proof⟩
```

Crucial secrecy property: Spy does not see the keys sent in msg YM3

```
lemma secrecy_lemma:
     "⟦A ∉ bad;  B ∉ bad;  evs ∈ yahalom⟧
      ⟹ Says Server A
            ⦃Crypt (shrK A) ⦃Agent B, Key K, na, nb⦄,
              Crypt (shrK B) ⦃Agent A, Key K⦄⦄
           ∈ set evs ⟶
          Key K ∉ analz (knows Spy evs)"
⟨proof⟩
```

Final version

```
lemma Spy_not_see_encrypted_key:
     "⟦Says Server A
            ⦃Crypt (shrK A) ⦃Agent B, Key K, na, nb⦄,
              Crypt (shrK B) ⦃Agent A, Key K⦄⦄
```

```
          ∈ set evs;
        A ∉ bad;  B ∉ bad;  evs ∈ yahalom⟧
     ⟹ Key K ∉ analz (knows Spy evs)"
```
⟨proof⟩

17.5 Security Guarantee for A upon receiving YM3

If the encrypted message appears then it originated with the Server

lemma *A_trusts_YM3:*
```
     "⟦Crypt (shrK A) ⦃Agent B, Key K, na, nb⦄ ∈ parts (knows Spy evs);
         A ∉ bad;  evs ∈ yahalom⟧
      ⟹ Says Server A
           ⦃Crypt (shrK A) ⦃Agent B, Key K, na, nb⦄,
             Crypt (shrK B) ⦃Agent A, Key K⦄⦄
          ∈ set evs"
```
⟨proof⟩

The obvious combination of `A_trusts_YM3` with `Spy_not_see_encrypted_key`

lemma *A_gets_good_key:*
```
     "⟦Crypt (shrK A) ⦃Agent B, Key K, na, nb⦄ ∈ parts (knows Spy evs);
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom⟧
      ⟹ Key K ∉ analz (knows Spy evs)"
```
⟨proof⟩

17.6 Security Guarantees for B upon receiving YM4

B knows, by the first part of A's message, that the Server distributed the key for A and B. But this part says nothing about nonces.

lemma *B_trusts_YM4_shrK:*
```
     "⟦Crypt (shrK B) ⦃Agent A, Key K⦄ ∈ parts (knows Spy evs);
         B ∉ bad;  evs ∈ yahalom⟧
      ⟹ ∃NA NB. Says Server A
                    ⦃Crypt (shrK A) ⦃Agent B, Key K, Nonce NA, Nonce NB⦄,
                      Crypt (shrK B) ⦃Agent A, Key K⦄⦄
                   ∈ set evs"
```
⟨proof⟩

17.7 The Flaw in the Model

Up to now, the reasoning is similar to standard Yahalom. Now the doubtful reasoning occurs. We should not be assuming that an unknown key is secure, but the model allows us to: there is no Oops rule to let session keys become compromised.

B knows, by the second part of A's message, that the Server distributed the key quoting nonce NB. This part says nothing about agent names. Secrecy of K is assumed; the valid Yahalom proof uses (and later proves) the secrecy of NB.

lemma *B_trusts_YM4_newK [rule_format]:*
```
     "⟦Key K ∉ analz (knows Spy evs);  evs ∈ yahalom⟧
      ⟹ Crypt K (Nonce NB) ∈ parts (knows Spy evs) ⟶
          (∃A B NA. Says Server A
```

```
                    {|Crypt (shrK A) {|Agent B, Key K,
                              Nonce NA, Nonce NB|},
                      Crypt (shrK B) {|Agent A, Key K|}|}
                   ∈ set evs)"
```
⟨*proof*⟩

B's session key guarantee from YM4. The two certificates contribute to a single conclusion about the Server's message.

lemma *B_trusts_YM4:*
```
     "[[Gets B {|Crypt (shrK B) {|Agent A, Key K|},
                 Crypt K (Nonce NB)|} ∈ set evs;
         Says B Server
           {|Agent B, Nonce NB, Crypt (shrK B) {|Agent A, Nonce NA|}|}
           ∈ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom]]
       ⟹ ∃na nb. Says Server A
                 {|Crypt (shrK A) {|Agent B, Key K, na, nb|},
                   Crypt (shrK B) {|Agent A, Key K|}|}
            ∈ set evs"
```
⟨*proof*⟩

The obvious combination of *B_trusts_YM4* with *Spy_not_see_encrypted_key*

lemma *B_gets_good_key:*
```
     "[[Gets B {|Crypt (shrK B) {|Agent A, Key K|},
                 Crypt K (Nonce NB)|} ∈ set evs;
         Says B Server
           {|Agent B, Nonce NB, Crypt (shrK B) {|Agent A, Nonce NA|}|}
           ∈ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom]]
      ⟹ Key K ∉ analz (knows Spy evs)"
```
⟨*proof*⟩

Assuming the session key is secure, if both certificates are present then A has said NB. We can't be sure about the rest of A's message, but only NB matters for freshness.

lemma *A_Said_YM3_lemma [rule_format]:*
```
     "evs ∈ yahalom
      ⟹ Key K ∉ analz (knows Spy evs) ⟶
          Crypt K (Nonce NB) ∈ parts (knows Spy evs) ⟶
          Crypt (shrK B) {|Agent A, Key K|} ∈ parts (knows Spy evs) ⟶
          B ∉ bad ⟶
          (∃X. Says A B {|X, Crypt K (Nonce NB)|} ∈ set evs)"
```
⟨*proof*⟩

If B receives YM4 then A has used nonce NB (and therefore is alive). Moreover, A associates K with NB (thus is talking about the same run). Other premises guarantee secrecy of K.

lemma *YM4_imp_A_Said_YM3 [rule_format]:*
```
     "[[Gets B {|Crypt (shrK B) {|Agent A, Key K|},
                 Crypt K (Nonce NB)|} ∈ set evs;
         Says B Server
           {|Agent B, Nonce NB, Crypt (shrK B) {|Agent A, Nonce NA|}|}
           ∈ set evs;
```

```
        A ∉ bad;  B ∉ bad;  evs ∈ yahalom⟧
      ⟹ ∃X. Says A B ⦃X, Crypt K (Nonce NB)⦄ ∈ set evs"
⟨proof⟩

end


theory ZhouGollmann imports Public begin

abbreviation
  TTP :: agent where "TTP == Server"

abbreviation f_sub :: nat where "f_sub == 5"
abbreviation f_nro :: nat where "f_nro == 2"
abbreviation f_nrr :: nat where "f_nrr == 3"
abbreviation f_con :: nat where "f_con == 4"


definition broken :: "agent set" where
    — the compromised honest agents; TTP is included as it's not allowed to use the
protocol
   "broken == bad - {Spy}"

declare broken_def [simp]

inductive_set zg :: "event list set"
  where

  Nil:  "[] ∈ zg"

| Fake: "⟦evsf ∈ zg;  X ∈ synth (analz (spies evsf))⟧
         ⟹ Says Spy B X  # evsf ∈ zg"

| Reception:  "⟦evsr ∈ zg; Says A B X ∈ set evsr⟧ ⟹ Gets B X # evsr ∈ zg"


| ZG1: "⟦evs1 ∈ zg;  Nonce L ∉ used evs1; C = Crypt K (Number m);
           K ∈ symKeys;
           NRO = Crypt (priK A) ⦃Number f_nro, Agent B, Nonce L, C⦄⟧
       ⟹ Says A B ⦃Number f_nro, Agent B, Nonce L, C, NRO⦄ # evs1 ∈ zg"


| ZG2: "⟦evs2 ∈ zg;
           Gets B ⦃Number f_nro, Agent B, Nonce L, C, NRO⦄ ∈ set evs2;
           NRO = Crypt (priK A) ⦃Number f_nro, Agent B, Nonce L, C⦄;
           NRR = Crypt (priK B) ⦃Number f_nrr, Agent A, Nonce L, C⦄⟧
       ⟹ Says B A ⦃Number f_nrr, Agent A, Nonce L, NRR⦄ # evs2  ∈  zg"


| ZG3: "⟦evs3 ∈ zg; C = Crypt K M; K ∈ symKeys;
           Says A B ⦃Number f_nro, Agent B, Nonce L, C, NRO⦄ ∈ set evs3;
           Gets A ⦃Number f_nrr, Agent A, Nonce L, NRR⦄ ∈ set evs3;
           NRR = Crypt (priK B) ⦃Number f_nrr, Agent A, Nonce L, C⦄;
           sub_K = Crypt (priK A) ⦃Number f_sub, Agent B, Nonce L, Key K⦄⟧
```

```
        ⟹ Says A TTP {|Number f_sub, Agent B, Nonce L, Key K, sub_K|}
          # evs3 ∈ zg"


| ZG4: "⟦evs4 ∈ zg; K ∈ symKeys;
          Gets TTP {|Number f_sub, Agent B, Nonce L, Key K, sub_K|}
            ∈ set evs4;
          sub_K = Crypt (priK A) {|Number f_sub, Agent B, Nonce L, Key K|};
          con_K = Crypt (priK TTP) {|Number f_con, Agent A, Agent B,
                                    Nonce L, Key K|}⟧
        ⟹ Says TTP Spy con_K
          #
          Notes TTP {|Number f_con, Agent A, Agent B, Nonce L, Key K, con_K|}
          # evs4 ∈ zg"
```

declare *Says_imp_knows_Spy [THEN analz.Inj, dest]*
declare *Fake_parts_insert_in_Un [dest]*
declare *analz_into_parts [dest]*

declare *symKey_neq_priEK [simp]*
declare *symKey_neq_priEK [THEN not_sym, simp]*

A "possibility property": there are traces that reach the end

```
lemma "⟦A ≠ B; TTP ≠ A; TTP ≠ B; K ∈ symKeys⟧ ⟹
     ∃L. ∃evs ∈ zg.
          Notes TTP {|Number f_con, Agent A, Agent B, Nonce L, Key K,
               Crypt (priK TTP) {|Number f_con, Agent A, Agent B, Nonce L,
Key K|}|}
               ∈ set evs"
```
⟨*proof*⟩

17.8 Basic Lemmas

lemma *Gets_imp_Says:*
"⟦Gets B X ∈ set evs; evs ∈ zg⟧ ⟹ ∃A. Says A B X ∈ set evs"
⟨*proof*⟩

lemma *Gets_imp_knows_Spy:*
"⟦Gets B X ∈ set evs; evs ∈ zg⟧ ⟹ X ∈ spies evs"
⟨*proof*⟩

Lets us replace proofs about *used evs* by simpler proofs about *parts (knows Spy evs)*.

lemma *Crypt_used_imp_spies:*
"⟦Crypt K X ∈ used evs; evs ∈ zg⟧
⟹ Crypt K X ∈ parts (spies evs)"
⟨*proof*⟩

```
lemma Notes_TTP_imp_Gets:
     "⟦Notes TTP {|Number f_con, Agent A, Agent B, Nonce L, Key K, con_K|}
          ∈ set evs;
        sub_K = Crypt (priK A) {|Number f_sub, Agent B, Nonce L, Key K|};
        evs ∈ zg⟧
```

```
    ⟹ Gets TTP ⦃Number f_sub, Agent B, Nonce L, Key K, sub_K⦄ ∈ set evs"
```
⟨proof⟩

For reasoning about C, which is encrypted in message ZG2

lemma *ZG2_msg_in_parts_spies:*
```
     "⟦Gets B ⦃F, B', L, C, X⦄ ∈ set evs; evs ∈ zg⟧
      ⟹ C ∈ parts (spies evs)"
```
⟨proof⟩

lemma *Spy_see_priK [simp]:*
```
     "evs ∈ zg ⟹ (Key (priK A) ∈ parts (spies evs)) = (A ∈ bad)"
```
⟨proof⟩

So that blast can use it too

declare *Spy_see_priK [THEN [2] rev_iffD1, dest!]*

lemma *Spy_analz_priK [simp]:*
```
     "evs ∈ zg ⟹ (Key (priK A) ∈ analz (spies evs)) = (A ∈ bad)"
```
⟨proof⟩

17.9 About NRO: Validity for *B*

Below we prove that if *NRO* exists then *A* definitely sent it, provided *A* is not broken.

Strong conclusion for a good agent

lemma *NRO_validity_good:*
```
     "⟦NRO = Crypt (priK A) ⦃Number f_nro, Agent B, Nonce L, C⦄;
        NRO ∈ parts (spies evs);
        A ∉ bad;  evs ∈ zg⟧
     ⟹ Says A B ⦃Number f_nro, Agent B, Nonce L, C, NRO⦄ ∈ set evs"
```
⟨proof⟩

lemma *NRO_sender:*
```
     "⟦Says A' B ⦃n, b, l, C, Crypt (priK A) X⦄ ∈ set evs; evs ∈ zg⟧
    ⟹ A' ∈ {A,Spy}"
```
⟨proof⟩

Holds also for *A = Spy*!

theorem *NRO_validity:*
```
     "⟦Gets B ⦃Number f_nro, Agent B, Nonce L, C, NRO⦄ ∈ set evs;
        NRO = Crypt (priK A) ⦃Number f_nro, Agent B, Nonce L, C⦄;
        A ∉ broken;  evs ∈ zg⟧
     ⟹ Says A B ⦃Number f_nro, Agent B, Nonce L, C, NRO⦄ ∈ set evs"
```
⟨proof⟩

17.10 About NRR: Validity for *A*

Below we prove that if *NRR* exists then *B* definitely sent it, provided *B* is not broken.

Strong conclusion for a good agent

```
lemma NRR_validity_good:
     "⟦NRR = Crypt (priK B) ⦃Number f_nrr, Agent A, Nonce L, C⦄;
        NRR ∈ parts (spies evs);
        B ∉ bad;  evs ∈ zg⟧
     ⟹ Says B A ⦃Number f_nrr, Agent A, Nonce L, NRR⦄ ∈ set evs"
⟨proof⟩
```

```
lemma NRR_sender:
     "⟦Says B' A ⦃n, a, l, Crypt (priK B) X⦄ ∈ set evs; evs ∈ zg⟧
    ⟹ B' ∈ {B,Spy}"
⟨proof⟩
```

Holds also for `B = Spy`!

```
theorem NRR_validity:
     "⟦Says B' A ⦃Number f_nrr, Agent A, Nonce L, NRR⦄ ∈ set evs;
        NRR = Crypt (priK B) ⦃Number f_nrr, Agent A, Nonce L, C⦄;
        B ∉ broken; evs ∈ zg⟧
    ⟹ Says B A ⦃Number f_nrr, Agent A, Nonce L, NRR⦄ ∈ set evs"
⟨proof⟩
```

17.11 Proofs About `sub_K`

Below we prove that if `sub_K` exists then `A` definitely sent it, provided `A` is not broken.

Strong conclusion for a good agent

```
lemma sub_K_validity_good:
     "⟦sub_K = Crypt (priK A) ⦃Number f_sub, Agent B, Nonce L, Key K⦄;
        sub_K ∈ parts (spies evs);
        A ∉ bad;  evs ∈ zg⟧
     ⟹ Says A TTP ⦃Number f_sub, Agent B, Nonce L, Key K, sub_K⦄ ∈ set evs"
⟨proof⟩
```

```
lemma sub_K_sender:
     "⟦Says A' TTP ⦃n, b, l, k, Crypt (priK A) X⦄ ∈ set evs;  evs ∈ zg⟧
    ⟹ A' ∈ {A,Spy}"
⟨proof⟩
```

Holds also for `A = Spy`!

```
theorem sub_K_validity:
     "⟦Gets TTP ⦃Number f_sub, Agent B, Nonce L, Key K, sub_K⦄ ∈ set evs;
        sub_K = Crypt (priK A) ⦃Number f_sub, Agent B, Nonce L, Key K⦄;
        A ∉ broken;  evs ∈ zg⟧
     ⟹ Says A TTP ⦃Number f_sub, Agent B, Nonce L, Key K, sub_K⦄ ∈ set evs"
⟨proof⟩
```

17.12 Proofs About `con_K`

Below we prove that if `con_K` exists, then `TTP` has it, and therefore `A` and `B`) can get it too. Moreover, we know that `A` sent `sub_K`

```
lemma con_K_validity:
     "⟦con_K ∈ used evs;
```

```
        con_K = Crypt (priK TTP)
                  {|Number f_con, Agent A, Agent B, Nonce L, Key K|};
        evs ∈ zg⟧
    ⟹ Notes TTP {|Number f_con, Agent A, Agent B, Nonce L, Key K, con_K|}
         ∈ set evs"
```
⟨*proof*⟩

If TTP holds con_K then A sent sub_K. We assume that A is not broken. Importantly, nothing needs to be assumed about the form of con_K!

lemma *Notes_TTP_imp_Says_A:*
```
     "⟦Notes TTP {|Number f_con, Agent A, Agent B, Nonce L, Key K, con_K|}
          ∈ set evs;
        sub_K = Crypt (priK A) {|Number f_sub, Agent B, Nonce L, Key K|};
        A ∉ broken; evs ∈ zg⟧
     ⟹ Says A TTP {|Number f_sub, Agent B, Nonce L, Key K, sub_K|} ∈ set evs"
```
⟨*proof*⟩

If con_K exists, then A sent sub_K. We again assume that A is not broken.

theorem *B_sub_K_validity:*
```
     "⟦con_K ∈ used evs;
        con_K = Crypt (priK TTP) {|Number f_con, Agent A, Agent B,
                                   Nonce L, Key K|};
        sub_K = Crypt (priK A) {|Number f_sub, Agent B, Nonce L, Key K|};
        A ∉ broken; evs ∈ zg⟧
     ⟹ Says A TTP {|Number f_sub, Agent B, Nonce L, Key K, sub_K|} ∈ set evs"
```
⟨*proof*⟩

17.13 Proving fairness

Cannot prove that, if B has NRO, then A has her NRR. It would appear that B has a small advantage, though it is useless to win disputes: B needs to present con_K as well.

Strange: unicity of the label protects A?

lemma *A_unicity:*
```
     "⟦NRO = Crypt (priK A) {|Number f_nro, Agent B, Nonce L, Crypt K M|};
        NRO ∈ parts (spies evs);
        Says A B {|Number f_nro, Agent B, Nonce L, Crypt K M', NRO'|}
          ∈ set evs;
        A ∉ bad; evs ∈ zg⟧
     ⟹ M'=M"
```
⟨*proof*⟩

Fairness lemma: if sub_K exists, then A holds NRR. Relies on unicity of labels.

lemma *sub_K_implies_NRR:*
```
     "⟦NRO = Crypt (priK A) {|Number f_nro, Agent B, Nonce L, Crypt K M|};
         NRR = Crypt (priK B) {|Number f_nrr, Agent A, Nonce L, Crypt K M|};
         sub_K ∈ parts (spies evs);
         NRO ∈ parts (spies evs);
         sub_K = Crypt (priK A) {|Number f_sub, Agent B, Nonce L, Key K|};
         A ∉ bad;  evs ∈ zg⟧
     ⟹ Gets A {|Number f_nrr, Agent A, Nonce L, NRR|} ∈ set evs"
```

⟨*proof*⟩

lemma *Crypt_used_imp_L_used:*

```
"⟦Crypt (priK TTP) {|F, A, B, L, K|} ∈ used evs; evs ∈ zg⟧
 ⟹ L ∈ used evs"
```

⟨*proof*⟩

Fairness for *A*: if *con_K* and *NRO* exist, then *A* holds NRR. *A* must be uncompromised, but there is no assumption about *B*.

theorem *A_fairness_NRO:*

```
"⟦con_K ∈ used evs;
   NRO ∈ parts (spies evs);
   con_K = Crypt (priK TTP)
              {|Number f_con, Agent A, Agent B, Nonce L, Key K|};
   NRO = Crypt (priK A) {|Number f_nro, Agent B, Nonce L, Crypt K M|};
   NRR = Crypt (priK B) {|Number f_nrr, Agent A, Nonce L, Crypt K M|};
   A ∉ bad;  evs ∈ zg⟧
 ⟹ Gets A {|Number f_nrr, Agent A, Nonce L, NRR|} ∈ set evs"
```

⟨*proof*⟩

Fairness for *B*: NRR exists at all, then *B* holds NRO. *B* must be uncompromised, but there is no assumption about *A*.

theorem *B_fairness_NRR:*

```
"⟦NRR ∈ used evs;
   NRR = Crypt (priK B) {|Number f_nrr, Agent A, Nonce L, C|};
   NRO = Crypt (priK A) {|Number f_nro, Agent B, Nonce L, C|};
   B ∉ bad; evs ∈ zg⟧
 ⟹ Gets B {|Number f_nro, Agent B, Nonce L, C, NRO|} ∈ set evs"
```

⟨*proof*⟩

If *con_K* exists at all, then *B* can get it, by *con_K_validity*. Cannot conclude that also NRO is available to *B*, because if *A* were unfair, *A* could build message 3 without building message 1, which contains NRO.

end

18 Conventional protocols: rely on conventional Message, Event and Public – Shared-key protocols

theory *Auth_Shared*
imports

```
NS_Shared
Kerberos_BAN
Kerberos_BAN_Gets
KerberosIV
KerberosIV_Gets
KerberosV
OtwayRees
OtwayRees_AN
OtwayRees_Bad
```

```
  OtwayReesBella
  WooLam
  Recur
  Yahalom
  Yahalom2
  Yahalom_Bad
  ZhouGollmann
begin

end
```

19 The Needham-Schroeder Public-Key Protocol (Flawed)

Flawed version, vulnerable to Lowe's attack. From Burrows, Abadi and Needham. A Logic of Authentication. Proc. Royal Soc. 426 (1989), p. 260

theory *NS_Public_Bad* **imports** *Public* **begin**

```
inductive_set ns_public :: "event list set"
  where
   Nil:  "[] ∈ ns_public"
   — Initial trace is empty
 | Fake: "⟦evsf ∈ ns_public;  X ∈ synth (analz (spies evsf))⟧
          ⟹ Says Spy B X  # evsf ∈ ns_public"
   — The spy can say almost anything.
 | NS1:  "⟦evs1 ∈ ns_public;  Nonce NA ∉ used evs1⟧
          ⟹ Says A B (Crypt (pubEK B) ⦃Nonce NA, Agent A⦄)
                # evs1  ∈  ns_public"
   — Alice initiates a protocol run, sending a nonce to Bob
 | NS2:  "⟦evs2 ∈ ns_public;  Nonce NB ∉ used evs2;
           Says A' B (Crypt (pubEK B) ⦃Nonce NA, Agent A⦄) ∈ set evs2⟧
          ⟹ Says B A (Crypt (pubEK A) ⦃Nonce NA, Nonce NB⦄)
                # evs2  ∈  ns_public"
   — Bob responds to Alice's message with a further nonce
 | NS3:  "⟦evs3 ∈ ns_public;
           Says A  B (Crypt (pubEK B) ⦃Nonce NA, Agent A⦄) ∈ set evs3;
           Says B' A (Crypt (pubEK A) ⦃Nonce NA, Nonce NB⦄) ∈ set evs3⟧
          ⟹ Says A B (Crypt (pubEK B) (Nonce NB)) # evs3 ∈ ns_public"
   — Alice proves her existence by sending NB back to Bob.

declare knows_Spy_partsEs [elim]
declare analz_into_parts [dest]
declare Fake_parts_insert_in_Un [dest]
```

A "possibility property": there are traces that reach the end

```
lemma "∃NB. ∃evs ∈ ns_public. Says A B (Crypt (pubEK B) (Nonce NB)) ∈ set
evs"
  ⟨proof⟩
```

19.1 Inductive proofs about *ns_public*

Spy never sees another agent's private key! (unless it's bad at start)

lemma *Spy_see_priEK [simp]:*
```
"evs ∈ ns_public ⟹ (Key (priEK A) ∈ parts (spies evs)) = (A ∈ bad)"
```
⟨*proof*⟩

lemma *Spy_analz_priEK [simp]:*
```
"evs ∈ ns_public ⟹ (Key (priEK A) ∈ analz (spies evs)) = (A ∈ bad)"
```
⟨*proof*⟩

19.2 Authenticity properties obtained from term NS1

It is impossible to re-use a nonce in both term NS1 and term NS2, provided the nonce is secret. (Honest users generate fresh nonces.)

lemma *no_nonce_NS1_NS2:*
```
"⟦evs ∈ ns_public;
  Crypt (pubEK C) ⦃NA', Nonce NA⦄ ∈ parts (spies evs);
  Crypt (pubEK B) ⦃Nonce NA, Agent A⦄ ∈ parts (spies evs)⟧
 ⟹ Nonce NA ∈ analz (spies evs)"
```
⟨*proof*⟩

Unicity for term NS1: nonce term NA identifies agents term A and term B

lemma *unique_NA:*
assumes *NA: "Crypt(pubEK B) ⦃Nonce NA, Agent A ⦄ ∈ parts(spies evs)"*
"Crypt(pubEK B') ⦃Nonce NA, Agent A'⦄ ∈ parts(spies evs)"
"Nonce NA ∉ analz (spies evs)"
and *evs: "evs ∈ ns_public"*
shows *"A=A' ∧ B=B'"*
⟨*proof*⟩

Secrecy: Spy does not see the nonce sent in msg term NS1 if term A and term B are secure The major premise "Says A B ..." makes it a dest-rule, hence the given assumption order.

theorem *Spy_not_see_NA:*
assumes *NA: "Says A B (Crypt(pubEK B) ⦃Nonce NA, Agent A⦄) ∈ set evs"*
"A ∉ bad" "B ∉ bad"
and *evs: "evs ∈ ns_public"*
shows *"Nonce NA ∉ analz (spies evs)"*
⟨*proof*⟩

Authentication for term A: if she receives message 2 and has used term NA to start a run, then term B has sent message 2.

lemma *A_trusts_NS2_lemma:*
```
"⟦evs ∈ ns_public;
  Crypt (pubEK A) ⦃Nonce NA, Nonce NB⦄ ∈ parts (spies evs);
  Says A B (Crypt(pubEK B) ⦃Nonce NA, Agent A⦄) ∈ set evs;
  A ∉ bad; B ∉ bad⟧
 ⟹ Says B A (Crypt(pubEK A) ⦃Nonce NA, Nonce NB⦄) ∈ set evs"
```
⟨*proof*⟩

theorem *A_trusts_NS2:*
```
"⟦Says A  B (Crypt(pubEK B) ⦃Nonce NA, Agent A⦄) ∈ set evs;
  Says B' A (Crypt(pubEK A) ⦃Nonce NA, Nonce NB⦄) ∈ set evs;
  A ∉ bad;  B ∉ bad;  evs ∈ ns_public⟧
```

```
      ⟹ Says B A (Crypt(pubEK A) {|Nonce NA, Nonce NB|}) ∈ set evs"
  ⟨proof⟩
```

If the encrypted message appears then it originated with Alice in term NS1

```
lemma B_trusts_NS1:
     "⟦evs ∈ ns_public;
       Crypt (pubEK B) {|Nonce NA, Agent A|} ∈ parts (spies evs);
       Nonce NA ∉ analz (spies evs)⟧
      ⟹ Says A B (Crypt (pubEK B) {|Nonce NA, Agent A|}) ∈ set evs"
  ⟨proof⟩
```

19.3 Authenticity properties obtained from term NS2

Unicity for term NS2: nonce term NB identifies nonce term NA and agent term A [proof closely follows that for `unique_NA`]

```
lemma unique_NB [dest]:
  assumes NB: "Crypt(pubEK A) {|Nonce NA, Nonce NB|} ∈ parts(spies evs)"
              "Crypt(pubEK A') {|Nonce NA', Nonce NB|} ∈ parts(spies evs)"
              "Nonce NB ∉ analz (spies evs)"
    and evs: "evs ∈ ns_public"
  shows "A=A' ∧ NA=NA'"
  ⟨proof⟩
```

term NB remains secret *provided* Alice never responds with round 3

```
theorem Spy_not_see_NB [dest]:
  assumes NB: "Says B A (Crypt (pubEK A) {|Nonce NA, Nonce NB|}) ∈ set evs"
              "∀C. Says A C (Crypt (pubEK C) (Nonce NB)) ∉ set evs"
              "A ∉ bad" "B ∉ bad"
    and evs: "evs ∈ ns_public"
  shows "Nonce NB ∉ analz (spies evs)"
  ⟨proof⟩
```

Authentication for term B: if he receives message 3 and has used term NB in message 2, then term A has sent message 3 (to somebody)

```
lemma B_trusts_NS3_lemma:
     "⟦evs ∈ ns_public;
       Crypt (pubEK B) (Nonce NB) ∈ parts (spies evs);
       Says B A (Crypt (pubEK A) {|Nonce NA, Nonce NB|}) ∈ set evs;
       A ∉ bad;  B ∉ bad⟧
      ⟹ ∃C. Says A C (Crypt (pubEK C) (Nonce NB)) ∈ set evs"
⟨proof⟩

theorem B_trusts_NS3:
     "⟦Says B A  (Crypt (pubEK A) {|Nonce NA, Nonce NB|}) ∈ set evs;
       Says A' B (Crypt (pubEK B) (Nonce NB)) ∈ set evs;
       A ∉ bad;  B ∉ bad;  evs ∈ ns_public⟧
      ⟹ ∃C. Says A C (Crypt (pubEK C) (Nonce NB)) ∈ set evs"
  ⟨proof⟩
```

Can we strengthen the secrecy theorem `Spy_not_see_NB`? NO

```
lemma "⟦evs ∈ ns_public;
        Says B A (Crypt (pubEK A) {|Nonce NA, Nonce NB|}) ∈ set evs;
```

```
    A ∉ bad; B ∉ bad⟧
  ⟹ Nonce NB ∉ analz (spies evs)"
```
⟨*proof*⟩

end

20 The Needham-Schroeder Public-Key Protocol

Flawed version, vulnerable to Lowe's attack. From Burrows, Abadi and Needham. A Logic of Authentication. Proc. Royal Soc. 426 (1989), p. 260

theory *NS_Public* **imports** *Public* **begin**

```
inductive_set ns_public :: "event list set"
  where
   Nil:  "[] ∈ ns_public"
   — Initial trace is empty
 | Fake: "⟦evsf ∈ ns_public;  X ∈ synth (analz (spies evsf))⟧
          ⟹ Says Spy B X  # evsf ∈ ns_public"
   — The spy can say almost anything.
 | NS1:  "⟦evs1 ∈ ns_public;  Nonce NA ∉ used evs1⟧
          ⟹ Says A B (Crypt (pubEK B) ⦃Nonce NA, Agent A⦄)
                # evs1  ∈  ns_public"
   — Alice initiates a protocol run, sending a nonce to Bob
 | NS2:  "⟦evs2 ∈ ns_public;  Nonce NB ∉ used evs2;
           Says A' B (Crypt (pubEK B) ⦃Nonce NA, Agent A⦄) ∈ set evs2⟧
          ⟹ Says B A (Crypt (pubEK A) ⦃Nonce NA, Nonce NB, Agent B⦄)
                # evs2  ∈  ns_public"
   — Bob responds to Alice's message with a further nonce
 | NS3:  "⟦evs3 ∈ ns_public;
           Says A  B (Crypt (pubEK B) ⦃Nonce NA, Agent A⦄) ∈ set evs3;
           Says B' A (Crypt (pubEK A) ⦃Nonce NA, Nonce NB, Agent B⦄) ∈ set
evs3⟧
          ⟹ Says A B (Crypt (pubEK B) (Nonce NB)) # evs3 ∈ ns_public"
   — Alice proves her existence by sending NB back to Bob.
```

declare *knows_Spy_partsEs [elim]*
declare *analz_into_parts [dest]*
declare *Fake_parts_insert_in_Un [dest]*

A "possibility property": there are traces that reach the end

lemma *"∃NB. ∃evs ∈ ns_public. Says A B (Crypt (pubEK B) (Nonce NB)) ∈ set evs"*
⟨*proof*⟩

20.1 Inductive proofs about *ns_public*

Spy never sees another agent's private key! (unless it's bad at start)

lemma *Spy_see_priEK [simp]:*

```
  "evs ∈ ns_public ⟹ (Key (priEK A) ∈ parts (spies evs)) = (A ∈ bad)"
  ⟨proof⟩
```

lemma *Spy_analz_priEK [simp]:*

```
  "evs ∈ ns_public ⟹ (Key (priEK A) ∈ analz (spies evs)) = (A ∈ bad)"
  ⟨proof⟩
```

20.2 Authenticity properties obtained from term NS1

It is impossible to re-use a nonce in both term NS1 and term NS2, provided the nonce is secret. (Honest users generate fresh nonces.)

lemma *no_nonce_NS1_NS2:*

```
      "⟦evs ∈ ns_public;
        Crypt (pubEK C) ⦃NA', Nonce NA, Agent D⦄ ∈ parts (spies evs);
        Crypt (pubEK B) ⦃Nonce NA, Agent A⦄ ∈ parts (spies evs)⟧
       ⟹ Nonce NA ∈ analz (spies evs)"
  ⟨proof⟩
```

Unicity for term NS1: nonce term NA identifies agents term A and term B

lemma *unique_NA:*

```
  assumes NA: "Crypt(pubEK B)  ⦃Nonce NA, Agent A ⦄ ∈ parts(spies evs)"
              "Crypt(pubEK B') ⦃Nonce NA, Agent A'⦄ ∈ parts(spies evs)"
              "Nonce NA ∉ analz (spies evs)"
    and evs: "evs ∈ ns_public"
  shows "A=A' ∧ B=B'"
  ⟨proof⟩
```

Secrecy: Spy does not see the nonce sent in msg term NS1 if term A and term B are secure The major premise "Says A B ..." makes it a dest-rule, hence the given assumption order.

theorem *Spy_not_see_NA:*

```
  assumes NA: "Says A B (Crypt(pubEK B) ⦃Nonce NA, Agent A⦄) ∈ set evs"
              "A ∉ bad" "B ∉ bad"
    and evs: "evs ∈ ns_public"
  shows "Nonce NA ∉ analz (spies evs)"
  ⟨proof⟩
```

Authentication for term A: if she receives message 2 and has used term NA to start a run, then term B has sent message 2.

lemma *A_trusts_NS2_lemma:*

```
    "⟦evs ∈ ns_public;
      Crypt (pubEK A) ⦃Nonce NA, Nonce NB, Agent B⦄ ∈ parts (spies evs);
      Says A B (Crypt(pubEK B) ⦃Nonce NA, Agent A⦄) ∈ set evs;
      A ∉ bad; B ∉ bad⟧
     ⟹ Says B A (Crypt(pubEK A) ⦃Nonce NA, Nonce NB, Agent B⦄) ∈ set evs"
  ⟨proof⟩
```

theorem *A_trusts_NS2:*

```
     "⟦Says A  B (Crypt(pubEK B) ⦃Nonce NA, Agent A⦄) ∈ set evs;
       Says B' A (Crypt(pubEK A) ⦃Nonce NA, Nonce NB, Agent B⦄) ∈ set evs;
       A ∉ bad;  B ∉ bad;  evs ∈ ns_public⟧
      ⟹ Says B A (Crypt(pubEK A) ⦃Nonce NA, Nonce NB, Agent B⦄) ∈ set evs"
```

⟨*proof*⟩

If the encrypted message appears then it originated with Alice in term NS1

```
lemma B_trusts_NS1:
     "⟦evs ∈ ns_public;
       Crypt (pubEK B) {|Nonce NA, Agent A|} ∈ parts (spies evs);
       Nonce NA ∉ analz (spies evs)⟧
      ⟹ Says A B (Crypt (pubEK B) {|Nonce NA, Agent A|}) ∈ set evs"
  ⟨proof⟩
```

20.3 Authenticity properties obtained from term NS2

Unicity for term NS2: nonce term NB identifies nonce term NA and agent term A [proof closely follows that for `unique_NA`]

```
lemma unique_NB [dest]:
  assumes NB: "Crypt(pubEK A) {|Nonce NA, Nonce NB, Agent B|} ∈ parts(spies
evs)"
             "Crypt(pubEK A') {|Nonce NA', Nonce NB, Agent B'|} ∈ parts(spies
evs)"
             "Nonce NB ∉ analz (spies evs)"
    and evs: "evs ∈ ns_public"
  shows "A=A' ∧ NA=NA' ∧ B=B'"
  ⟨proof⟩
```

term NB remains secret

```
theorem Spy_not_see_NB [dest]:
  assumes NB: "Says B A (Crypt (pubEK A) {|Nonce NA, Nonce NB, Agent B|}) ∈
set evs"
             "A ∉ bad" "B ∉ bad"
    and evs: "evs ∈ ns_public"
  shows "Nonce NB ∉ analz (spies evs)"
  ⟨proof⟩
```

Authentication for term B: if he receives message 3 and has used term NB in message 2, then term A has sent message 3.

```
lemma B_trusts_NS3_lemma:
     "⟦evs ∈ ns_public;
       Crypt (pubEK B) (Nonce NB) ∈ parts (spies evs);
       Says B A (Crypt (pubEK A) {|Nonce NA, Nonce NB, Agent B|}) ∈ set evs;
       A ∉ bad;  B ∉ bad⟧
      ⟹ Says A B (Crypt (pubEK B) (Nonce NB)) ∈ set evs"
⟨proof⟩

theorem B_trusts_NS3:
     "⟦Says B A  (Crypt (pubEK A) {|Nonce NA, Nonce NB, Agent B|}) ∈ set evs;
       Says A' B (Crypt (pubEK B) (Nonce NB)) ∈ set evs;
       A ∉ bad;  B ∉ bad;  evs ∈ ns_public⟧
      ⟹ Says A B (Crypt (pubEK B) (Nonce NB)) ∈ set evs"
  ⟨proof⟩
```

20.4 Overall guarantee for term B

If NS3 has been sent and the nonce NB agrees with the nonce B joined with NA, then A initiated the run using NA.

```
theorem B_trusts_protocol:
     "⟦A ∉ bad;  B ∉ bad;  evs ∈ ns_public⟧ ⟹
      Crypt (pubEK B) (Nonce NB) ∈ parts (spies evs) ⟶
      Says B A  (Crypt (pubEK A) {|Nonce NA, Nonce NB, Agent B|}) ∈ set evs
⟶
      Says A B (Crypt (pubEK B) {|Nonce NA, Agent A|}) ∈ set evs"
⟨proof⟩

end
```

21 The TLS Protocol: Transport Layer Security

```
theory TLS imports Public "HOL-Library.Nat_Bijection" begin

definition certificate :: "[agent,key] ⇒ msg" where
    "certificate A KA == Crypt (priSK Server) {|Agent A, Key KA|}"
```

TLS apparently does not require separate keypairs for encryption and signature. Therefore, we formalize signature as encryption using the private encryption key.

```
datatype role = ClientRole | ServerRole

consts

  PRF  :: "nat*nat*nat ⇒ nat"


  sessionK :: "(nat*nat*nat) * role ⇒ key"

abbreviation
  clientK :: "nat*nat*nat ⇒ key" where
  "clientK X == sessionK(X, ClientRole)"

abbreviation
  serverK :: "nat*nat*nat ⇒ key" where
  "serverK X == sessionK(X, ServerRole)"


specification (PRF)
  inj_PRF: "inj PRF"
  — the pseudo-random function is collision-free
   ⟨proof⟩

specification (sessionK)
  inj_sessionK: "inj sessionK"
  — sessionK is collision-free; also, no clientK clashes with any serverK.
   ⟨proof⟩

axiomatization where
  — sessionK makes symmetric keys
```

```
  isSym_sessionK: "sessionK nonces ∈ symKeys" and
```

— sessionK never clashes with a long-term symmetric key (they don't exist in TLS anyway)

```
  sessionK_neq_shrK [iff]: "sessionK nonces ≠ shrK A"
```

inductive_set *tls :: "event list set"*
where

```
  Nil:   — The initial, empty trace
         "[] ∈ tls"
```

| *Fake:* — The Spy may say anything he can say. The sender field is correct, but agents don't use that information.

```
         "⟦evsf ∈ tls;  X ∈ synth (analz (spies evsf))⟧
          ⟹ Says Spy B X # evsf ∈ tls"
```

| *SpyKeys:* — The spy may apply *PRF* and *sessionK* to available nonces

```
         "⟦evsSK ∈ tls;
             {Nonce NA, Nonce NB, Nonce M} ⊆ analz (spies evsSK)⟧
          ⟹ Notes Spy ⦃ Nonce (PRF(M,NA,NB)),
                         Key (sessionK((NA,NB,M),role))⦄ # evsSK ∈ tls"
```

| *ClientHello:*

— (7.4.1.2) PA represents *CLIENT_VERSION*, *CIPHER_SUITES* and *COMPRESSION_METHODS*. It is uninterpreted but will be confirmed in the FINISHED messages. NA is CLIENT RANDOM, while SID is *SESSION_ID*. UNIX TIME is omitted because the protocol doesn't use it. May assume *NA ∉ range PRF* because CLIENT RANDOM is 28 bytes while MASTER SECRET is 48 bytes

```
         "⟦evsCH ∈ tls;  Nonce NA ∉ used evsCH;  NA ∉ range PRF⟧
          ⟹ Says A B ⦃Agent A, Nonce NA, Number SID, Number PA⦄
               # evsCH  ∈  tls"
```

| *ServerHello:*

— 7.4.1.3 of the TLS Internet-Draft PB represents *CLIENT_VERSION*, *CIPHER_SUITE* and *COMPRESSION_METHOD*. SERVER CERTIFICATE (7.4.2) is always present. *CERTIFICATE_REQUEST* (7.4.4) is implied.

```
         "⟦evsSH ∈ tls;  Nonce NB ∉ used evsSH;  NB ∉ range PRF;
             Says A' B ⦃Agent A, Nonce NA, Number SID, Number PA⦄
               ∈ set evsSH⟧
          ⟹ Says B A ⦃Nonce NB, Number SID, Number PB⦄ # evsSH  ∈  tls"
```

| *Certificate:*

— SERVER (7.4.2) or CLIENT (7.4.6) CERTIFICATE.

```
         "evsC ∈ tls ⟹ Says B A (certificate B (pubK B)) # evsC  ∈  tls"
```

| *ClientKeyExch:*

— CLIENT KEY EXCHANGE (7.4.7). The client, A, chooses PMS, the PREMASTER SECRET. She encrypts PMS using the supplied KB, which ought to be pubK B. We assume *PMS ∉ range PRF* because a clash betweem the PMS and another MASTER SECRET is highly unlikely (even though both items have the same length, 48 bytes). The Note event records in the trace that she knows PMS (see REMARK at top).

```
         "⟦evsCX ∈ tls;  Nonce PMS ∉ used evsCX;  PMS ∉ range PRF;
```

```
          Says B' A (certificate B KB) ∈ set evsCX⟧
        ⟹ Says A B (Crypt KB (Nonce PMS))
            # Notes A {|Agent B, Nonce PMS|}
            # evsCX  ∈  tls"
```

| CertVerify:

— The optional Certificate Verify (7.4.8) message contains the specific components listed in the security analysis, F.1.1.2. It adds the pre-master-secret, which is also essential! Checking the signature, which is the only use of A's certificate, assures B of A's presence

```
        "⟦evsCV ∈ tls;
            Says B' A {|Nonce NB, Number SID, Number PB|} ∈ set evsCV;
            Notes A {|Agent B, Nonce PMS|} ∈ set evsCV⟧
         ⟹ Says A B (Crypt (priK A) (Hash{|Nonce NB, Agent B, Nonce PMS|}))
             # evsCV  ∈  tls"
```

— Finally come the FINISHED messages (7.4.8), confirming PA and PB among other things. The master-secret is PRF(PMS,NA,NB). Either party may send its message first.

| ClientFinished:

— The occurrence of `Notes A {|Agent B, Nonce PMS|}` stops the rule's applying when the Spy has satisfied the `Says A B` by repaying messages sent by the true client; in that case, the Spy does not know PMS and could not send ClientFinished. One could simply put `A ≠ Spy` into the rule, but one should not expect the spy to be well-behaved.

```
        "⟦evsCF ∈ tls;
            Says A  B {|Agent A, Nonce NA, Number SID, Number PA|}
              ∈ set evsCF;
            Says B' A {|Nonce NB, Number SID, Number PB|} ∈ set evsCF;
            Notes A {|Agent B, Nonce PMS|} ∈ set evsCF;
            M = PRF(PMS,NA,NB)⟧
         ⟹ Says A B (Crypt (clientK(NA,NB,M))
                       (Hash{|Number SID, Nonce M,
                              Nonce NA, Number PA, Agent A,
                              Nonce NB, Number PB, Agent B|}))
             # evsCF  ∈  tls"
```

| ServerFinished:

— Keeping A' and A" distinct means B cannot even check that the two messages originate from the same source.

```
        "⟦evsSF ∈ tls;
            Says A' B  {|Agent A, Nonce NA, Number SID, Number PA|}
              ∈ set evsSF;
            Says B  A  {|Nonce NB, Number SID, Number PB|} ∈ set evsSF;
            Says A'' B (Crypt (pubK B) (Nonce PMS)) ∈ set evsSF;
            M = PRF(PMS,NA,NB)⟧
         ⟹ Says B A (Crypt (serverK(NA,NB,M))
                       (Hash{|Number SID, Nonce M,
                              Nonce NA, Number PA, Agent A,
                              Nonce NB, Number PB, Agent B|}))
             # evsSF  ∈  tls"
```

| ClientAccepts:

— Having transmitted ClientFinished and received an identical message encrypted with serverK, the client stores the parameters needed to resume this session. The "Notes A ..." premise is used to prove *Notes_master_imp_Crypt_PMS*.

```
"⟦evsCA ∈ tls;
    Notes A {|Agent B, Nonce PMS|} ∈ set evsCA;
    M = PRF(PMS,NA,NB);
    X = Hash{|Number SID, Nonce M,
              Nonce NA, Number PA, Agent A,
              Nonce NB, Number PB, Agent B|};
    Says A  B (Crypt (clientK(NA,NB,M)) X) ∈ set evsCA;
    Says B' A (Crypt (serverK(NA,NB,M)) X) ∈ set evsCA⟧
  ⟹
    Notes A {|Number SID, Agent A, Agent B, Nonce M|} # evsCA  ∈  tls"
```

| *ServerAccepts:*

— Having transmitted ServerFinished and received an identical message encrypted with clientK, the server stores the parameters needed to resume this session. The "Says A'' B ..." premise is used to prove *Notes_master_imp_Crypt_PMS*.

```
"⟦evsSA ∈ tls;
    A ≠ B;
    Says A'' B (Crypt (pubK B) (Nonce PMS)) ∈ set evsSA;
    M = PRF(PMS,NA,NB);
    X = Hash{|Number SID, Nonce M,
              Nonce NA, Number PA, Agent A,
              Nonce NB, Number PB, Agent B|};
    Says B  A (Crypt (serverK(NA,NB,M)) X) ∈ set evsSA;
    Says A' B (Crypt (clientK(NA,NB,M)) X) ∈ set evsSA⟧
  ⟹
    Notes B {|Number SID, Agent A, Agent B, Nonce M|} # evsSA  ∈  tls"
```

| *ClientResume:*

— If A recalls the *SESSION_ID*, then she sends a FINISHED message using the new nonces and stored MASTER SECRET.

```
"⟦evsCR ∈ tls;
    Says A  B {|Agent A, Nonce NA, Number SID, Number PA|} ∈ set evsCR;
    Says B' A {|Nonce NB, Number SID, Number PB|} ∈ set evsCR;
    Notes A {|Number SID, Agent A, Agent B, Nonce M|} ∈ set evsCR⟧
  ⟹ Says A B (Crypt (clientK(NA,NB,M))
              (Hash{|Number SID, Nonce M,
                     Nonce NA, Number PA, Agent A,
                     Nonce NB, Number PB, Agent B|}))
     # evsCR  ∈  tls"
```

| *ServerResume:*

— Resumption (7.3): If B finds the *SESSION_ID* then he can send a FINISHED message using the recovered MASTER SECRET

```
"⟦evsSR ∈ tls;
    Says A' B {|Agent A, Nonce NA, Number SID, Number PA|} ∈ set evsSR;
    Says B  A {|Nonce NB, Number SID, Number PB|} ∈ set evsSR;
    Notes B {|Number SID, Agent A, Agent B, Nonce M|} ∈ set evsSR⟧
  ⟹ Says B A (Crypt (serverK(NA,NB,M))
              (Hash{|Number SID, Nonce M,
                     Nonce NA, Number PA, Agent A,
                     Nonce NB, Number PB, Agent B|})) # evsSR
```

```
                ∈  tls"

 | Oops:
```

— The most plausible compromise is of an old session key. Losing the MASTER SECRET or PREMASTER SECRET is more serious but rather unlikely. The assumption *A ≠ Spy* is essential: otherwise the Spy could learn session keys merely by replaying messages!

```
"⟦evso ∈ tls;  A ≠ Spy;
    Says A B (Crypt (sessionK((NA,NB,M),role)) X) ∈ set evso⟧
 ⟹ Says A Spy (Key (sessionK((NA,NB,M),role))) # evso  ∈  tls"
```

declare *Says_imp_knows_Spy [THEN analz.Inj, dest]*
declare *parts.Body [dest]*
declare *analz_into_parts [dest]*
declare *Fake_parts_insert_in_Un [dest]*

Automatically unfold the definition of "certificate"

declare *certificate_def [simp]*

Injectiveness of key-generating functions

declare *inj_PRF [THEN inj_eq, iff]*
declare *inj_sessionK [THEN inj_eq, iff]*
declare *isSym_sessionK [simp]*

lemma *pubK_neq_sessionK [iff]: "publicKey b A ≠ sessionK arg"*
⟨*proof*⟩

declare *pubK_neq_sessionK [THEN not_sym, iff]*

lemma *priK_neq_sessionK [iff]: "invKey (publicKey b A) ≠ sessionK arg"*
⟨*proof*⟩

declare *priK_neq_sessionK [THEN not_sym, iff]*

lemmas *keys_distinct = pubK_neq_sessionK priK_neq_sessionK*

21.1 Protocol Proofs

Possibility properties state that some traces run the protocol to the end. Four paths and 12 rules are considered.

Possibility property ending with ClientAccepts.

lemma *"⟦∀evs. (SOME N. Nonce N ∉ used evs) ∉ range PRF; A ≠ B⟧*
⟹ ∃SID M. ∃evs ∈ tls.
Notes A ⦃Number SID, Agent A, Agent B, Nonce M⦄ ∈ set evs"
⟨*proof*⟩

And one for ServerAccepts. Either FINISHED message may come first.

```
lemma "⟦∀evs. (SOME N. Nonce N ∉ used evs) ∉ range PRF; A ≠ B⟧
      ⟹ ∃SID NA PA NB PB M. ∃evs ∈ tls.
           Notes B {|Number SID, Agent A, Agent B, Nonce M|} ∈ set evs"
⟨proof⟩
```

Another one, for CertVerify (which is optional)

```
lemma "⟦∀evs. (SOME N. Nonce N ∉ used evs) ∉ range PRF;  A ≠ B⟧
       ⟹ ∃NB PMS. ∃evs ∈ tls.
              Says A B (Crypt (priK A) (Hash{|Nonce NB, Agent B, Nonce PMS|}))

                ∈ set evs"
⟨proof⟩
```

Another one, for session resumption (both ServerResume and ClientResume). NO tls.Nil here: we refer to a previous session, not the empty trace.

```
lemma "⟦evs0 ∈ tls;
          Notes A {|Number SID, Agent A, Agent B, Nonce M|} ∈ set evs0;
          Notes B {|Number SID, Agent A, Agent B, Nonce M|} ∈ set evs0;
          ∀evs. (SOME N. Nonce N ∉ used evs) ∉ range PRF;
          A ≠ B⟧
      ⟹ ∃NA PA NB PB X. ∃evs ∈ tls.
                X = Hash{|Number SID, Nonce M,
                          Nonce NA, Number PA, Agent A,
                          Nonce NB, Number PB, Agent B|}  ∧
                Says A B (Crypt (clientK(NA,NB,M)) X) ∈ set evs  ∧
                Says B A (Crypt (serverK(NA,NB,M)) X) ∈ set evs"
⟨proof⟩
```

21.2 Inductive proofs about tls

Spy never sees a good agent's private key!

```
lemma Spy_see_priK [simp]:
     "evs ∈ tls ⟹ (Key (privateKey b A) ∈ parts (spies evs)) = (A ∈ bad)"
⟨proof⟩

lemma Spy_analz_priK [simp]:
     "evs ∈ tls ⟹ (Key (privateKey b A) ∈ analz (spies evs)) = (A ∈ bad)"
⟨proof⟩

lemma Spy_see_priK_D [dest!]:
    "⟦Key (privateKey b A) ∈ parts (knows Spy evs);  evs ∈ tls⟧ ⟹ A ∈ bad"
⟨proof⟩
```

This lemma says that no false certificates exist. One might extend the model to include bogus certificates for the agents, but there seems little point in doing so: the loss of their private keys is a worse breach of security.

```
lemma certificate_valid:
    "⟦certificate B KB ∈ parts (spies evs);  evs ∈ tls⟧ ⟹ KB = pubK B"
⟨proof⟩

lemmas CX_KB_is_pubKB = Says_imp_spies [THEN parts.Inj, THEN certificate_valid]
```

21.2.1 Properties of items found in Notes

```
lemma Notes_Crypt_parts_spies:
    "⟦Notes A {|Agent B, X|} ∈ set evs;  evs ∈ tls⟧
     ⟹ Crypt (pubK B) X ∈ parts (spies evs)"
⟨proof⟩
```

C may be either A or B

```
lemma Notes_master_imp_Crypt_PMS:
    "⟦Notes C {|s, Agent A, Agent B, Nonce(PRF(PMS,NA,NB))|} ∈ set evs;
       evs ∈ tls⟧
     ⟹ Crypt (pubK B) (Nonce PMS) ∈ parts (spies evs)"
⟨proof⟩
```

Compared with the theorem above, both premise and conclusion are stronger

```
lemma Notes_master_imp_Notes_PMS:
    "⟦Notes A {|s, Agent A, Agent B, Nonce(PRF(PMS,NA,NB))|} ∈ set evs;
       evs ∈ tls⟧
     ⟹ Notes A {|Agent B, Nonce PMS|} ∈ set evs"
⟨proof⟩
```

21.2.2 Protocol goal: if B receives CertVerify, then A sent it

B can check A's signature if he has received A's certificate.

```
lemma TrustCertVerify_lemma:
    "⟦X ∈ parts (spies evs);
       X = Crypt (priK A) (Hash{|nb, Agent B, pms|});
       evs ∈ tls;  A ∉ bad⟧
     ⟹ Says A B X ∈ set evs"
⟨proof⟩
```

Final version: B checks X using the distributed KA instead of priK A

```
lemma TrustCertVerify:
    "⟦X ∈ parts (spies evs);
       X = Crypt (invKey KA) (Hash{|nb, Agent B, pms|});
       certificate A KA ∈ parts (spies evs);
       evs ∈ tls;  A ∉ bad⟧
     ⟹ Says A B X ∈ set evs"
⟨proof⟩
```

If CertVerify is present then A has chosen PMS.

```
lemma UseCertVerify_lemma:
    "⟦Crypt (priK A) (Hash{|nb, Agent B, Nonce PMS|}) ∈ parts (spies evs);
       evs ∈ tls;  A ∉ bad⟧
     ⟹ Notes A {|Agent B, Nonce PMS|} ∈ set evs"
⟨proof⟩
```

Final version using the distributed KA instead of priK A

```
lemma UseCertVerify:
    "⟦Crypt (invKey KA) (Hash{|nb, Agent B, Nonce PMS|})
         ∈ parts (spies evs);
       certificate A KA ∈ parts (spies evs);
```

```
        evs ∈ tls;  A ∉ bad⟧
      ⟹ Notes A ⦃Agent B, Nonce PMS⦄ ∈ set evs"
```
⟨proof⟩

lemma *no_Notes_A_PRF [simp]:*
```
    "evs ∈ tls ⟹ Notes A ⦃Agent B, Nonce (PRF x)⦄ ∉ set evs"
```
⟨proof⟩

lemma *MS_imp_PMS [dest!]:*
```
    "⟦Nonce (PRF (PMS,NA,NB)) ∈ parts (spies evs);  evs ∈ tls⟧
     ⟹ Nonce PMS ∈ parts (spies evs)"
```
⟨proof⟩

21.2.3 Unicity results for PMS, the pre-master-secret

PMS determines B.

lemma *Crypt_unique_PMS:*
```
    "⟦Crypt(pubK B)  (Nonce PMS) ∈ parts (spies evs);
        Crypt(pubK B') (Nonce PMS) ∈ parts (spies evs);
        Nonce PMS ∉ analz (spies evs);
        evs ∈ tls⟧
     ⟹ B=B'"
```
⟨proof⟩

In A's internal Note, PMS determines A and B.

lemma *Notes_unique_PMS:*
```
    "⟦Notes A  ⦃Agent B,  Nonce PMS⦄ ∈ set evs;
        Notes A' ⦃Agent B', Nonce PMS⦄ ∈ set evs;
        evs ∈ tls⟧
     ⟹ A=A' ∧ B=B'"
```
⟨proof⟩

21.3 Secrecy Theorems

Key compromise lemma needed to prove `analz_image_keys`. No collection of keys can help the spy get new private keys.

lemma *analz_image_priK [rule_format]:*
```
    "evs ∈ tls
     ⟹ ∀KK. (Key(priK B) ∈ analz (Key`KK ∪ (spies evs))) =
          (priK B ∈ KK | B ∈ bad)"
```
⟨proof⟩

slightly speeds up the big simplification below

lemma *range_sessionkeys_not_priK:*
```
    "KK ⊆ range sessionK ⟹ priK B ∉ KK"
```
⟨proof⟩

Lemma for the trivial direction of the if-and-only-if

lemma *analz_image_keys_lemma:*
```
    "(X ∈ analz (G ∪ H)) ⟶ (X ∈ analz H)  ⟹
```

```
    (X ∈ analz (G ∪ H))  =  (X ∈ analz H)"
```
⟨proof⟩

lemma analz_image_keys [rule_format]:
```
   "evs ∈ tls ⟹
    ∀KK. KK ⊆ range sessionK ⟶
          (Nonce N ∈ analz (Key‘KK ∪ (spies evs))) =
          (Nonce N ∈ analz (spies evs))"
```
⟨proof⟩

Knowing some session keys is no help in getting new nonces

lemma analz_insert_key [simp]:
```
   "evs ∈ tls ⟹
    (Nonce N ∈ analz (insert (Key (sessionK z)) (spies evs))) =
    (Nonce N ∈ analz (spies evs))"
```
⟨proof⟩

21.3.1 Protocol goal: serverK(Na,Nb,M) and clientK(Na,Nb,M) remain secure

Lemma: session keys are never used if PMS is fresh. Nonces don't have to agree, allowing session resumption. Converse doesn't hold; revealing PMS doesn't force the keys to be sent. THEY ARE NOT SUITABLE AS SAFE ELIM RULES.

lemma PMS_lemma:
```
   "⟦Nonce PMS ∉ parts (spies evs);
      K = sessionK((Na, Nb, PRF(PMS,NA,NB)), role);
      evs ∈ tls⟧
 ⟹ Key K ∉ parts (spies evs) ∧ (∀Y. Crypt K Y ∉ parts (spies evs))"
```
⟨proof⟩

lemma PMS_sessionK_not_spied:
```
   "⟦Key (sessionK((Na, Nb, PRF(PMS,NA,NB)), role)) ∈ parts (spies evs);
      evs ∈ tls⟧
    ⟹ Nonce PMS ∈ parts (spies evs)"
```
⟨proof⟩

lemma PMS_Crypt_sessionK_not_spied:
```
   "⟦Crypt (sessionK((Na, Nb, PRF(PMS,NA,NB)), role)) Y
        ∈ parts (spies evs);  evs ∈ tls⟧
    ⟹ Nonce PMS ∈ parts (spies evs)"
```
⟨proof⟩

Write keys are never sent if M (MASTER SECRET) is secure. Converse fails; betraying M doesn't force the keys to be sent! The strong Oops condition can be weakened later by unicity reasoning, with some effort. NO LONGER USED: see `clientK_not_spied` and `serverK_not_spied`

lemma sessionK_not_spied:
```
   "⟦∀A. Says A Spy (Key (sessionK((NA,NB,M),role))) ∉ set evs;
      Nonce M ∉ analz (spies evs);  evs ∈ tls⟧
    ⟹ Key (sessionK((NA,NB,M),role)) ∉ parts (spies evs)"
```

⟨proof⟩

If A sends ClientKeyExch to an honest B, then the PMS will stay secret.

lemma *Spy_not_see_PMS:*
```
"⟦Notes A {|Agent B, Nonce PMS|} ∈ set evs;
    evs ∈ tls;  A ∉ bad;  B ∉ bad⟧
 ⟹ Nonce PMS ∉ analz (spies evs)"
```
⟨proof⟩

If A sends ClientKeyExch to an honest B, then the MASTER SECRET will stay secret.

lemma *Spy_not_see_MS:*
```
"⟦Notes A {|Agent B, Nonce PMS|} ∈ set evs;
    evs ∈ tls;  A ∉ bad;  B ∉ bad⟧
 ⟹ Nonce (PRF(PMS,NA,NB)) ∉ analz (spies evs)"
```
⟨proof⟩

21.3.2 Weakening the Oops conditions for leakage of clientK

If A created PMS then nobody else (except the Spy in replays) would send a message using a clientK generated from that PMS.

lemma *Says_clientK_unique:*
```
"⟦Says A' B' (Crypt (clientK(Na,Nb,PRF(PMS,NA,NB))) Y) ∈ set evs;
    Notes A {|Agent B, Nonce PMS|} ∈ set evs;
    evs ∈ tls;  A' ≠ Spy⟧
 ⟹ A = A'"
```
⟨proof⟩

If A created PMS and has not leaked her clientK to the Spy, then it is completely secure: not even in parts!

lemma *clientK_not_spied:*
```
"⟦Notes A {|Agent B, Nonce PMS|} ∈ set evs;
    Says A Spy (Key (clientK(Na,Nb,PRF(PMS,NA,NB)))) ∉ set evs;
    A ∉ bad;  B ∉ bad;
    evs ∈ tls⟧
 ⟹ Key (clientK(Na,Nb,PRF(PMS,NA,NB))) ∉ parts (spies evs)"
```
⟨proof⟩

21.3.3 Weakening the Oops conditions for leakage of serverK

If A created PMS for B, then nobody other than B or the Spy would send a message using a serverK generated from that PMS.

lemma *Says_serverK_unique:*
```
"⟦Says B' A' (Crypt (serverK(Na,Nb,PRF(PMS,NA,NB))) Y) ∈ set evs;
    Notes A {|Agent B, Nonce PMS|} ∈ set evs;
    evs ∈ tls;  A ∉ bad;  B ∉ bad;  B' ≠ Spy⟧
 ⟹ B = B'"
```
⟨proof⟩

If A created PMS for B, and B has not leaked his serverK to the Spy, then it is completely secure: not even in parts!

```
lemma serverK_not_spied:
    "⟦Notes A {|Agent B, Nonce PMS|} ∈ set evs;
       Says B Spy (Key(serverK(Na,Nb,PRF(PMS,NA,NB)))) ∉ set evs;
       A ∉ bad;  B ∉ bad;  evs ∈ tls⟧
     ⟹ Key (serverK(Na,Nb,PRF(PMS,NA,NB))) ∉ parts (spies evs)"
⟨proof⟩
```

21.3.4 Protocol goals: if A receives ServerFinished, then B is present and has used the quoted values PA, PB, etc. Note that it is up to A to compare PA with what she originally sent.

The mention of her name (A) in X assures A that B knows who she is.

```
lemma TrustServerFinished [rule_format]:
    "⟦X = Crypt (serverK(Na,Nb,M))
              (Hash{|Number SID, Nonce M,
                     Nonce Na, Number PA, Agent A,
                     Nonce Nb, Number PB, Agent B|});
       M = PRF(PMS,NA,NB);
       evs ∈ tls;  A ∉ bad;  B ∉ bad⟧
     ⟹ Says B Spy (Key(serverK(Na,Nb,M))) ∉ set evs ⟶
        Notes A {|Agent B, Nonce PMS|} ∈ set evs ⟶
        X ∈ parts (spies evs) ⟶ Says B A X ∈ set evs"
⟨proof⟩
```

This version refers not to ServerFinished but to any message from B. We don't assume B has received CertVerify, and an intruder could have changed A's identity in all other messages, so we can't be sure that B sends his message to A. If CLIENT KEY EXCHANGE were augmented to bind A's identity with PMS, then we could replace A' by A below.

```
lemma TrustServerMsg [rule_format]:
    "⟦M = PRF(PMS,NA,NB);  evs ∈ tls;  A ∉ bad;  B ∉ bad⟧
     ⟹ Says B Spy (Key(serverK(Na,Nb,M))) ∉ set evs ⟶
        Notes A {|Agent B, Nonce PMS|} ∈ set evs ⟶
        Crypt (serverK(Na,Nb,M)) Y ∈ parts (spies evs)  ⟶
        (∃A'. Says B A' (Crypt (serverK(Na,Nb,M)) Y) ∈ set evs)"
⟨proof⟩
```

21.3.5 Protocol goal: if B receives any message encrypted with clientK then A has sent it

ASSUMING that A chose PMS. Authentication is assumed here; B cannot verify it. But if the message is ClientFinished, then B can then check the quoted values PA, PB, etc.

```
lemma TrustClientMsg [rule_format]:
    "⟦M = PRF(PMS,NA,NB);  evs ∈ tls;  A ∉ bad;  B ∉ bad⟧
     ⟹ Says A Spy (Key(clientK(Na,Nb,M))) ∉ set evs ⟶
        Notes A {|Agent B, Nonce PMS|} ∈ set evs ⟶
        Crypt (clientK(Na,Nb,M)) Y ∈ parts (spies evs) ⟶
        Says A B (Crypt (clientK(Na,Nb,M)) Y) ∈ set evs"
⟨proof⟩
```

21.3.6 Protocol goal: if B receives ClientFinished, and if B is able to check a CertVerify from A, then A has used the quoted values PA, PB, etc. Even this one requires A to be uncompromised.

```
lemma AuthClientFinished:
     "⟦M = PRF(PMS,NA,NB);
         Says A Spy (Key(clientK(Na,Nb,M))) ∉ set evs;
         Says A' B (Crypt (clientK(Na,Nb,M)) Y) ∈ set evs;
         certificate A KA ∈ parts (spies evs);
         Says A'' B (Crypt (invKey KA) (Hash{|nb, Agent B, Nonce PMS|}))
           ∈ set evs;
         evs ∈ tls;  A ∉ bad;  B ∉ bad⟧
      ⟹ Says A B (Crypt (clientK(Na,Nb,M)) Y) ∈ set evs"
⟨proof⟩
```

end

22 The Certified Electronic Mail Protocol by Abadi et al.

theory *CertifiedEmail* **imports** *Public* **begin**

```
abbreviation
  TTP :: agent where
  "TTP == Server"

abbreviation
  RPwd :: "agent ⇒ key" where
  "RPwd == shrK"

consts
  NoAuth   :: nat
  TTPAuth  :: nat
  SAuth    :: nat
  BothAuth :: nat
```

We formalize a fixed way of computing responses. Could be better.

```
definition "response" :: "agent ⇒ agent ⇒ nat ⇒ msg" where
  "response S R q == Hash {|Agent S, Key (shrK R), Nonce q|}"


inductive_set certified_mail :: "event list set"
  where

  Nil: — The empty trace
     "[] ∈ certified_mail"

| Fake: — The Spy may say anything he can say. The sender field is correct, but
agents don't use that information.
      "[[evsf ∈ certified_mail; X ∈ synth(analz(spies evsf))]]
       ⟹ Says Spy B X # evsf ∈ certified_mail"

| FakeSSL: — The Spy may open SSL sessions with TTP, who is the only agent
equipped with the necessary credentials to serve as an SSL server.
         "[[evsfssl ∈ certified_mail; X ∈ synth(analz(spies evsfssl))]]
          ⟹ Notes TTP {|Agent Spy, Agent TTP, X|} # evsfssl ∈ certified_mail"

| CM1: — The sender approaches the recipient. The message is a number.
 "[[evs1 ∈ certified_mail;
    Key K ∉ used evs1;
    K ∈ symKeys;
    Nonce q ∉ used evs1;
    hs = Hash{|Number cleartext, Nonce q, response S R q, Crypt K (Number m)|};
    S2TTP = Crypt(pubEK TTP) {|Agent S, Number BothAuth, Key K, Agent R, hs|}]]
  ⟹ Says S R {|Agent S, Agent TTP, Crypt K (Number m), Number BothAuth,
                 Number cleartext, Nonce q, S2TTP|} # evs1
        ∈ certified_mail"

| CM2: — The recipient records S2TTP while transmitting it and her password to TTP
over an SSL channel.
 "[[evs2 ∈ certified_mail;
    Gets R {|Agent S, Agent TTP, em, Number BothAuth, Number cleartext,
             Nonce q, S2TTP|} ∈ set evs2;
    TTP ≠ R;
    hr = Hash {|Number cleartext, Nonce q, response S R q, em|}]]
  ⟹
   Notes TTP {|Agent R, Agent TTP, S2TTP, Key(RPwd R), hr|} # evs2
      ∈ certified_mail"

| CM3: — TTP simultaneously reveals the key to the recipient and gives a receipt to
the sender. The SSL channel does not authenticate the client (R), but TTP accepts the
message only if the given password is that of the claimed sender, R. He replies over the
established SSL channel.
 "[[evs3 ∈ certified_mail;
    Notes TTP {|Agent R, Agent TTP, S2TTP, Key(RPwd R), hr|} ∈ set evs3;
    S2TTP = Crypt (pubEK TTP)
                     {|Agent S, Number BothAuth, Key k, Agent R, hs|};
    TTP ≠ R;  hs = hr;  k ∈ symKeys]]
  ⟹
   Notes R {|Agent TTP, Agent R, Key k, hr|} #
   Gets S (Crypt (priSK TTP) S2TTP) #
```

```
   Says TTP S (Crypt (priSK TTP) S2TTP) # evs3 ∈ certified_mail"

| Reception:
 "⟦evsr ∈ certified_mail; Says A B X ∈ set evsr⟧
  ⟹ Gets B X#evsr ∈ certified_mail"
```

declare *Says_imp_knows_Spy [THEN analz.Inj, dest]*
declare *analz_into_parts [dest]*

```
lemma "⟦Key K ∉ used []; K ∈ symKeys⟧ ⟹
       ∃S2TTP. ∃evs ∈ certified_mail.
           Says TTP S (Crypt (priSK TTP) S2TTP) ∈ set evs"
⟨proof⟩
```

```
lemma Gets_imp_Says:
 "⟦Gets B X ∈ set evs; evs ∈ certified_mail⟧ ⟹ ∃A. Says A B X ∈ set evs"
⟨proof⟩
```

```
lemma Gets_imp_parts_knows_Spy:
     "⟦Gets A X ∈ set evs; evs ∈ certified_mail⟧ ⟹ X ∈ parts(spies evs)"
⟨proof⟩
```

```
lemma CM2_S2TTP_analz_knows_Spy:
 "⟦Gets R {|Agent A, Agent B, em, Number AO, Number cleartext,
              Nonce q, S2TTP|} ∈ set evs;
    evs ∈ certified_mail⟧
  ⟹ S2TTP ∈ analz(spies evs)"
⟨proof⟩
```

```
lemmas CM2_S2TTP_parts_knows_Spy =
    CM2_S2TTP_analz_knows_Spy [THEN analz_subset_parts [THEN subsetD]]
```

```
lemma hr_form_lemma [rule_format]:
 "evs ∈ certified_mail
  ⟹ hr ∉ synth (analz (spies evs)) ⟶
      (∀S2TTP. Notes TTP {|Agent R, Agent TTP, S2TTP, pwd, hr|}
          ∈ set evs ⟶
      (∃clt q S em. hr = Hash {|Number clt, Nonce q, response S R q, em|}))"
⟨proof⟩
```

Cannot strengthen the first disjunct to $R \neq Spy$ because the fakessl rule allows Spy to spoof the sender's name. Maybe can strengthen the second disjunct with $R \neq Spy$.

```
lemma hr_form:
 "⟦Notes TTP {|Agent R, Agent TTP, S2TTP, pwd, hr|} ∈ set evs;
    evs ∈ certified_mail⟧
  ⟹ hr ∈ synth (analz (spies evs)) |
      (∃clt q S em. hr = Hash {|Number clt, Nonce q, response S R q, em|})"
⟨proof⟩
```

lemma *Spy_dont_know_private_keys [dest!]:*
```
"⟦Key (privateKey b A) ∈ parts (spies evs); evs ∈ certified_mail⟧
 ⟹ A ∈ bad"
```
⟨*proof*⟩

lemma *Spy_know_private_keys_iff [simp]:*
```
"evs ∈ certified_mail
 ⟹ (Key (privateKey b A) ∈ parts (spies evs)) = (A ∈ bad)"
```
⟨*proof*⟩

lemma *Spy_dont_know_TTPKey_parts [simp]:*
```
"evs ∈ certified_mail ⟹ Key (privateKey b TTP) ∉ parts(spies evs)"
```

⟨*proof*⟩

lemma *Spy_dont_know_TTPKey_analz [simp]:*
```
"evs ∈ certified_mail ⟹ Key (privateKey b TTP) ∉ analz(spies evs)"
```
⟨*proof*⟩

Thus, prove any goal that assumes that *Spy* knows a private key belonging to *TTP*

declare *Spy_dont_know_TTPKey_parts [THEN [2] rev_notE, elim!]*

lemma *CM3_k_parts_knows_Spy:*
```
"⟦evs ∈ certified_mail;
  Notes TTP ⦃Agent A, Agent TTP,
              Crypt (pubEK TTP) ⦃Agent S, Number AO, Key K,
              Agent R, hs⦄, Key (RPwd R), hs⦄ ∈ set evs⟧
 ⟹ Key K ∈ parts(spies evs)"
```
⟨*proof*⟩

lemma *Spy_dont_know_RPwd [rule_format]:*
```
"evs ∈ certified_mail ⟹ Key (RPwd A) ∈ parts(spies evs) ⟶ A ∈ bad"
```
⟨*proof*⟩

lemma *Spy_know_RPwd_iff [simp]:*
```
"evs ∈ certified_mail ⟹ (Key (RPwd A) ∈ parts(spies evs)) = (A∈bad)"
```
⟨*proof*⟩

lemma *Spy_analz_RPwd_iff [simp]:*
```
"evs ∈ certified_mail ⟹ (Key (RPwd A) ∈ analz(spies evs)) = (A∈bad)"
```
⟨*proof*⟩

Unused, but a guarantee of sorts

theorem *CertAutenticity:*
```
"⟦Crypt (priSK TTP) X ∈ parts (spies evs); evs ∈ certified_mail⟧
 ⟹ ∃A. Says TTP A (Crypt (priSK TTP) X) ∈ set evs"
```
⟨*proof*⟩

22.1 Proving Confidentiality Results

lemma *analz_image_freshK [rule_format]:*

```
"evs ∈ certified_mail ⟹
  ∀K KK. invKey (pubEK TTP) ∉ KK ⟶
        (Key K ∈ analz (Key`KK ∪ (spies evs))) =
        (K ∈ KK | Key K ∈ analz (spies evs))"
```
⟨proof⟩

lemma *analz_insert_freshK:*
```
  "⟦evs ∈ certified_mail;  KAB ≠ invKey (pubEK TTP)⟧ ⟹
      (Key K ∈ analz (insert (Key KAB) (spies evs))) =
      (K = KAB | Key K ∈ analz (spies evs))"
```
⟨proof⟩

S2TTP must have originated from a valid sender provided *K* is secure. Proof is surprisingly hard.

lemma *Notes_SSL_imp_used:*
```
     "⟦Notes B {|Agent A, Agent B, X|} ∈ set evs⟧ ⟹ X ∈ used evs"
```
⟨proof⟩

lemma *S2TTP_sender_lemma [rule_format]:*
```
 "evs ∈ certified_mail ⟹
    Key K ∉ analz (spies evs) ⟶
    (∀AO. Crypt (pubEK TTP)
           {|Agent S, Number AO, Key K, Agent R, hs|} ∈ used evs ⟶
    (∃m ctxt q.
        hs = Hash{|Number ctxt, Nonce q, response S R q, Crypt K (Number m)|}
∧
        Says S R
           {|Agent S, Agent TTP, Crypt K (Number m), Number AO,
             Number ctxt, Nonce q,
             Crypt (pubEK TTP)
              {|Agent S, Number AO, Key K, Agent R, hs |}|} ∈ set evs))"
```
⟨proof⟩

lemma *S2TTP_sender:*
```
 "⟦Crypt (pubEK TTP) {|Agent S, Number AO, Key K, Agent R, hs|} ∈ used evs;
    Key K ∉ analz (spies evs);
    evs ∈ certified_mail⟧
  ⟹ ∃m ctxt q.
        hs = Hash{|Number ctxt, Nonce q, response S R q, Crypt K (Number m)|}
∧
        Says S R
           {|Agent S, Agent TTP, Crypt K (Number m), Number AO,
             Number ctxt, Nonce q,
             Crypt (pubEK TTP)
              {|Agent S, Number AO, Key K, Agent R, hs|}|} ∈ set evs"
```
⟨proof⟩

Nobody can have used non-existent keys!

lemma *new_keys_not_used [simp]:*
```
    "⟦Key K ∉ used evs; K ∈ symKeys; evs ∈ certified_mail⟧
     ⟹ K ∉ keysFor (parts (spies evs))"
```

⟨*proof*⟩

Less easy to prove `m' = m`. Maybe needs a separate unicity theorem for ciphertexts of the form `Crypt K (Number m)`, where `K` is secure.

```
lemma Key_unique_lemma [rule_format]:
     "evs ∈ certified_mail ⟹
       Key K ∉ analz (spies evs) ⟶
       (∀m cleartext q hs.
        Says S R
           {|Agent S, Agent TTP, Crypt K (Number m), Number AO,
             Number cleartext, Nonce q,
             Crypt (pubEK TTP) {|Agent S, Number AO, Key K, Agent R, hs|}|}
          ∈ set evs ⟶
       (∀m' cleartext' q' hs'.
       Says S' R'
           {|Agent S', Agent TTP, Crypt K (Number m'), Number AO',
             Number cleartext', Nonce q',
             Crypt (pubEK TTP) {|Agent S', Number AO', Key K, Agent R', hs'|}|}
          ∈ set evs ⟶ R' = R ∧ S' = S ∧ AO' = AO ∧ hs' = hs))"
```

⟨*proof*⟩

The key determines the sender, recipient and protocol options.

```
lemma Key_unique:
      "⟦Says S R
           {|Agent S, Agent TTP, Crypt K (Number m), Number AO,
             Number cleartext, Nonce q,
             Crypt (pubEK TTP) {|Agent S, Number AO, Key K, Agent R, hs|}|}
          ∈ set evs;
         Says S' R'
           {|Agent S', Agent TTP, Crypt K (Number m'), Number AO',
             Number cleartext', Nonce q',
             Crypt (pubEK TTP) {|Agent S', Number AO', Key K, Agent R', hs'|}|}
          ∈ set evs;
         Key K ∉ analz (spies evs);
         evs ∈ certified_mail⟧
       ⟹ R' = R ∧ S' = S ∧ AO' = AO ∧ hs' = hs"
```

⟨*proof*⟩

22.2 The Guarantees for Sender and Recipient

A Sender's guarantee: If Spy gets the key then `R` is bad and `S` moreover gets his return receipt (and therefore has no grounds for complaint).

```
theorem S_fairness_bad_R:
      "⟦Says S R {|Agent S, Agent TTP, Crypt K (Number m), Number AO,
                    Number cleartext, Nonce q, S2TTP|} ∈ set evs;
         S2TTP = Crypt (pubEK TTP) {|Agent S, Number AO, Key K, Agent R, hs|};
         Key K ∈ analz (spies evs);
         evs ∈ certified_mail;
         S≠Spy⟧
      ⟹ R ∈ bad ∧ Gets S (Crypt (priSK TTP) S2TTP) ∈ set evs"
```

⟨*proof*⟩

Confidentially for the symmetric key

theorem *Spy_not_see_encrypted_key:*
```
"⟦Says S R ⦃Agent S, Agent TTP, Crypt K (Number m), Number AO,
             Number cleartext, Nonce q, S2TTP⦄ ∈ set evs;
  S2TTP = Crypt (pubEK TTP) ⦃Agent S, Number AO, Key K, Agent R, hs⦄;
  evs ∈ certified_mail;
  S≠Spy; R ∉ bad⟧
 ⟹ Key K ∉ analz(spies evs)"
```
⟨*proof*⟩

Agent R, who may be the Spy, doesn't receive the key until S has access to the return receipt.

theorem *S_guarantee:*
```
"⟦Says S R ⦃Agent S, Agent TTP, Crypt K (Number m), Number AO,
             Number cleartext, Nonce q, S2TTP⦄ ∈ set evs;
  S2TTP = Crypt (pubEK TTP) ⦃Agent S, Number AO, Key K, Agent R, hs⦄;
  Notes R ⦃Agent TTP, Agent R, Key K, hs⦄ ∈ set evs;
  S≠Spy;  evs ∈ certified_mail⟧
 ⟹ Gets S (Crypt (priSK TTP) S2TTP) ∈ set evs"
```
⟨*proof*⟩

If R sends message 2, and a delivery certificate exists, then R receives the necessary key. This result is also important to S, as it confirms the validity of the return receipt.

theorem *RR_validity:*
```
"⟦Crypt (priSK TTP) S2TTP ∈ used evs;
  S2TTP = Crypt (pubEK TTP)
           ⦃Agent S, Number AO, Key K, Agent R,
             Hash ⦃Number cleartext, Nonce q, r, em⦄⦄;
  hr = Hash ⦃Number cleartext, Nonce q, r, em⦄;
  R≠Spy;  evs ∈ certified_mail⟧
 ⟹ Notes R ⦃Agent TTP, Agent R, Key K, hr⦄ ∈ set evs"
```
⟨*proof*⟩

end

23 Conventional protocols: rely on conventional Message, Event and Public – Public-key protocols

theory *Auth_Public*
imports
  NS_Public_Bad
  NS_Public
  TLS
  CertifiedEmail
begin

end

24 Theory of Events for Security Protocols that use smartcards

```
theory EventSC
imports
  "../Message"
  "HOL-Library.Simps_Case_Conv"
begin

consts
  initState :: "agent => msg set"

datatype card = Card agent
```

Four new events express the traffic between an agent and his card

```
datatype
  event = Says  agent agent msg
        | Notes agent       msg
        | Gets  agent       msg
        | Inputs agent card msg
        | C_Gets card       msg
        | Outpts card agent msg
        | A_Gets agent      msg

consts
 bad     :: "agent set"
 stolen    :: "card set"
 cloned  :: "card set"
 secureM :: "bool"

abbreviation
  insecureM :: bool where
  "insecureM == ¬secureM"
```

Spy has access to his own key for spoof messages, but Server is secure

```
specification (bad)
  Spy_in_bad      [iff]: "Spy ∈ bad"
  Server_not_bad [iff]: "Server ∉ bad"
  ⟨proof⟩

specification (stolen)

  Card_Server_not_stolen [iff]: "Card Server ∉ stolen"
  Card_Spy_not_stolen    [iff]: "Card Spy ∉ stolen"
  ⟨proof⟩

specification (cloned)

  Card_Server_not_cloned [iff]: "Card Server ∉ cloned"
  Card_Spy_not_cloned    [iff]: "Card Spy ∉ cloned"
  ⟨proof⟩

primrec
```

```
  knows   :: "agent => event list => msg set"  where
  knows_Nil:   "knows A [] = initState A" |
  knows_Cons:  "knows A (ev # evs) =
         (case ev of
            Says A' B X =>
                if (A=A' | A=Spy) then insert X (knows A evs) else knows A
evs
          | Notes A' X  =>
                if (A=A' | (A=Spy & A'∈bad)) then insert X (knows A evs)
                                             else knows A evs
          | Gets A' X   =>
                if (A=A' & A ≠ Spy) then insert X (knows A evs)
                                     else knows A evs
          | Inputs A' C X =>
              if secureM then
                if A=A' then insert X (knows A evs) else knows A evs
              else
                if (A=A' | A=Spy) then insert X (knows A evs) else knows A
evs
          | C_Gets C X   => knows A evs
          | Outpts C A' X =>
              if secureM then
                if A=A' then insert X (knows A evs) else knows A evs
              else
                if A=Spy then insert X (knows A evs) else knows A evs
          | A_Gets A' X   =>
                if (A=A' & A ≠ Spy) then insert X (knows A evs)
                                     else knows A evs)"
```

primrec

```
  used :: "event list => msg set" where
  used_Nil:   "used []         = (UN B. parts (initState B))" |
  used_Cons:  "used (ev # evs) =
                 (case ev of
                    Says A B X => parts {X} ∪ (used evs)
                  | Notes A X  => parts {X} ∪ (used evs)
                  | Gets A X   => used evs
                  | Inputs A C X  => parts{X} ∪ (used evs)
                  | C_Gets C X   => used evs
                  | Outpts C A X  => parts{X} ∪ (used evs)
                  | A_Gets A X   => used evs)"
```

— *Gets* always follows *Says* in real protocols. Likewise, *C_Gets* will always have to follow *Inputs* and *A_Gets* will always have to follow *Outpts*

lemma *Notes_imp_used [rule_format]: "Notes A X ∈ set evs ⟶ X ∈ used evs"*
⟨*proof*⟩

lemma *Says_imp_used [rule_format]: "Says A B X ∈ set evs ⟶ X ∈ used evs"*
⟨*proof*⟩

lemma *MPair_used [rule_format]:*

```
"MPair X Y ∈ used evs ⟶ X ∈ used evs & Y ∈ used evs"
```
⟨proof⟩

24.1 Function knows

lemmas *parts_insert_knows_A = parts_insert [of _ "knows A evs"]* **for** *A evs*

lemma *knows_Spy_Says [simp]:*
```
"knows Spy (Says A B X # evs) = insert X (knows Spy evs)"
```
⟨proof⟩

Letting the Spy see "bad" agents' notes avoids redundant case-splits on whether *A = Spy* and whether *A ∈ bad*

lemma *knows_Spy_Notes [simp]:*
```
"knows Spy (Notes A X # evs) =
     (if A∈bad then insert X (knows Spy evs) else knows Spy evs)"
```
⟨proof⟩

lemma *knows_Spy_Gets [simp]: "knows Spy (Gets A X # evs) = knows Spy evs"*
⟨proof⟩

lemma *knows_Spy_Inputs_secureM [simp]:*
```
"secureM ⟹ knows Spy (Inputs A C X # evs) =
   (if A=Spy then insert X (knows Spy evs) else knows Spy evs)"
```
⟨proof⟩

lemma *knows_Spy_Inputs_insecureM [simp]:*
```
"insecureM ⟹ knows Spy (Inputs A C X # evs) = insert X (knows Spy evs)"
```
⟨proof⟩

lemma *knows_Spy_C_Gets [simp]: "knows Spy (C_Gets C X # evs) = knows Spy evs"*
⟨proof⟩

lemma *knows_Spy_Outpts_secureM [simp]:*
```
"secureM ⟹ knows Spy (Outpts C A X # evs) =
   (if A=Spy then insert X (knows Spy evs) else knows Spy evs)"
```
⟨proof⟩

lemma *knows_Spy_Outpts_insecureM [simp]:*
```
"insecureM ⟹ knows Spy (Outpts C A X # evs) = insert X (knows Spy evs)"
```
⟨proof⟩

lemma *knows_Spy_A_Gets [simp]: "knows Spy (A_Gets A X # evs) = knows Spy evs"*
⟨proof⟩

lemma *knows_Spy_subset_knows_Spy_Says:*
```
"knows Spy evs ⊆ knows Spy (Says A B X # evs)"
```
⟨proof⟩

lemma *knows_Spy_subset_knows_Spy_Notes:*
"knows Spy evs ⊆ knows Spy (Notes A X # evs)"
⟨*proof*⟩

lemma *knows_Spy_subset_knows_Spy_Gets:*
"knows Spy evs ⊆ knows Spy (Gets A X # evs)"
⟨*proof*⟩

lemma *knows_Spy_subset_knows_Spy_Inputs:*
"knows Spy evs ⊆ knows Spy (Inputs A C X # evs)"
⟨*proof*⟩

lemma *knows_Spy_equals_knows_Spy_Gets:*
"knows Spy evs = knows Spy (C_Gets C X # evs)"
⟨*proof*⟩

lemma *knows_Spy_subset_knows_Spy_Outpts: "knows Spy evs ⊆ knows Spy (Outpts C A X # evs)"*
⟨*proof*⟩

lemma *knows_Spy_subset_knows_Spy_A_Gets: "knows Spy evs ⊆ knows Spy (A_Gets A X # evs)"*
⟨*proof*⟩

Spy sees what is sent on the traffic

lemma *Says_imp_knows_Spy [rule_format]:*
"Says A B X ∈ set evs ⟶ X ∈ knows Spy evs"
⟨*proof*⟩

lemma *Notes_imp_knows_Spy [rule_format]:*
"Notes A X ∈ set evs ⟶ A∈ bad ⟶ X ∈ knows Spy evs"
⟨*proof*⟩

lemma *Inputs_imp_knows_Spy_secureM [rule_format (no_asm)]:*
"Inputs Spy C X ∈ set evs ⟶ secureM ⟶ X ∈ knows Spy evs"
⟨*proof*⟩

lemma *Inputs_imp_knows_Spy_insecureM [rule_format (no_asm)]:*
"Inputs A C X ∈ set evs ⟶ insecureM ⟶ X ∈ knows Spy evs"
⟨*proof*⟩

lemma *Outpts_imp_knows_Spy_secureM [rule_format (no_asm)]:*
"Outpts C Spy X ∈ set evs ⟶ secureM ⟶ X ∈ knows Spy evs"
⟨*proof*⟩

lemma *Outpts_imp_knows_Spy_insecureM [rule_format (no_asm)]:*
"Outpts C A X ∈ set evs ⟶ insecureM ⟶ X ∈ knows Spy evs"
⟨*proof*⟩

Elimination rules: derive contradictions from old Says events containing items known to be fresh

lemmas *knows_Spy_partsEs =*
    Says_imp_knows_Spy [THEN parts.Inj, elim_format]
    parts.Body [elim_format]

24.2 Knowledge of Agents

lemma *knows_Inputs: "knows A (Inputs A C X # evs) = insert X (knows A evs)"*
⟨*proof*⟩

lemma *knows_C_Gets: "knows A (C_Gets C X # evs) = knows A evs"*
⟨*proof*⟩

lemma *knows_Outpts_secureM:*
    "secureM ⟶ knows A (Outpts C A X # evs) = insert X (knows A evs)"
⟨*proof*⟩

lemma *knows_Outpts_insecureM:*
    "insecureM ⟶ knows Spy (Outpts C A X # evs) = insert X (knows Spy evs)"
⟨*proof*⟩

lemma *knows_subset_knows_Says: "knows A evs ⊆ knows A (Says A' B X # evs)"*
⟨*proof*⟩

lemma *knows_subset_knows_Notes: "knows A evs ⊆ knows A (Notes A' X # evs)"*
⟨*proof*⟩

lemma *knows_subset_knows_Gets: "knows A evs ⊆ knows A (Gets A' X # evs)"*
⟨*proof*⟩

lemma *knows_subset_knows_Inputs: "knows A evs ⊆ knows A (Inputs A' C X # evs)"*
⟨*proof*⟩

lemma *knows_subset_knows_C_Gets: "knows A evs ⊆ knows A (C_Gets C X # evs)"*
⟨*proof*⟩

lemma *knows_subset_knows_Outpts: "knows A evs ⊆ knows A (Outpts C A' X # evs)"*
⟨*proof*⟩

lemma *knows_subset_knows_A_Gets: "knows A evs ⊆ knows A (A_Gets A' X # evs)"*
⟨*proof*⟩

Agents know what they say

lemma *Says_imp_knows [rule_format]: "Says A B X ∈ set evs ⟶ X ∈ knows A evs"*

⟨*proof*⟩

Agents know what they note

lemma *Notes_imp_knows [rule_format]: "Notes A X ∈ set evs ⟶ X ∈ knows A evs"*
⟨*proof*⟩

Agents know what they receive

lemma *Gets_imp_knows_agents [rule_format]:*
"A ≠ Spy ⟶ Gets A X ∈ set evs ⟶ X ∈ knows A evs"
⟨*proof*⟩

lemma *Inputs_imp_knows_agents [rule_format (no_asm)]:*
"Inputs A (Card A) X ∈ set evs ⟶ X ∈ knows A evs"
⟨*proof*⟩

lemma *Outpts_imp_knows_agents_secureM [rule_format (no_asm)]:*
"secureM ⟶ Outpts (Card A) A X ∈ set evs ⟶ X ∈ knows A evs"
⟨*proof*⟩

lemma *Outpts_imp_knows_agents_insecureM [rule_format (no_asm)]:*
"insecureM ⟶ Outpts (Card A) A X ∈ set evs ⟶ X ∈ knows Spy evs"
⟨*proof*⟩

lemma *parts_knows_Spy_subset_used: "parts (knows Spy evs) ⊆ used evs"*
⟨*proof*⟩

lemmas *usedI = parts_knows_Spy_subset_used [THEN subsetD, intro]*

lemma *initState_into_used: "X ∈ parts (initState B) ⟹ X ∈ used evs"*
⟨*proof*⟩

simps_of_case *used_Cons_simps[simp]: used_Cons*

lemma *used_nil_subset: "used [] ⊆ used evs"*
⟨*proof*⟩

lemma *Says_parts_used [rule_format (no_asm)]:*
"Says A B X ∈ set evs ⟶ (parts {X}) ⊆ used evs"
⟨*proof*⟩

lemma *Notes_parts_used [rule_format (no_asm)]:*

```
  "Notes A X ∈ set evs ⟶ (parts {X}) ⊆ used evs"
⟨proof⟩

lemma Outpts_parts_used [rule_format (no_asm)]:
  "Outpts C A X ∈ set evs ⟶ (parts {X}) ⊆ used evs"
⟨proof⟩

lemma Inputs_parts_used [rule_format (no_asm)]:
  "Inputs A C X ∈ set evs ⟶ (parts {X}) ⊆ used evs"
⟨proof⟩
```

NOTE REMOVAL–laws above are cleaner, as they don't involve "case"

```
declare knows_Cons [simp del]
        used_Nil [simp del] used_Cons [simp del]


lemma knows_subset_knows_Cons: "knows A evs ⊆ knows A (e # evs)"
⟨proof⟩

lemma initState_subset_knows: "initState A ⊆ knows A evs"
⟨proof⟩
```

For proving `new_keys_not_used`

```
lemma keysFor_parts_insert:
  "⟦ K ∈ keysFor (parts (insert X G)); X ∈ synth (analz H) ⟧
   ⟹ K ∈ keysFor (parts (G ∪ H)) ∨ Key (invKey K) ∈ parts H"
⟨proof⟩

end
theory All_Symmetric
imports Message
begin
```

All keys are symmetric

```
overloading all_symmetric ≡ all_symmetric
begin
  definition "all_symmetric ≡ True"
end

lemma isSym_keys: "K ∈ symKeys"
  ⟨proof⟩

end
```

25 Theory of smartcards

```
theory Smartcard
imports EventSC "../All_Symmetric"
begin
```

As smartcards handle long-term (symmetric) keys, this theoy extends and supersedes theory Private.thy

An agent is bad if she reveals her PIN to the spy, not the shared key that is embedded in her card. An agent's being bad implies nothing about her smartcard, which independently may be stolen or cloned.

```
axiomatization
  shrK    :: "agent => key" and
  crdK    :: "card  => key" and
  pin     :: "agent => key" and


  Pairkey :: "agent * agent => nat" and
  pairK   :: "agent * agent => key"
where
  inj_shrK: "inj shrK" and   — No two smartcards store the same key
  inj_crdK: "inj crdK" and   — Nor do two cards
  inj_pin : "inj pin" and    — Nor do two agents have the same pin


  inj_pairK    [iff]: "(pairK(A,B) = pairK(A',B')) = (A = A' & B = B')" and
  comm_Pairkey [iff]: "Pairkey(A,B) = Pairkey(B,A)" and


  pairK_disj_crdK [iff]: "pairK(A,B) ≠ crdK C" and
  pairK_disj_shrK [iff]: "pairK(A,B) ≠ shrK P" and
  pairK_disj_pin [iff]:  "pairK(A,B) ≠ pin P" and
  shrK_disj_crdK [iff]:  "shrK P ≠ crdK C" and
  shrK_disj_pin [iff]:  "shrK P ≠ pin Q" and
  crdK_disj_pin [iff]:   "crdK C ≠ pin P"

definition legalUse :: "card => bool" (‹legalUse (_)›) where
  "legalUse C == C ∉ stolen"

primrec illegalUse :: "card  => bool" where
  illegalUse_def: "illegalUse (Card A) = ( (Card A ∈ stolen ∧ A ∈ bad)  ∨
Card A ∈ cloned )"
```

initState must be defined with care

```
overloading
  initState ≡ initState
begin

primrec initState where

  initState_Server:  "initState Server =
       (Key‘(range shrK ∪ range crdK ∪ range pin ∪ range pairK)) ∪
       (Nonce‘(range Pairkey))" |


  initState_Friend:  "initState (Friend i) = {Key (pin (Friend i))}" |


  initState_Spy: "initState Spy  =
               (Key‘((pin‘bad) ∪ (pin ‘{A. Card A ∈ cloned}) ∪
                                  (shrK‘{A. Card A ∈ cloned}) ∪
```

```
                        (crdK`cloned) ∪
                        (pairK`{(X,A). Card A ∈ cloned}))
            ∪ (Nonce`(Pairkey`{(A,B). Card A ∈ cloned & Card B ∈ cloned}))"
```

end

Still relying on axioms

axiomatization where
  Key_supply_ax: "finite KK ⟹ ∃ K. K ∉ KK & Key K ∉ used evs" **and**

  Nonce_supply_ax: "finite NN ⟹ ∃ N. N ∉ NN & Nonce N ∉ used evs"

25.1 Basic properties of shrK

declare *inj_shrK [THEN inj_eq, iff]*
declare *inj_crdK [THEN inj_eq, iff]*
declare *inj_pin [THEN inj_eq, iff]*

lemma *invKey_K [simp]: "invKey K = K"*
⟨*proof*⟩

lemma *analz_Decrypt' [dest]:*
  "⟦ Crypt K X ∈ analz H; Key K ∈ analz H ⟧ ⟹ X ∈ analz H"
⟨*proof*⟩

Now cancel the *dest* attribute given to *analz.Decrypt* in its declaration.

declare *analz.Decrypt [rule del]*

Rewrites should not refer to *initState (Friend i)* because that expression is not in normal form.

Added to extend initstate with set of nonces

lemma *parts_image_Nonce [simp]: "parts (Nonce`N) = Nonce`N"*
  ⟨*proof*⟩

lemma *keysFor_parts_initState [simp]: "keysFor (parts (initState C)) = {}"*
⟨*proof*⟩

lemma *keysFor_parts_insert:*
  "⟦ K ∈ keysFor (parts (insert X G)); X ∈ synth (analz H) ⟧
  ⟹ K ∈ keysFor (parts (G ∪ H)) | Key K ∈ parts H"
⟨*proof*⟩

lemma *Crypt_imp_keysFor: "Crypt K X ∈ H ⟹ K ∈ keysFor H"*
⟨*proof*⟩

25.2 Function "knows"

lemma *Spy_knows_bad [intro!]: "A ∈ bad ⟹ Key (pin A) ∈ knows Spy evs"*
⟨*proof*⟩

```
lemma Spy_knows_cloned [intro!]:
     "Card A ∈ cloned ⟹  Key (crdK (Card A)) ∈ knows Spy evs &
                         Key (shrK A) ∈ knows Spy evs &
                         Key (pin A)  ∈ knows Spy evs &
                        (∀ B. Key (pairK(B,A)) ∈ knows Spy evs)"
⟨proof⟩

lemma Spy_knows_cloned1 [intro!]: "C ∈ cloned ⟹ Key (crdK C) ∈ knows Spy
evs"
⟨proof⟩

lemma Spy_knows_cloned2 [intro!]: "⟦ Card A ∈ cloned; Card B ∈ cloned ⟧

   ⟹ Nonce (Pairkey(A,B))∈ knows Spy evs"
⟨proof⟩

lemma Spy_knows_Spy_bad [intro!]: "A∈ bad ⟹ Key (pin A) ∈ knows Spy evs"
⟨proof⟩

lemma Crypt_Spy_analz_bad:
  "⟦ Crypt (pin A) X ∈ analz (knows Spy evs);  A∈bad ⟧
      ⟹ X ∈ analz (knows Spy evs)"
⟨proof⟩

lemma shrK_in_initState [iff]: "Key (shrK A) ∈ initState Server"
⟨proof⟩

lemma shrK_in_used [iff]: "Key (shrK A) ∈ used evs"
⟨proof⟩

lemma crdK_in_initState [iff]: "Key (crdK A) ∈ initState Server"
⟨proof⟩

lemma crdK_in_used [iff]: "Key (crdK A) ∈ used evs"
⟨proof⟩

lemma pin_in_initState [iff]: "Key (pin A) ∈ initState A"
⟨proof⟩

lemma pin_in_used [iff]: "Key (pin A) ∈ used evs"
⟨proof⟩

lemma pairK_in_initState [iff]: "Key (pairK X) ∈ initState Server"
⟨proof⟩

lemma pairK_in_used [iff]: "Key (pairK X) ∈ used evs"
⟨proof⟩
```

lemma *Key_not_used [simp]: "Key K ∉ used evs ⟹ K ∉ range shrK"*
⟨*proof*⟩

lemma *shrK_neq [simp]: "Key K ∉ used evs ⟹ shrK B ≠ K"*
⟨*proof*⟩

lemma *crdK_not_used [simp]: "Key K ∉ used evs ⟹ K ∉ range crdK"*
⟨*proof*⟩

lemma *crdK_neq [simp]: "Key K ∉ used evs ⟹ crdK C ≠ K"*
⟨*proof*⟩

lemma *pin_not_used [simp]: "Key K ∉ used evs ⟹ K ∉ range pin"*
⟨*proof*⟩

lemma *pin_neq [simp]: "Key K ∉ used evs ⟹ pin A ≠ K"*
⟨*proof*⟩

lemma *pairK_not_used [simp]: "Key K ∉ used evs ⟹ K ∉ range pairK"*
⟨*proof*⟩

lemma *pairK_neq [simp]: "Key K ∉ used evs ⟹ pairK(A,B) ≠ K"*
⟨*proof*⟩

declare *shrK_neq [THEN not_sym, simp]*
declare *crdK_neq [THEN not_sym, simp]*
declare *pin_neq [THEN not_sym, simp]*
declare *pairK_neq [THEN not_sym, simp]*

25.3 Fresh nonces

lemma *Nonce_notin_initState [iff]: "Nonce N ∉ parts (initState (Friend i))"*
⟨*proof*⟩

25.4 Supply fresh nonces for possibility theorems.

lemma *Nonce_supply1: "∃N. Nonce N ∉ used evs"*
⟨*proof*⟩

lemma *Nonce_supply2:*
"∃N N'. Nonce N ∉ used evs & Nonce N' ∉ used evs' & N ≠ N'"
⟨*proof*⟩

lemma *Nonce_supply3: "∃N N' N''. Nonce N ∉ used evs & Nonce N' ∉ used evs' &*
Nonce N'' ∉ used evs'' & N ≠ N' & N' ≠ N'' & N ≠ N''"
⟨*proof*⟩

lemma *Nonce_supply: "Nonce (SOME N. Nonce N ∉ used evs) ∉ used evs"*

⟨*proof*⟩

Unlike the corresponding property of nonces, we cannot prove *finite KK* ⟹ ∃*K. K* ∉ *KK* ∧ *Key K* ∉ *used evs*. We have infinitely many agents and there is nothing to stop their long-term keys from exhausting all the natural numbers. Instead, possibility theorems must assume the existence of a few keys.

25.5 Specialized Rewriting for Theorems About *analz* and Image

lemma *subset_Compl_range_shrK: "A ⊆ - (range shrK) ⟹ shrK x ∉ A"*
⟨*proof*⟩

lemma *subset_Compl_range_crdK: "A ⊆ - (range crdK) ⟹ crdK x ∉ A"*
⟨*proof*⟩

lemma *subset_Compl_range_pin: "A ⊆ - (range pin) ⟹ pin x ∉ A"*
⟨*proof*⟩

lemma *subset_Compl_range_pairK: "A ⊆ - (range pairK) ⟹ pairK x ∉ A"*
⟨*proof*⟩
lemma *insert_Key_singleton: "insert (Key K) H = Key ‘ {K} ∪ H"*
⟨*proof*⟩

lemma *insert_Key_image: "insert (Key K) (Key‘KK ∪ C) = Key‘(insert K KK) ∪ C"*
⟨*proof*⟩

lemmas *analz_image_freshK_simps =*
simp_thms mem_simps — these two allow its use with *only:*
disj_comms
image_insert [THEN sym] image_Un [THEN sym] empty_subsetI insert_subset
analz_insert_eq Un_upper2 [THEN analz_mono, THEN [2] rev_subsetD]
insert_Key_singleton subset_Compl_range_shrK subset_Compl_range_crdK
subset_Compl_range_pin subset_Compl_range_pairK
Key_not_used insert_Key_image Un_assoc [THEN sym]

lemma *analz_image_freshK_lemma:*
"(Key K ∈ analz (Key‘nE ∪ H)) ⟶ (K ∈ nE | Key K ∈ analz H) ⟹
(Key K ∈ analz (Key‘nE ∪ H)) = (K ∈ nE | Key K ∈ analz H)"
⟨*proof*⟩

25.6 Tactics for possibility theorems

⟨*ML*⟩

lemma *invKey_shrK_iff [iff]:*
"(Key (invKey K) ∈ X) = (Key K ∈ X)"

⟨*proof*⟩

⟨*ML*⟩

lemma *knows_subset_knows_Cons: "knows A evs ⊆ knows A (e # evs)"*
⟨*proof*⟩

declare *shrK_disj_crdK[THEN not_sym, iff]*
declare *shrK_disj_pin[THEN not_sym, iff]*
declare *pairK_disj_shrK[THEN not_sym, iff]*
declare *pairK_disj_crdK[THEN not_sym, iff]*
declare *pairK_disj_pin[THEN not_sym, iff]*
declare *crdK_disj_pin[THEN not_sym, iff]*

declare *legalUse_def [iff] illegalUse_def [iff]*

end

26 Original Shoup-Rubin protocol

theory *ShoupRubin* **imports** *Smartcard* **begin**

```
axiomatization sesK :: "nat*key => key"
where

  inj_sesK [iff]: "(sesK(m,k) = sesK(m',k')) = (m = m' ∧ k = k')" and

  shrK_disj_sesK [iff]: "shrK A ≠ sesK(m,pk)" and
  crdK_disj_sesK [iff]: "crdK C ≠ sesK(m,pk)" and
  pin_disj_sesK  [iff]: "pin P ≠ sesK(m,pk)" and
  pairK_disj_sesK[iff]:"pairK(A,B) ≠ sesK(m,pk)" and


  Atomic_distrib [iff]: "Atomic`(KEY`K ∪ NONCE`N) =
                 Atomic`(KEY`K) ∪ Atomic`(NONCE`N)" and


  shouprubin_assumes_securemeans [iff]: "evs ∈ sr ⟹ secureM"

definition Unique :: "[event, event list] => bool" (‹Unique _ on _›) where
  "Unique ev on evs ==
     ev ∉ set (tl (dropWhile (% z. z ≠ ev) evs))"


inductive_set sr :: "event list set"
  where

   Nil:  "[]∈ sr"
```

```
| Fake: "⟦ evsF∈ sr;  X∈ synth (analz (knows Spy evsF));
           illegalUse(Card B) ⟧
        ⟹ Says Spy A X #
            Inputs Spy (Card B) X # evsF ∈ sr"


| Forge:
       "⟦ evsFo ∈ sr; Nonce Nb ∈ analz (knows Spy evsFo);
           Key (pairK(A,B)) ∈ knows Spy evsFo ⟧
        ⟹ Notes Spy (Key (sesK(Nb,pairK(A,B)))) # evsFo ∈ sr"


| Reception: "⟦ evsR∈ sr; Says A B X ∈ set evsR ⟧
            ⟹ Gets B X # evsR ∈ sr"


| SR1:  "⟦ evs1∈ sr; A ≠ Server⟧
        ⟹ Says A Server ⦃Agent A, Agent B⦄
              # evs1 ∈ sr"

| SR2:  "⟦ evs2∈ sr;
           Gets Server ⦃Agent A, Agent B⦄ ∈ set evs2 ⟧
        ⟹ Says Server A ⦃Nonce (Pairkey(A,B)),
                         Crypt (shrK A) ⦃Nonce (Pairkey(A,B)), Agent B⦄
               ⦄
              # evs2 ∈ sr"


| SR3:  "⟦ evs3∈ sr; legalUse(Card A);
           Says A Server ⦃Agent A, Agent B⦄ ∈ set evs3;
           Gets A ⦃Nonce Pk, Certificate⦄ ∈ set evs3 ⟧
        ⟹ Inputs A (Card A) (Agent A)
              # evs3 ∈ sr"


| SR4:  "⟦ evs4∈ sr;  A ≠ Server;
           Nonce Na ∉ used evs4; legalUse(Card A);
           Inputs A (Card A) (Agent A) ∈ set evs4 ⟧
     ⟹ Outpts (Card A) A ⦃Nonce Na, Crypt (crdK (Card A)) (Nonce Na)⦄
            # evs4 ∈ sr"


| SR4Fake: "⟦ evs4F∈ sr; Nonce Na ∉ used evs4F;
              illegalUse(Card A);
              Inputs Spy (Card A) (Agent A) ∈ set evs4F ⟧
```

```
    ⟹ Outpts (Card A) Spy {|Nonce Na, Crypt (crdK (Card A)) (Nonce Na)|}
         # evs4F ∈ sr"

| SR5:  "⟦ evs5∈ sr;
           Outpts (Card A) A {|Nonce Na, Certificate|} ∈ set evs5;
           ∀ p q. Certificate ≠ {|p, q|} ⟧
        ⟹ Says A B {|Agent A, Nonce Na|} # evs5 ∈ sr"

| SR6:  "⟦ evs6∈ sr; legalUse(Card B);
           Gets B {|Agent A, Nonce Na|} ∈ set evs6 ⟧
        ⟹ Inputs B (Card B) {|Agent A, Nonce Na|}
             # evs6 ∈ sr"

| SR7:  "⟦ evs7∈ sr;
           Nonce Nb ∉ used evs7; legalUse(Card B); B ≠ Server;
           K = sesK(Nb,pairK(A,B));
           Key K ∉ used evs7;
           Inputs B (Card B) {|Agent A, Nonce Na|} ∈ set evs7⟧
  ⟹ Outpts (Card B) B {|Nonce Nb, Key K,
                          Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|},
                          Crypt (pairK(A,B)) (Nonce Nb)|}
             # evs7 ∈ sr"

| SR7Fake:  "⟦ evs7F∈ sr; Nonce Nb ∉ used evs7F;
               illegalUse(Card B);
               K = sesK(Nb,pairK(A,B));
               Key K ∉ used evs7F;
               Inputs Spy (Card B) {|Agent A, Nonce Na|} ∈ set evs7F ⟧
        ⟹ Outpts (Card B) Spy {|Nonce Nb, Key K,
                          Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|},
                          Crypt (pairK(A,B)) (Nonce Nb)|}
             # evs7F ∈ sr"

| SR8:  "⟦ evs8∈ sr;
           Inputs B (Card B) {|Agent A, Nonce Na|} ∈ set evs8;
           Outpts (Card B) B {|Nonce Nb, Key K,
                                Cert1, Cert2|} ∈ set evs8 ⟧
```

```
         ⟹ Says B A {|Nonce Nb, Cert1|} # evs8 ∈ sr"



  | SR9:  "⟦ evs9∈ sr; legalUse(Card A);
            Gets A {|Nonce Pk, Cert1|} ∈ set evs9;
            Outpts (Card A) A {|Nonce Na, Cert2|} ∈ set evs9;
            Gets A {|Nonce Nb, Cert3|} ∈ set evs9;
            ∀ p q. Cert2 ≠ {|p, q|} ⟧
         ⟹ Inputs A (Card A)
               {|Agent B, Nonce Na, Nonce Nb, Nonce Pk,
                Cert1, Cert3, Cert2|}
              # evs9 ∈ sr"


  | SR10: "⟦ evs10∈ sr; legalUse(Card A); A ≠ Server;
            K = sesK(Nb,pairK(A,B));
            Inputs A (Card A) {|Agent B, Nonce Na, Nonce Nb,
                                Nonce (Pairkey(A,B)),
                                Crypt (shrK A) {|Nonce (Pairkey(A,B)),
                                                  Agent B|},
                                Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|},

                                Crypt (crdK (Card A)) (Nonce Na)|}
             ∈ set evs10 ⟧
         ⟹ Outpts (Card A) A {|Key K, Crypt (pairK(A,B)) (Nonce Nb)|}
               # evs10 ∈ sr"


  | SR10Fake: "⟦ evs10F∈ sr;
                illegalUse(Card A);
                K = sesK(Nb,pairK(A,B));
                Inputs Spy (Card A) {|Agent B, Nonce Na, Nonce Nb,
                                      Nonce (Pairkey(A,B)),
                                      Crypt (shrK A) {|Nonce (Pairkey(A,B)),

                                                       Agent B|},
                                      Crypt (pairK(A,B)) {|Nonce Na, Nonce
Nb|},
                                      Crypt (crdK (Card A)) (Nonce Na)|}
                 ∈ set evs10F ⟧
         ⟹ Outpts (Card A) Spy {|Key K, Crypt (pairK(A,B)) (Nonce Nb)|}
               # evs10F ∈ sr"




  | SR11: "⟦ evs11∈ sr;
```

```
          Says A Server {|Agent A, Agent B|} ∈ set evs11;
          Outpts (Card A) A {|Key K, Certificate|} ∈ set evs11 ⟧
       ⟹ Says A B (Certificate)
              # evs11 ∈ sr"
```

```
  | Oops1:
     "⟦ evsO1 ∈ sr;
         Outpts (Card B) B {|Nonce Nb, Key K, Certificate,
                             Crypt (pairK(A,B)) (Nonce Nb)|} ∈ set evsO1 ⟧
     ⟹ Notes Spy {|Key K, Nonce Nb, Agent A, Agent B|} # evsO1 ∈ sr"

  | Oops2:
     "⟦ evsO2 ∈ sr;
         Outpts (Card A) A {|Key K, Crypt (pairK(A,B)) (Nonce Nb)|}
           ∈ set evsO2 ⟧
    ⟹ Notes Spy {|Key K, Nonce Nb, Agent A, Agent B|} # evsO2 ∈ sr"
```

declare *Fake_parts_insert_in_Un [dest]*
declare *analz_into_parts [dest]*

lemma *Gets_imp_Says:*
"⟦ Gets B X ∈ set evs; evs ∈ sr ⟧ ⟹ ∃ A. Says A B X ∈ set evs"
⟨*proof*⟩

lemma *Gets_imp_knows_Spy:*
"⟦ Gets B X ∈ set evs; evs ∈ sr ⟧ ⟹ X ∈ knows Spy evs"
⟨*proof*⟩

lemma *Gets_imp_knows_Spy_parts_Snd:*
"⟦ Gets B {|X, Y|} ∈ set evs; evs ∈ sr ⟧ ⟹ Y ∈ parts (knows Spy evs)"
⟨*proof*⟩

lemma *Gets_imp_knows_Spy_analz_Snd:*
"⟦ Gets B {|X, Y|} ∈ set evs; evs ∈ sr ⟧ ⟹ Y ∈ analz (knows Spy evs)"
⟨*proof*⟩

lemma *Inputs_imp_knows_Spy_secureM_sr:*
"⟦ Inputs Spy C X ∈ set evs; evs ∈ sr ⟧ ⟹ X ∈ knows Spy evs"
⟨*proof*⟩

lemma *knows_Spy_Inputs_secureM_sr_Spy:*
"evs ∈sr ⟹ knows Spy (Inputs Spy C X # evs) = insert X (knows Spy evs)"
⟨*proof*⟩

lemma *knows_Spy_Inputs_secureM_sr:*
"⟦ A ≠ Spy; evs ∈sr ⟧ ⟹ knows Spy (Inputs A C X # evs) = knows Spy evs"
⟨*proof*⟩

lemma *knows_Spy_Outpts_secureM_sr_Spy:*
"evs ∈sr ⟹ knows Spy (Outpts C Spy X # evs) = insert X (knows Spy evs)"
⟨*proof*⟩

lemma *knows_Spy_Outpts_secureM_sr:*
"⟦ A ≠ Spy; evs ∈sr ⟧ ⟹ knows Spy (Outpts C A X # evs) = knows Spy evs"
⟨*proof*⟩

lemma *Inputs_A_Card_3:*
"⟦ Inputs A C (Agent A) ∈ set evs; A ≠ Spy; evs ∈ sr ⟧
⟹ legalUse(C) ∧ C = (Card A) ∧
(∃ Pk Certificate. Gets A ⦃Pk, Certificate⦄ ∈ set evs)"
⟨*proof*⟩

lemma *Inputs_B_Card_6:*
"⟦ Inputs B C ⦃Agent A, Nonce Na⦄ ∈ set evs; B ≠ Spy; evs ∈ sr ⟧
⟹ legalUse(C) ∧ C = (Card B) ∧ Gets B ⦃Agent A, Nonce Na⦄ ∈ set evs"
⟨*proof*⟩

lemma *Inputs_A_Card_9:*
"⟦ Inputs A C ⦃Agent B, Nonce Na, Nonce Nb, Nonce Pk,
Cert1, Cert2, Cert3⦄ ∈ set evs;

A ≠ Spy; evs ∈ sr ⟧
⟹ legalUse(C) ∧ C = (Card A) ∧
Gets A ⦃Nonce Pk, Cert1⦄ ∈ set evs ∧
Outpts (Card A) A ⦃Nonce Na, Cert3⦄ ∈ set evs ∧
Gets A ⦃Nonce Nb, Cert2⦄ ∈ set evs"

⟨*proof*⟩

lemma *Outpts_A_Card_4:*

```
"⟦ Outpts C A ⦃Nonce Na, (Crypt (crdK (Card A)) (Nonce Na))⦄ ∈ set evs;

    evs ∈ sr ⟧
⟹ legalUse(C) ∧ C = (Card A) ∧
    Inputs A (Card A) (Agent A) ∈ set evs"
```

⟨*proof*⟩

lemma *Outpts_B_Card_7:*

```
 "⟦ Outpts C B ⦃Nonce Nb, Key K,
               Crypt (pairK(A,B)) ⦃Nonce Na, Nonce Nb⦄,
               Cert2⦄ ∈ set evs;
    evs ∈ sr ⟧
⟹ legalUse(C) ∧ C = (Card B) ∧
    Inputs B (Card B) ⦃Agent A, Nonce Na⦄ ∈ set evs"
```

⟨*proof*⟩

lemma *Outpts_A_Card_10:*

```
"⟦ Outpts C A ⦃Key K, (Crypt (pairK(A,B)) (Nonce Nb))⦄ ∈ set evs;
    evs ∈ sr ⟧
⟹ legalUse(C) ∧ C = (Card A) ∧
    (∃ Na Ver1 Ver2 Ver3.
  Inputs A (Card A) ⦃Agent B, Nonce Na, Nonce Nb, Nonce (Pairkey(A,B)),

                           Ver1, Ver2, Ver3⦄ ∈ set evs)"
```

⟨*proof*⟩

lemma *Outpts_A_Card_10_imp_Inputs:*

```
"⟦ Outpts (Card A) A ⦃Key K, Certificate⦄ ∈ set evs; evs ∈ sr ⟧
⟹ (∃ B Na Nb Ver1 Ver2 Ver3.
  Inputs A (Card A) ⦃Agent B, Nonce Na, Nonce Nb, Nonce (Pairkey(A,B)),

                           Ver1, Ver2, Ver3⦄ ∈ set evs)"
```

⟨*proof*⟩

lemma *Outpts_honest_A_Card_4:*

```
"⟦ Outpts C A ⦃Nonce Na, Crypt K X⦄ ∈set evs;
    A ≠ Spy;  evs ∈ sr ⟧
⟹ legalUse(C) ∧ C = (Card A) ∧
    Inputs A (Card A) (Agent A) ∈ set evs"
```

⟨*proof*⟩

lemma *Outpts_honest_B_Card_7:*
```
"⟦ Outpts C B {|Nonce Nb, Key K, Cert1, Cert2|} ∈ set evs;
    B ≠ Spy; evs ∈ sr ⟧
 ⟹ legalUse(C) ∧ C = (Card B) ∧
    (∃ A Na. Inputs B (Card B) {|Agent A, Nonce Na|} ∈ set evs)"
```
⟨*proof*⟩

lemma *Outpts_honest_A_Card_10:*
```
"⟦ Outpts C A {|Key K, Certificate|} ∈ set evs;
    A ≠ Spy; evs ∈ sr ⟧
 ⟹ legalUse (C) ∧ C = (Card A) ∧
    (∃ B Na Nb Pk Ver1 Ver2 Ver3.
     Inputs A (Card A) {|Agent B, Nonce Na, Nonce Nb, Pk,
                         Ver1, Ver2, Ver3|} ∈ set evs)"
```
⟨*proof*⟩

lemma *Outpts_which_Card_4:*
```
"⟦ Outpts (Card A) A {|Nonce Na, Crypt K X|} ∈ set evs; evs ∈ sr ⟧
 ⟹ Inputs A (Card A) (Agent A) ∈ set evs"
```
⟨*proof*⟩

lemma *Outpts_which_Card_7:*
```
"⟦ Outpts (Card B) B {|Nonce Nb, Key K, Cert1, Cert2|} ∈ set evs;
    evs ∈ sr ⟧
 ⟹ ∃ A Na. Inputs B (Card B) {|Agent A, Nonce Na|} ∈ set evs"
```
⟨*proof*⟩

lemma *Outpts_which_Card_10:*
```
"⟦ Outpts (Card A) A {|Key (sesK(Nb,pairK(A,B))),
                       Crypt (pairK(A,B)) (Nonce Nb) |} ∈ set evs;
    evs ∈ sr ⟧
 ⟹ ∃ Na. Inputs A (Card A) {|Agent B, Nonce Na, Nonce Nb, Nonce (Pairkey(A,B)),

                      Crypt (shrK A) {|Nonce (Pairkey(A,B)), Agent B|},

                      Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|},
                      Crypt (crdK (Card A)) (Nonce Na) |} ∈ set evs"
```
⟨*proof*⟩

lemma *Outpts_A_Card_form_4:*
```
"⟦ Outpts (Card A) A {|Nonce Na, Certificate|} ∈ set evs;
    ∀ p q. Certificate ≠ {|p, q|}; evs ∈ sr ⟧
```

```
     ⟹ Certificate = (Crypt (crdK (Card A)) (Nonce Na))"
```
⟨*proof*⟩

lemma *Outpts_B_Card_form_7:*
```
   "⟦ Outpts (Card B) B {|Nonce Nb, Key K, Cert1, Cert2|} ∈ set evs;
         evs ∈ sr ⟧
      ⟹ ∃ A Na.
          K = sesK(Nb,pairK(A,B)) ∧
          Cert1 = (Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|}) ∧
          Cert2 = (Crypt (pairK(A,B)) (Nonce Nb))"
```
⟨*proof*⟩

lemma *Outpts_A_Card_form_10:*
```
   "⟦ Outpts (Card A) A {|Key K, Certificate|} ∈ set evs; evs ∈ sr ⟧
      ⟹ ∃ B Nb.
          K = sesK(Nb,pairK(A,B)) ∧
          Certificate = (Crypt (pairK(A,B)) (Nonce Nb))"
```
⟨*proof*⟩

lemma *Outpts_A_Card_form_bis:*
```
  "⟦ Outpts (Card A') A' {|Key (sesK(Nb,pairK(A,B))), Certificate|} ∈ set evs;

         evs ∈ sr ⟧
      ⟹ A' = A ∧
          Certificate = (Crypt (pairK(A,B)) (Nonce Nb))"
```
⟨*proof*⟩

lemma *Inputs_A_Card_form_9:*
```
     "⟦ Inputs A (Card A) {|Agent B, Nonce Na, Nonce Nb, Nonce Pk,
                              Cert1, Cert2, Cert3|} ∈ set evs;
         evs ∈ sr ⟧
  ⟹     Cert3 = Crypt (crdK (Card A)) (Nonce Na)"
```
⟨*proof*⟩

lemma *Inputs_Card_legalUse:*
```
  "⟦ Inputs A (Card A) X ∈ set evs; evs ∈ sr ⟧ ⟹ legalUse(Card A)"
```
⟨*proof*⟩

lemma *Outpts_Card_legalUse:*
```
  "⟦ Outpts (Card A) A X ∈ set evs; evs ∈ sr ⟧ ⟹ legalUse(Card A)"
```
⟨*proof*⟩

lemma *Inputs_Card:* `"⟦ Inputs A C X ∈ set evs; A ≠ Spy; evs ∈ sr ⟧`
```
      ⟹ C = (Card A) ∧ legalUse(C)"
```

⟨*proof*⟩

lemma *Outpts_Card: "⟦ Outpts C A X ∈ set evs; A ≠ Spy; evs ∈ sr ⟧
⟹ C = (Card A) ∧ legalUse(C)"*
⟨*proof*⟩

lemma *Inputs_Outpts_Card:
"⟦ Inputs A C X ∈ set evs ∨ Outpts C A Y ∈ set evs;
A ≠ Spy; evs ∈ sr ⟧
⟹ C = (Card A) ∧ legalUse(Card A)"*
⟨*proof*⟩

lemma *Inputs_Card_Spy:
"⟦ Inputs Spy C X ∈ set evs ∨ Outpts C Spy X ∈ set evs; evs ∈ sr ⟧
⟹ C = (Card Spy) ∧ legalUse(Card Spy) ∨
(∃ A. C = (Card A) ∧ illegalUse(Card A))"*
⟨*proof*⟩

lemma *Outpts_A_Card_unique_nonce:
"⟦ Outpts (Card A) A {|Nonce Na, Crypt (crdK (Card A)) (Nonce Na)|}
∈ set evs;
Outpts (Card A') A' {|Nonce Na, Crypt (crdK (Card A')) (Nonce Na)|}
∈ set evs;
evs ∈ sr ⟧ ⟹ A=A'"*
⟨*proof*⟩

lemma *Outpts_B_Card_unique_nonce:
"⟦ Outpts (Card B) B {|Nonce Nb, Key SK, Cert1, Cert2|} ∈ set evs;
Outpts (Card B') B' {|Nonce Nb, Key SK', Cert1', Cert2'|} ∈ set evs;
evs ∈ sr ⟧ ⟹ B=B' ∧ SK=SK' ∧ Cert1=Cert1' ∧ Cert2=Cert2'"*
⟨*proof*⟩

lemma *Outpts_B_Card_unique_key:
"⟦ Outpts (Card B) B {|Nonce Nb, Key SK, Cert1, Cert2|} ∈ set evs;*

Outpts (Card B') B' ⦃Nonce Nb', Key SK, Cert1', Cert2'⦄ ∈ set evs;

evs ∈ sr ⟧ ⟹ B=B' ∧ Nb=Nb' ∧ Cert1=Cert1' ∧ Cert2=Cert2'"
⟨*proof*⟩

lemma *Outpts_A_Card_unique_key: "⟦ Outpts (Card A) A ⦃Key K, V⦄ ∈ set evs;*

Outpts (Card A') A' ⦃Key K, V'⦄ ∈ set evs;
evs ∈ sr ⟧ ⟹ A=A' ∧ V=V'"
⟨*proof*⟩

lemma *Outpts_A_Card_Unique:*
"⟦ Outpts (Card A) A ⦃Nonce Na, rest⦄ ∈ set evs; evs ∈ sr ⟧
⟹ Unique (Outpts (Card A) A ⦃Nonce Na, rest⦄) on evs"
⟨*proof*⟩

lemma *Spy_knows_Na:*
"⟦ Says A B ⦃Agent A, Nonce Na⦄ ∈ set evs; evs ∈ sr ⟧
⟹ Nonce Na ∈ analz (knows Spy evs)"
⟨*proof*⟩

lemma *Spy_knows_Nb:*
"⟦ Says B A ⦃Nonce Nb, Certificate⦄ ∈ set evs; evs ∈ sr ⟧
⟹ Nonce Nb ∈ analz (knows Spy evs)"
⟨*proof*⟩

lemma *Pairkey_Gets_analz_knows_Spy:*
"⟦ Gets A ⦃Nonce (Pairkey(A,B)), Certificate⦄ ∈ set evs; evs ∈ sr ⟧

⟹ Nonce (Pairkey(A,B)) ∈ analz (knows Spy evs)"
⟨*proof*⟩

lemma *Pairkey_Inputs_imp_Gets:*
"⟦ Inputs A (Card A)
⦃Agent B, Nonce Na, Nonce Nb, Nonce (Pairkey(A,B)),
Cert1, Cert3, Cert2⦄ ∈ set evs;
A ≠ Spy; evs ∈ sr ⟧
⟹ Gets A ⦃Nonce (Pairkey(A,B)), Cert1⦄ ∈ set evs"
⟨*proof*⟩

lemma *Pairkey_Inputs_analz_knows_Spy:*
```
    "⟦ Inputs A (Card A)
         ⦃Agent B, Nonce Na, Nonce Nb, Nonce (Pairkey(A,B)),
           Cert1, Cert3, Cert2⦄ ∈ set evs;
       evs ∈ sr ⟧
    ⟹ Nonce (Pairkey(A,B)) ∈ analz (knows Spy evs)"
```
⟨*proof*⟩

declare *shrK_disj_sesK [THEN not_sym, iff]*
declare *pin_disj_sesK [THEN not_sym, iff]*
declare *crdK_disj_sesK [THEN not_sym, iff]*
declare *pairK_disj_sesK [THEN not_sym, iff]*

⟨*ML*⟩

lemma *Spy_parts_keys [simp]: "evs ∈ sr ⟹*
```
  (Key (shrK P) ∈ parts (knows Spy evs)) = (Card P ∈ cloned) ∧
  (Key (pin P) ∈ parts (knows Spy evs)) = (P ∈ bad ∨ Card P ∈ cloned) ∧

  (Key (crdK C) ∈ parts (knows Spy evs)) = (C ∈ cloned) ∧
  (Key (pairK(A,B)) ∈ parts (knows Spy evs)) = (Card B ∈ cloned)"
```
⟨*proof*⟩

lemma *Spy_analz_shrK[simp]: "evs ∈ sr ⟹*
```
  (Key (shrK P) ∈ analz (knows Spy evs)) = (Card P ∈ cloned)"
```
⟨*proof*⟩

lemma *Spy_analz_crdK[simp]: "evs ∈ sr ⟹*
```
  (Key (crdK C) ∈ analz (knows Spy evs)) = (C ∈ cloned)"
```
⟨*proof*⟩

lemma *Spy_analz_pairK[simp]: "evs ∈ sr ⟹*
```
  (Key (pairK(A,B)) ∈ analz (knows Spy evs)) = (Card B ∈ cloned)"
```
⟨*proof*⟩

lemma *analz_image_Key_Un_Nonce:*
```
  "analz (Key ‘ K ∪ Nonce ‘ N) = Key ‘ K ∪ Nonce ‘ N"
```
⟨*proof*⟩

⟨*ML*⟩

lemma *analz_image_freshK [rule_format]:*
```
    "evs ∈ sr ⟹      ∀ K KK.
          (Key K ∈ analz (Key‘KK ∪ (knows Spy evs))) =
          (K ∈ KK ∨ Key K ∈ analz (knows Spy evs))"
```
⟨*proof*⟩

lemma *analz_insert_freshK: "evs ∈ sr ⟹*
```
        Key K ∈ analz (insert (Key K’) (knows Spy evs)) =
        (K = K’ ∨ Key K ∈ analz (knows Spy evs))"
```
⟨*proof*⟩

lemma *Na_Nb_certificate_authentic:*
```
    "⟦ Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|} ∈ parts (knows Spy evs);
        ¬illegalUse(Card B);
        evs ∈ sr ⟧
    ⟹ Outpts (Card B) B {|Nonce Nb, Key (sesK(Nb,pairK(A,B))),
                Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|},
                Crypt (pairK(A,B)) (Nonce Nb)|} ∈ set evs"
```
⟨*proof*⟩

lemma *Nb_certificate_authentic:*
```
     "⟦ Crypt (pairK(A,B)) (Nonce Nb) ∈ parts (knows Spy evs);
        B ≠ Spy; ¬illegalUse(Card A); ¬illegalUse(Card B);
        evs ∈ sr ⟧
    ⟹ Outpts (Card A) A {|Key (sesK(Nb,pairK(A,B))),
                          Crypt (pairK(A,B)) (Nonce Nb)|} ∈ set evs"
```
⟨*proof*⟩

lemma *Outpts_A_Card_imp_pairK_parts:*
```
    "⟦ Outpts (Card A) A
        {|Key K, Crypt (pairK(A,B)) (Nonce Nb)|} ∈ set evs;
```

```
        evs ∈ sr ⟧
     ⟹ ∃ Na. Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|} ∈ parts (knows Spy
evs)"
```
⟨proof⟩

lemma *Nb_certificate_authentic_bis:*
```
     "⟦ Crypt (pairK(A,B)) (Nonce Nb) ∈ parts (knows Spy evs);
        B ≠ Spy; ¬illegalUse(Card B);
        evs ∈ sr ⟧
     ⟹ ∃ Na. Outpts (Card B) B {|Nonce Nb, Key (sesK(Nb,pairK(A,B))),
                  Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|},
                  Crypt (pairK(A,B)) (Nonce Nb)|} ∈ set evs"
```
⟨proof⟩

lemma *Pairkey_certificate_authentic:*
```
    "⟦ Crypt (shrK A) {|Nonce Pk, Agent B|} ∈ parts (knows Spy evs);
        Card A ∉ cloned; evs ∈ sr ⟧
     ⟹ Pk = Pairkey(A,B) ∧
        Says Server A {|Nonce Pk,
                       Crypt (shrK A) {|Nonce Pk, Agent B|}|}
          ∈ set evs"
```
⟨proof⟩

lemma *sesK_authentic:*
```
     "⟦ Key (sesK(Nb,pairK(A,B))) ∈ parts (knows Spy evs);
        A ≠ Spy; B ≠ Spy; ¬illegalUse(Card A); ¬illegalUse(Card B);
        evs ∈ sr ⟧
      ⟹ Notes Spy {|Key (sesK(Nb,pairK(A,B))), Nonce Nb, Agent A, Agent B|}

          ∈ set evs"
```
⟨proof⟩

lemma *Confidentiality:*
```
     "⟦ Notes Spy {|Key (sesK(Nb,pairK(A,B))), Nonce Nb, Agent A, Agent B|}

          ∉ set evs;
       A ≠ Spy; B ≠ Spy; ¬illegalUse(Card A); ¬illegalUse(Card B);
       evs ∈ sr ⟧
      ⟹ Key (sesK(Nb,pairK(A,B))) ∉ analz (knows Spy evs)"
```
⟨proof⟩

```
lemma Confidentiality_B:
  "⟦ Outpts (Card B) B ⦃Nonce Nb, Key K, Certificate,
                    Crypt (pairK(A,B)) (Nonce Nb)⦄ ∈ set evs;
     Notes Spy ⦃Key K, Nonce Nb, Agent A, Agent B⦄ ∉ set evs;
     A ≠ Spy; B ≠ Spy; ¬illegalUse(Card A); Card B ∉ cloned;
     evs ∈ sr ⟧
   ⟹ Key K ∉ analz (knows Spy evs)"
⟨proof⟩
```

```
lemma A_authenticates_B:
  "⟦ Outpts (Card A) A ⦃Key K, Crypt (pairK(A,B)) (Nonce Nb)⦄ ∈ set evs;

     ¬illegalUse(Card B);
     evs ∈ sr ⟧
  ⟹ ∃ Na.
       Outpts (Card B) B ⦃Nonce Nb, Key K,
          Crypt (pairK(A,B)) ⦃Nonce Na, Nonce Nb⦄,
          Crypt (pairK(A,B)) (Nonce Nb)⦄ ∈ set evs"
⟨proof⟩
```

```
lemma A_authenticates_B_Gets:
  "⟦ Gets A ⦃Nonce Nb, Crypt (pairK(A,B)) ⦃Nonce Na, Nonce Nb⦄⦄
       ∈ set evs;
     ¬illegalUse(Card B);
     evs ∈ sr ⟧
  ⟹ Outpts (Card B) B ⦃Nonce Nb, Key (sesK(Nb, pairK (A, B))),
                    Crypt (pairK(A,B)) ⦃Nonce Na, Nonce Nb⦄,
                    Crypt (pairK(A,B)) (Nonce Nb)⦄ ∈ set evs"
⟨proof⟩
```

```
lemma B_authenticates_A:
  "⟦ Gets B (Crypt (pairK(A,B)) (Nonce Nb)) ∈ set evs;
     B ≠ Spy; ¬illegalUse(Card A); ¬illegalUse(Card B);
     evs ∈ sr ⟧
   ⟹ Outpts (Card A) A
    ⦃Key (sesK(Nb,pairK(A,B))), Crypt (pairK(A,B)) (Nonce Nb)⦄ ∈ set evs"
⟨proof⟩
```

```
lemma Confidentiality_A: "⟦ Outpts (Card A) A
```

```
          {|Key K, Crypt (pairK(A,B)) (Nonce Nb)|} ∈ set evs;
        Notes Spy {|Key K, Nonce Nb, Agent A, Agent B|} ∉ set evs;
        A ≠ Spy; B ≠ Spy; ¬illegalUse(Card A); ¬illegalUse(Card B);
        evs ∈ sr ⟧
     ⟹ Key K ∉ analz (knows Spy evs)"
```

⟨proof⟩

lemma *Outpts_imp_knows_agents_secureM_sr:*

```
   "⟦ Outpts (Card A) A X ∈ set evs; evs ∈ sr ⟧ ⟹ X ∈ knows A evs"
```

⟨proof⟩

lemma *A_keydist_to_B:*

```
     "⟦ Outpts (Card A) A
          {|Key K, Crypt (pairK(A,B)) (Nonce Nb)|} ∈ set evs;
        ¬illegalUse(Card B);
        evs ∈ sr ⟧
     ⟹ Key K ∈ analz (knows B evs)"
```

⟨proof⟩

lemma *B_keydist_to_A:*

```
     "⟦ Outpts (Card B) B {|Nonce Nb, Key K, Certificate,
                             (Crypt (pairK(A,B)) (Nonce Nb))|} ∈ set evs;
        Gets B (Crypt (pairK(A,B)) (Nonce Nb)) ∈ set evs;
        B ≠ Spy; ¬illegalUse(Card A); ¬illegalUse(Card B);
        evs ∈ sr ⟧
     ⟹ Key K ∈ analz (knows A evs)"
```

⟨proof⟩

lemma *Nb_certificate_authentic_B:*

```
     "⟦ Gets B (Crypt (pairK(A,B)) (Nonce Nb)) ∈ set evs;
       B ≠ Spy; ¬illegalUse(Card B);
       evs ∈ sr ⟧
    ⟹ ∃ Na.
          Outpts (Card B) B {|Nonce Nb, Key (sesK(Nb,pairK(A,B))),
              Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|},
```

```
                 Crypt (pairK(A,B)) (Nonce Nb)⦄ ∈ set evs"
```
⟨proof⟩

lemma *Pairkey_certificate_authentic_A_Card:*
```
     "⟦ Inputs A (Card A)
            ⦃Agent B, Nonce Na, Nonce Nb, Nonce Pk,
              Crypt (shrK A) ⦃Nonce Pk, Agent B⦄,
              Cert2, Cert3⦄ ∈ set evs;
        A ≠ Spy; Card A ∉ cloned; evs ∈ sr ⟧
     ⟹ Pk = Pairkey(A,B) ∧
        Says Server A ⦃Nonce (Pairkey(A,B)),
                 Crypt (shrK A) ⦃Nonce (Pairkey(A,B)), Agent B⦄⦄
          ∈ set evs "
```
⟨proof⟩

lemma *Na_Nb_certificate_authentic_A_Card:*
```
      "⟦ Inputs A (Card A)
            ⦃Agent B, Nonce Na, Nonce Nb, Nonce Pk,
              Cert1,
              Crypt (pairK(A,B)) ⦃Nonce Na, Nonce Nb⦄, Cert3⦄ ∈ set evs;

      A ≠ Spy; ¬illegalUse(Card B); evs ∈ sr ⟧
     ⟹ Outpts (Card B) B ⦃Nonce Nb, Key (sesK(Nb, pairK (A, B))),
                             Crypt (pairK(A,B)) ⦃Nonce Na, Nonce Nb⦄,
                             Crypt (pairK(A,B)) (Nonce Nb)⦄
          ∈ set evs "
```
⟨proof⟩

lemma *Na_authentic_A_Card:*
```
     "⟦ Inputs A (Card A)
            ⦃Agent B, Nonce Na, Nonce Nb, Nonce Pk,
               Cert1, Cert2, Cert3⦄ ∈ set evs;
        A ≠ Spy; evs ∈ sr ⟧
     ⟹ Outpts (Card A) A ⦃Nonce Na, Cert3⦄
          ∈ set evs"
```
⟨proof⟩

lemma *Inputs_A_Card_9_authentic:*
```
  "⟦ Inputs A (Card A)
            ⦃Agent B, Nonce Na, Nonce Nb, Nonce Pk,
              Crypt (shrK A) ⦃Nonce Pk, Agent B⦄,
              Crypt (pairK(A,B)) ⦃Nonce Na, Nonce Nb⦄, Cert3⦄ ∈ set evs;
```

```
    A ≠ Spy; Card A ∉ cloned;¬illegalUse(Card B); evs ∈ sr ⟧
    ⟹  Says Server A ⦃Nonce Pk, Crypt (shrK A) ⦃Nonce Pk, Agent B⦄⦄
           ∈ set evs  ∧
         Outpts (Card B) B ⦃Nonce Nb, Key (sesK(Nb, pairK (A, B))),
                             Crypt (pairK(A,B)) ⦃Nonce Na, Nonce Nb⦄,
                             Crypt (pairK(A,B)) (Nonce Nb)⦄
           ∈ set evs  ∧
         Outpts (Card A) A ⦃Nonce Na, Cert3⦄
           ∈ set evs"
```
⟨*proof*⟩

lemma *SR4_imp:*

```
     "⟦ Outpts (Card A) A ⦃Nonce Na, Crypt (crdK (Card A)) (Nonce Na)⦄
           ∈ set evs;
         A ≠ Spy; evs ∈ sr ⟧
     ⟹ ∃ Pk V. Gets A ⦃Pk, V⦄ ∈ set evs"
```
⟨*proof*⟩

lemma *SR7_imp:*

```
     "⟦ Outpts (Card B) B ⦃Nonce Nb, Key K,
                      Crypt (pairK(A,B)) ⦃Nonce Na, Nonce Nb⦄,
                      Cert2⦄ ∈ set evs;
         B ≠ Spy; evs ∈ sr ⟧
     ⟹ Gets B ⦃Agent A, Nonce Na⦄ ∈ set evs"
```
⟨*proof*⟩

lemma *SR10_imp:*

```
     "⟦ Outpts (Card A) A ⦃Key K, Crypt (pairK(A,B)) (Nonce Nb)⦄
           ∈ set evs;
         A ≠ Spy; evs ∈ sr ⟧
     ⟹ ∃ Cert1 Cert2.
                 Gets A ⦃Nonce (Pairkey (A, B)), Cert1⦄ ∈ set evs ∧
                 Gets A ⦃Nonce Nb, Cert2⦄ ∈ set evs"
```
⟨*proof*⟩

lemma *Outpts_Server_not_evs: "evs ∈ sr ⟹ Outpts (Card Server) P X ∉ set evs"*
⟨*proof*⟩

step2_integrity also is a reliability theorem

lemma *Says_Server_message_form:*
```
"⟦ Says Server A {|Pk, Certificate|} ∈ set evs;
    evs ∈ sr ⟧
 ⟹ ∃ B. Pk = Nonce (Pairkey(A,B)) ∧
    Certificate = Crypt (shrK A) {|Nonce (Pairkey(A,B)), Agent B|}"
```
⟨*proof*⟩

step4integrity is *Outpts_A_Card_form_4*

step7integrity is *Outpts_B_Card_form_7*

lemma *step8_integrity:*
```
"⟦ Says B A {|Nonce Nb, Certificate|} ∈ set evs;
    B ≠ Server; B ≠ Spy; evs ∈ sr ⟧
 ⟹ ∃ Cert2 K.
     Outpts (Card B) B {|Nonce Nb, Key K, Certificate, Cert2|} ∈ set evs"
```
⟨*proof*⟩

step9integrity is *Inputs_A_Card_form_9*

step10integrity is *Outpts_A_Card_form_10*.

lemma *step11_integrity:*
```
"⟦ Says A B (Certificate) ∈ set evs;
    ∀ p q. Certificate ≠ {|p, q|};
    A ≠ Spy; evs ∈ sr ⟧
 ⟹ ∃ K.
       Outpts (Card A) A {|Key K, Certificate|} ∈ set evs"
```
⟨*proof*⟩

end

27 Bella's modification of the Shoup-Rubin protocol

theory *ShoupRubinBella* **imports** *Smartcard* **begin**

The modifications are that message 7 now mentions A, while message 10 now mentions Nb and B. The lack of explicitness of the original version was discovered by investigating adherence to the principle of Goal Availability. Only the updated version makes the goals of confidentiality, authentication and key distribution available to both peers.

axiomatization *sesK :: "nat*key => key"*
where

inj_sesK [iff]: "(sesK(m,k) = sesK(m',k')) = (m = m' ∧ k = k')" **and**

```
  shrK_disj_sesK [iff]: "shrK A ≠ sesK(m,pk)" and
  crdK_disj_sesK [iff]: "crdK C ≠ sesK(m,pk)" and
  pin_disj_sesK  [iff]: "pin P ≠ sesK(m,pk)" and
  pairK_disj_sesK[iff]: "pairK(A,B) ≠ sesK(m,pk)" and


  Atomic_distrib [iff]: "Atomic`(KEY`K ∪ NONCE`N) =
                 Atomic`(KEY`K) ∪ Atomic`(NONCE`N)" and


  shouprubin_assumes_securemeans [iff]: "evs ∈ srb ⟹ secureM"

definition Unique :: "[event, event list] => bool" (<Unique _ on _>) where
  "Unique ev on evs ==
     ev ∉ set (tl (dropWhile (% z. z ≠ ev) evs))"


inductive_set srb :: "event list set"
  where

   Nil:  "[]∈ srb"



  | Fake: "⟦ evsF ∈ srb;  X ∈ synth (analz (knows Spy evsF));
           illegalUse(Card B) ⟧
         ⟹ Says Spy A X #
             Inputs Spy (Card B) X # evsF ∈ srb"


  | Forge:
         "⟦ evsFo ∈ srb; Nonce Nb ∈ analz (knows Spy evsFo);
           Key (pairK(A,B)) ∈ knows Spy evsFo ⟧
         ⟹ Notes Spy (Key (sesK(Nb,pairK(A,B)))) # evsFo ∈ srb"



  | Reception: "⟦ evsrb∈ srb; Says A B X ∈ set evsrb ⟧
             ⟹ Gets B X # evsrb ∈ srb"




  | SR_U1:  "⟦ evs1 ∈ srb; A ≠ Server ⟧
         ⟹ Says A Server {|Agent A, Agent B|}
               # evs1 ∈ srb"

  | SR_U2:  "⟦ evs2 ∈ srb;
           Gets Server {|Agent A, Agent B|} ∈ set evs2 ⟧
         ⟹ Says Server A {|Nonce (Pairkey(A,B)),
                       Crypt (shrK A) {|Nonce (Pairkey(A,B)), Agent B|}
               |}
               # evs2 ∈ srb"
```

```
| SR_U3:  "⟦ evs3 ∈ srb; legalUse(Card A);
           Says A Server {|Agent A, Agent B|} ∈ set evs3;
           Gets A {|Nonce Pk, Certificate|} ∈ set evs3 ⟧
        ⟹ Inputs A (Card A) (Agent A)
             # evs3 ∈ srb"

| SR_U4:  "⟦ evs4 ∈ srb;
           Nonce Na ∉ used evs4; legalUse(Card A); A ≠ Server;
           Inputs A (Card A) (Agent A) ∈ set evs4 ⟧
     ⟹ Outpts (Card A) A {|Nonce Na, Crypt (crdK (Card A)) (Nonce Na)|}
           # evs4 ∈ srb"

| SR_U4Fake: "⟦ evs4F ∈ srb; Nonce Na ∉ used evs4F;
           illegalUse(Card A);
           Inputs Spy (Card A) (Agent A) ∈ set evs4F ⟧
    ⟹ Outpts (Card A) Spy {|Nonce Na, Crypt (crdK (Card A)) (Nonce Na)|}
          # evs4F ∈ srb"

| SR_U5:  "⟦ evs5 ∈ srb;
           Outpts (Card A) A {|Nonce Na, Certificate|} ∈ set evs5;
           ∀ p q. Certificate ≠ {|p, q|} ⟧
        ⟹ Says A B {|Agent A, Nonce Na|} # evs5 ∈ srb"

| SR_U6:  "⟦ evs6 ∈ srb; legalUse(Card B);
           Gets B {|Agent A, Nonce Na|} ∈ set evs6 ⟧
        ⟹ Inputs B (Card B) {|Agent A, Nonce Na|}
             # evs6 ∈ srb"

| SR_U7:  "⟦ evs7 ∈ srb;
           Nonce Nb ∉ used evs7; legalUse(Card B); B ≠ Server;
           K = sesK(Nb,pairK(A,B));
           Key K ∉ used evs7;
           Inputs B (Card B) {|Agent A, Nonce Na|} ∈ set evs7⟧
  ⟹ Outpts (Card B) B {|Nonce Nb, Agent A, Key K,
                          Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|},
                          Crypt (pairK(A,B)) (Nonce Nb)|}
```

```
              # evs7 ∈ srb"

| SR_U7Fake:  "⟦ evs7F ∈ srb; Nonce Nb ∉ used evs7F;
           illegalUse(Card B);
           K = sesK(Nb,pairK(A,B));
           Key K ∉ used evs7F;
           Inputs Spy (Card B) {|Agent A, Nonce Na|} ∈ set evs7F ⟧
        ⟹ Outpts (Card B) Spy {|Nonce Nb, Agent A, Key K,
                        Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|},
                        Crypt (pairK(A,B)) (Nonce Nb)|}
              # evs7F ∈ srb"

| SR_U8:  "⟦ evs8 ∈ srb;
           Inputs B (Card B) {|Agent A, Nonce Na|} ∈ set evs8;
           Outpts (Card B) B {|Nonce Nb, Agent A, Key K,
                             Cert1, Cert2|} ∈ set evs8 ⟧
        ⟹ Says B A {|Nonce Nb, Cert1|} # evs8 ∈ srb"

| SR_U9:  "⟦ evs9 ∈ srb; legalUse(Card A);
           Gets A {|Nonce Pk, Cert1|} ∈ set evs9;
           Outpts (Card A) A {|Nonce Na, Cert2|} ∈ set evs9;
           Gets A {|Nonce Nb, Cert3|} ∈ set evs9;
           ∀ p q. Cert2 ≠ {|p, q|} ⟧
        ⟹ Inputs A (Card A)
               {|Agent B, Nonce Na, Nonce Nb, Nonce Pk,
                Cert1, Cert3, Cert2|}
              # evs9 ∈ srb"

| SR_U10: "⟦ evs10 ∈ srb; legalUse(Card A); A ≠ Server;
           K = sesK(Nb,pairK(A,B));
           Inputs A (Card A) {|Agent B, Nonce Na, Nonce Nb,
                             Nonce (Pairkey(A,B)),
                             Crypt (shrK A) {|Nonce (Pairkey(A,B)),
                                              Agent B|},
                             Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|},

                             Crypt (crdK (Card A)) (Nonce Na)|}
             ∈ set evs10 ⟧
        ⟹ Outpts (Card A) A {|Agent B, Nonce Nb,
                             Key K, Crypt (pairK(A,B)) (Nonce Nb)|}
               # evs10 ∈ srb"
```

```
| SR_U10Fake: "⟦ evs10F ∈ srb;
           illegalUse(Card A);
           K = sesK(Nb,pairK(A,B));
           Inputs Spy (Card A) {|Agent B, Nonce Na, Nonce Nb,
                                 Nonce (Pairkey(A,B)),
                                 Crypt (shrK A) {|Nonce (Pairkey(A,B)),
                                                  Agent B|},
                                 Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|},

                                 Crypt (crdK (Card A)) (Nonce Na)|}
             ∈ set evs10F ⟧
        ⟹ Outpts (Card A) Spy {|Agent B, Nonce Nb,
                                 Key K, Crypt (pairK(A,B)) (Nonce Nb)|}
               # evs10F ∈ srb"
```

```
| SR_U11: "⟦ evs11 ∈ srb;
           Says A Server {|Agent A, Agent B|} ∈ set evs11;
           Outpts (Card A) A {|Agent B, Nonce Nb, Key K, Certificate|}
             ∈ set evs11 ⟧
        ⟹ Says A B (Certificate)
               # evs11 ∈ srb"
```

```
| Oops1:
   "⟦ evsO1 ∈ srb;
       Outpts (Card B) B {|Nonce Nb, Agent A, Key K, Cert1, Cert2|}
         ∈ set evsO1 ⟧
   ⟹ Notes Spy {|Key K, Nonce Nb, Agent A, Agent B|} # evsO1 ∈ srb"

| Oops2:
   "⟦ evsO2 ∈ srb;
       Outpts (Card A) A {|Agent B, Nonce Nb, Key K, Certificate|}
         ∈ set evsO2 ⟧
  ⟹ Notes Spy {|Key K, Nonce Nb, Agent A, Agent B|} # evsO2 ∈ srb"
```

declare *Fake_parts_insert_in_Un [dest]*
declare *analz_into_parts [dest]*

lemma *Gets_imp_Says:*
"⟦ Gets B X ∈ set evs; evs ∈ srb ⟧ ⟹ ∃ A. Says A B X ∈ set evs"
⟨*proof*⟩

lemma *Gets_imp_knows_Spy:*
"⟦ Gets B X ∈ set evs; evs ∈ srb ⟧ ⟹ X ∈ knows Spy evs"
⟨*proof*⟩

lemma *Gets_imp_knows_Spy_parts_Snd:*
"⟦ Gets B {|X, Y|} ∈ set evs; evs ∈ srb ⟧ ⟹ Y ∈ parts (knows Spy evs)"
⟨*proof*⟩

lemma *Gets_imp_knows_Spy_analz_Snd:*
"⟦ Gets B {|X, Y|} ∈ set evs; evs ∈ srb ⟧ ⟹ Y ∈ analz (knows Spy evs)"
⟨*proof*⟩

lemma *Inputs_imp_knows_Spy_secureM_srb:*
"⟦ Inputs Spy C X ∈ set evs; evs ∈ srb ⟧ ⟹ X ∈ knows Spy evs"
⟨*proof*⟩

lemma *knows_Spy_Inputs_secureM_srb_Spy:*
"evs ∈srb ⟹ knows Spy (Inputs Spy C X # evs) = insert X (knows Spy evs)"
⟨*proof*⟩

lemma *knows_Spy_Inputs_secureM_srb:*
"⟦ A ≠ Spy; evs ∈srb ⟧ ⟹ knows Spy (Inputs A C X # evs) = knows Spy evs"
⟨*proof*⟩

lemma *knows_Spy_Outpts_secureM_srb_Spy:*
"evs ∈srb ⟹ knows Spy (Outpts C Spy X # evs) = insert X (knows Spy evs)"
⟨*proof*⟩

lemma *knows_Spy_Outpts_secureM_srb:*
"⟦ A ≠ Spy; evs ∈srb ⟧ ⟹ knows Spy (Outpts C A X # evs) = knows Spy evs"
⟨*proof*⟩

lemma *Inputs_A_Card_3:*

```
    "⟦ Inputs A C (Agent A) ∈ set evs; A ≠ Spy; evs ∈ srb ⟧
     ⟹ legalUse(C) ∧ C = (Card A) ∧
      (∃ Pk Certificate. Gets A {|Pk, Certificate|} ∈ set evs)"
```

⟨*proof*⟩

lemma *Inputs_B_Card_6:*

```
     "⟦ Inputs B C {|Agent A, Nonce Na|} ∈ set evs; B ≠ Spy; evs ∈ srb ⟧
      ⟹ legalUse(C) ∧ C = (Card B) ∧ Gets B {|Agent A, Nonce Na|} ∈ set evs"
```

⟨*proof*⟩

lemma *Inputs_A_Card_9:*

```
     "⟦ Inputs A C {|Agent B, Nonce Na, Nonce Nb, Nonce Pk,
                                           Cert1, Cert2, Cert3|} ∈ set evs;

         A ≠ Spy; evs ∈ srb ⟧
  ⟹ legalUse(C) ∧ C = (Card A) ∧
      Gets A {|Nonce Pk, Cert1|} ∈ set evs     ∧
      Outpts (Card A) A {|Nonce Na, Cert3|} ∈ set evs        ∧
      Gets A {|Nonce Nb, Cert2|} ∈ set evs"
```

⟨*proof*⟩

lemma *Outpts_A_Card_4:*

```
     "⟦ Outpts C A {|Nonce Na, (Crypt (crdK (Card A)) (Nonce Na))|} ∈ set evs;

         evs ∈ srb ⟧
     ⟹ legalUse(C) ∧ C = (Card A) ∧
         Inputs A (Card A) (Agent A) ∈ set evs"
```

⟨*proof*⟩

lemma *Outpts_B_Card_7:*

```
      "⟦ Outpts C B {|Nonce Nb, Agent A, Key K,
                     Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|},
                     Cert2|} ∈ set evs;
         evs ∈ srb ⟧
     ⟹ legalUse(C) ∧ C = (Card B) ∧
         Inputs B (Card B) {|Agent A, Nonce Na|} ∈ set evs"
```

⟨*proof*⟩

lemma *Outpts_A_Card_10:*

```
     "⟦ Outpts C A {|Agent B, Nonce Nb,
                   Key K, (Crypt (pairK(A,B)) (Nonce Nb))|} ∈ set evs;
         evs ∈ srb ⟧
     ⟹ legalUse(C) ∧ C = (Card A) ∧
         (∃ Na Ver1 Ver2 Ver3.
       Inputs A (Card A) {|Agent B, Nonce Na, Nonce Nb, Nonce (Pairkey(A,B)),

                                 Ver1, Ver2, Ver3|} ∈ set evs)"
```

⟨*proof*⟩

```
lemma Outpts_A_Card_10_imp_Inputs:
     "⟦ Outpts (Card A) A ⦃Agent B, Nonce Nb, Key K, Certificate⦄
         ∈ set evs; evs ∈ srb ⟧
     ⟹ (∃ Na Ver1 Ver2 Ver3.
       Inputs A (Card A) ⦃Agent B, Nonce Na, Nonce Nb, Nonce (Pairkey(A,B)),

                               Ver1, Ver2, Ver3⦄ ∈ set evs)"
```
⟨*proof*⟩

```
lemma Outpts_honest_A_Card_4:
     "⟦ Outpts C A ⦃Nonce Na, Crypt K X⦄ ∈set evs;
         A ≠ Spy;  evs ∈ srb ⟧
     ⟹ legalUse(C) ∧ C = (Card A) ∧
         Inputs A (Card A) (Agent A) ∈ set evs"
```
⟨*proof*⟩

```
lemma Outpts_honest_B_Card_7:
    "⟦ Outpts C B ⦃Nonce Nb, Agent A, Key K, Cert1, Cert2⦄ ∈ set evs;
       B ≠ Spy; evs ∈ srb ⟧
   ⟹ legalUse(C) ∧ C = (Card B) ∧
       (∃ Na. Inputs B (Card B) ⦃Agent A, Nonce Na⦄ ∈ set evs)"
```
⟨*proof*⟩

```
lemma Outpts_honest_A_Card_10:
     "⟦ Outpts C A ⦃Agent B, Nonce Nb, Key K, Certificate⦄ ∈ set evs;
         A ≠ Spy; evs ∈ srb ⟧
     ⟹ legalUse (C) ∧ C = (Card A) ∧
         (∃ Na Pk Ver1 Ver2 Ver3.
          Inputs A (Card A) ⦃Agent B, Nonce Na, Nonce Nb, Pk,
                               Ver1, Ver2, Ver3⦄ ∈ set evs)"
```
⟨*proof*⟩

```
lemma Outpts_which_Card_4:
    "⟦ Outpts (Card A) A ⦃Nonce Na, Crypt K X⦄ ∈ set evs; evs ∈ srb ⟧
    ⟹ Inputs A (Card A) (Agent A) ∈ set evs"
```
⟨*proof*⟩

```
lemma Outpts_which_Card_7:
  "⟦ Outpts (Card B) B ⦃Nonce Nb, Agent A, Key K, Cert1, Cert2⦄
```

```
     ∈ set evs;  evs ∈ srb ⟧
   ⟹ ∃ Na. Inputs B (Card B) ⦃Agent A, Nonce Na⦄ ∈ set evs"
```
⟨proof⟩

lemma *Outpts_which_Card_10:*
```
  "⟦ Outpts (Card A) A ⦃Agent B, Nonce Nb, Key K, Certificate ⦄ ∈ set evs;
     evs ∈ srb ⟧
  ⟹ ∃ Na. Inputs A (Card A) ⦃Agent B, Nonce Na, Nonce Nb, Nonce (Pairkey(A,B)),

                           Crypt (shrK A) ⦃Nonce (Pairkey(A,B)), Agent B⦄,

                           Crypt (pairK(A,B)) ⦃Nonce Na, Nonce Nb⦄,
                           Crypt (crdK (Card A)) (Nonce Na) ⦄ ∈ set evs"
```
⟨proof⟩

lemma *Outpts_A_Card_form_4:*
```
  "⟦ Outpts (Card A) A ⦃Nonce Na, Certificate⦄ ∈ set evs;
       ∀ p q. Certificate ≠ ⦃p, q⦄; evs ∈ srb ⟧
    ⟹ Certificate = (Crypt (crdK (Card A)) (Nonce Na))"
```
⟨proof⟩

lemma *Outpts_B_Card_form_7:*
```
  "⟦ Outpts (Card B) B ⦃Nonce Nb, Agent A, Key K, Cert1, Cert2⦄
      ∈ set evs; evs ∈ srb ⟧
    ⟹ ∃ Na.
        K = sesK(Nb,pairK(A,B)) ∧
        Cert1 = (Crypt (pairK(A,B)) ⦃Nonce Na, Nonce Nb⦄) ∧
        Cert2 = (Crypt (pairK(A,B)) (Nonce Nb))"
```
⟨proof⟩

lemma *Outpts_A_Card_form_10:*
```
  "⟦ Outpts (Card A) A ⦃Agent B, Nonce Nb, Key K, Certificate⦄
      ∈ set evs; evs ∈ srb ⟧
    ⟹ K = sesK(Nb,pairK(A,B)) ∧
        Certificate = (Crypt (pairK(A,B)) (Nonce Nb))"
```
⟨proof⟩

lemma *Outpts_A_Card_form_bis:*
```
  "⟦ Outpts (Card A') A' ⦃Agent B', Nonce Nb', Key (sesK(Nb,pairK(A,B))),

    Certificate⦄ ∈ set evs;
       evs ∈ srb ⟧
    ⟹ A' = A ∧ B' = B ∧ Nb = Nb' ∧
        Certificate = (Crypt (pairK(A,B)) (Nonce Nb))"
```
⟨proof⟩

lemma *Inputs_A_Card_form_9:*

```
    "⟦ Inputs A (Card A) ⦃Agent B, Nonce Na, Nonce Nb, Nonce Pk,
                          Cert1, Cert2, Cert3⦄ ∈ set evs;
       evs ∈ srb ⟧
  ⟹    Cert3 = Crypt (crdK (Card A)) (Nonce Na)"
```

⟨*proof*⟩

lemma *Inputs_Card_legalUse:*

```
  "⟦ Inputs A (Card A) X ∈ set evs; evs ∈ srb ⟧ ⟹ legalUse(Card A)"
```

⟨*proof*⟩

lemma *Outpts_Card_legalUse:*

```
  "⟦ Outpts (Card A) A X ∈ set evs; evs ∈ srb ⟧ ⟹ legalUse(Card A)"
```

⟨*proof*⟩

lemma *Inputs_Card:* `"⟦ Inputs A C X ∈ set evs; A ≠ Spy; evs ∈ srb ⟧`

```
      ⟹ C = (Card A) ∧ legalUse(C)"
```

⟨*proof*⟩

lemma *Outpts_Card:* `"⟦ Outpts C A X ∈ set evs; A ≠ Spy; evs ∈ srb ⟧`

```
      ⟹ C = (Card A) ∧ legalUse(C)"
```

⟨*proof*⟩

lemma *Inputs_Outpts_Card:*

```
    "⟦ Inputs A C X ∈ set evs ∨ Outpts C A Y ∈ set evs;
       A ≠ Spy; evs ∈ srb ⟧
    ⟹ C = (Card A) ∧ legalUse(Card A)"
```

⟨*proof*⟩

lemma *Inputs_Card_Spy:*

```
  "⟦ Inputs Spy C X ∈ set evs ∨ Outpts C Spy X ∈ set evs; evs ∈ srb ⟧
      ⟹ C = (Card Spy) ∧ legalUse(Card Spy) ∨
         (∃ A. C = (Card A) ∧ illegalUse(Card A))"
```

⟨*proof*⟩

lemma *Outpts_A_Card_unique_nonce:*

```
"⟦ Outpts (Card A) A ⦃Nonce Na, Crypt (crdK (Card A)) (Nonce Na)⦄
      ∈ set evs;
    Outpts (Card A') A' ⦃Nonce Na, Crypt (crdK (Card A')) (Nonce Na)⦄

      ∈ set evs;
    evs ∈ srb ⟧ ⟹ A=A'"
```

⟨*proof*⟩

lemma *Outpts_B_Card_unique_nonce:*

```
"⟦ Outpts (Card B) B ⦃Nonce Nb, Agent A, Key SK, Cert1, Cert2⦄ ∈ set
evs;
 Outpts (Card B') B' ⦃Nonce Nb, Agent A', Key SK', Cert1', Cert2'⦄ ∈
set evs;
  evs ∈ srb ⟧ ⟹ B=B' ∧ A=A' ∧ SK=SK' ∧ Cert1=Cert1' ∧ Cert2=Cert2'"
```

⟨*proof*⟩

lemma *Outpts_B_Card_unique_key:*

```
"⟦ Outpts (Card B) B ⦃Nonce Nb, Agent A, Key SK, Cert1, Cert2⦄ ∈ set
evs;
 Outpts (Card B') B' ⦃Nonce Nb', Agent A', Key SK, Cert1', Cert2'⦄ ∈
set evs;
  evs ∈ srb ⟧ ⟹ B=B' ∧ A=A' ∧ Nb=Nb' ∧ Cert1=Cert1' ∧ Cert2=Cert2'"
```

⟨*proof*⟩

lemma *Outpts_A_Card_unique_key:*

```
"⟦ Outpts (Card A) A ⦃Agent B, Nonce Nb, Key K, V⦄ ∈ set evs;
  Outpts (Card A') A' ⦃Agent B', Nonce Nb', Key K, V'⦄ ∈ set evs;
    evs ∈ srb ⟧ ⟹ A=A' ∧ B=B' ∧ Nb=Nb' ∧ V=V'"
```

⟨*proof*⟩

lemma *Outpts_A_Card_Unique:*

```
"⟦ Outpts (Card A) A ⦃Nonce Na, rest⦄ ∈ set evs; evs ∈ srb ⟧
  ⟹ Unique (Outpts (Card A) A ⦃Nonce Na, rest⦄) on evs"
```

⟨*proof*⟩

lemma *Spy_knows_Na:*
```
"⟦ Says A B {|Agent A, Nonce Na|} ∈ set evs; evs ∈ srb ⟧
⟹ Nonce Na ∈ analz (knows Spy evs)"
```
⟨*proof*⟩

lemma *Spy_knows_Nb:*
```
"⟦ Says B A {|Nonce Nb, Certificate|} ∈ set evs; evs ∈ srb ⟧
⟹ Nonce Nb ∈ analz (knows Spy evs)"
```
⟨*proof*⟩

lemma *Pairkey_Gets_analz_knows_Spy:*
```
"⟦ Gets A {|Nonce (Pairkey(A,B)), Certificate|} ∈ set evs; evs ∈ srb
⟧
⟹ Nonce (Pairkey(A,B)) ∈ analz (knows Spy evs)"
```
⟨*proof*⟩

lemma *Pairkey_Inputs_imp_Gets:*
```
"⟦ Inputs A (Card A)
      {|Agent B, Nonce Na, Nonce Nb, Nonce (Pairkey(A,B)),
        Cert1, Cert3, Cert2|} ∈ set evs;
    A ≠ Spy; evs ∈ srb ⟧
⟹ Gets A {|Nonce (Pairkey(A,B)), Cert1|} ∈ set evs"
```
⟨*proof*⟩

lemma *Pairkey_Inputs_analz_knows_Spy:*
```
"⟦ Inputs A (Card A)
      {|Agent B, Nonce Na, Nonce Nb, Nonce (Pairkey(A,B)),
        Cert1, Cert3, Cert2|} ∈ set evs;
    evs ∈ srb ⟧
⟹ Nonce (Pairkey(A,B)) ∈ analz (knows Spy evs)"
```
⟨*proof*⟩

declare *shrK_disj_sesK [THEN not_sym, iff]*
declare *pin_disj_sesK [THEN not_sym, iff]*
declare *crdK_disj_sesK [THEN not_sym, iff]*
declare *pairK_disj_sesK [THEN not_sym, iff]*

⟨*ML*⟩

lemma *Spy_parts_keys [simp]: "evs ∈ srb ⟹
  (Key (shrK P) ∈ parts (knows Spy evs)) = (Card P ∈ cloned) ∧
  (Key (pin P) ∈ parts (knows Spy evs)) = (P ∈ bad ∨ Card P ∈ cloned) ∧

  (Key (crdK C) ∈ parts (knows Spy evs)) = (C ∈ cloned) ∧
  (Key (pairK(A,B)) ∈ parts (knows Spy evs)) = (Card B ∈ cloned)"*
⟨*proof*⟩

lemma *Spy_analz_shrK[simp]: "evs ∈ srb ⟹
  (Key (shrK P) ∈ analz (knows Spy evs)) = (Card P ∈ cloned)"*
⟨*proof*⟩

lemma *Spy_analz_crdK[simp]: "evs ∈ srb ⟹
  (Key (crdK C) ∈ analz (knows Spy evs)) = (C ∈ cloned)"*
⟨*proof*⟩

lemma *Spy_analz_pairK[simp]: "evs ∈ srb ⟹
  (Key (pairK(A,B)) ∈ analz (knows Spy evs)) = (Card B ∈ cloned)"*
⟨*proof*⟩

lemma *analz_image_Key_Un_Nonce:
  "analz (Key ‘ K ∪ Nonce ‘ N) = Key ‘ K ∪ Nonce ‘ N"*
  ⟨*proof*⟩

⟨*ML*⟩

lemma *analz_image_freshK [rule_format]:
    "evs ∈ srb ⟹   ∀ K KK.
        (Key K ∈ analz (Key‘KK ∪ (knows Spy evs))) =
        (K ∈ KK ∨ Key K ∈ analz (knows Spy evs))"*
⟨*proof*⟩

lemma *analz_insert_freshK: "evs ∈ srb ⟹
      Key K ∈ analz (insert (Key K’) (knows Spy evs)) =
      (K = K’ ∨ Key K ∈ analz (knows Spy evs))"*
⟨*proof*⟩

lemma *Na_Nb_certificate_authentic:*
```
"⟦ Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|} ∈ parts (knows Spy evs);
    ¬illegalUse(Card B);
    evs ∈ srb ⟧
 ⟹ Outpts (Card B) B {|Nonce Nb, Agent A, Key (sesK(Nb,pairK(A,B))),

          Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|},
          Crypt (pairK(A,B)) (Nonce Nb)|} ∈ set evs"
```
⟨proof⟩

lemma *Nb_certificate_authentic:*
```
"⟦ Crypt (pairK(A,B)) (Nonce Nb) ∈ parts (knows Spy evs);
    B ≠ Spy; ¬illegalUse(Card A); ¬illegalUse(Card B);
    evs ∈ srb ⟧
 ⟹ Outpts (Card A) A {|Agent B, Nonce Nb, Key (sesK(Nb,pairK(A,B))),

                    Crypt (pairK(A,B)) (Nonce Nb)|} ∈ set evs"
```
⟨proof⟩

lemma *Outpts_A_Card_imp_pairK_parts:*
```
"⟦ Outpts (Card A) A  {|Agent B, Nonce Nb,
                Key K, Certificate|} ∈ set evs;
    evs ∈ srb ⟧
 ⟹ ∃ Na. Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|} ∈ parts (knows Spy
evs)"
```
⟨proof⟩

lemma *Nb_certificate_authentic_bis:*
```
"⟦ Crypt (pairK(A,B)) (Nonce Nb) ∈ parts (knows Spy evs);
    B ≠ Spy; ¬illegalUse(Card B);
    evs ∈ srb ⟧
⟹ ∃ Na. Outpts (Card B) B {|Nonce Nb, Agent A, Key (sesK(Nb,pairK(A,B))),

          Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|},
          Crypt (pairK(A,B)) (Nonce Nb)|} ∈ set evs"
```
⟨proof⟩

lemma *Pairkey_certificate_authentic:*
```
"⟦ Crypt (shrK A) {|Nonce Pk, Agent B|} ∈ parts (knows Spy evs);
    Card A ∉ cloned; evs ∈ srb ⟧
 ⟹ Pk = Pairkey(A,B) ∧
    Says Server A {|Nonce Pk,
              Crypt (shrK A) {|Nonce Pk, Agent B|}|}
      ∈ set evs"
```
⟨proof⟩

lemma *sesK_authentic:*
```
"⟦ Key (sesK(Nb,pairK(A,B))) ∈ parts (knows Spy evs);
   A ≠ Spy; B ≠ Spy; ¬illegalUse(Card A); ¬illegalUse(Card B);
   evs ∈ srb ⟧
 ⟹ Notes Spy ⦃Key (sesK(Nb,pairK(A,B))), Nonce Nb, Agent A, Agent B⦄

     ∈ set evs"
```
⟨*proof*⟩

lemma *Confidentiality:*
```
"⟦ Notes Spy ⦃Key (sesK(Nb,pairK(A,B))), Nonce Nb, Agent A, Agent B⦄

     ∉ set evs;
   A ≠ Spy; B ≠ Spy; ¬illegalUse(Card A); ¬illegalUse(Card B);
   evs ∈ srb ⟧
 ⟹ Key (sesK(Nb,pairK(A,B))) ∉ analz (knows Spy evs)"
```
⟨*proof*⟩

lemma *Confidentiality_B:*
```
"⟦ Outpts (Card B) B ⦃Nonce Nb, Agent A, Key K, Cert1, Cert2⦄
     ∈ set evs;
   Notes Spy ⦃Key K, Nonce Nb, Agent A, Agent B⦄ ∉ set evs;
   A ≠ Spy; B ≠ Spy; ¬illegalUse(Card A); Card B ∉ cloned;
   evs ∈ srb ⟧
 ⟹ Key K ∉ analz (knows Spy evs)"
```
⟨*proof*⟩

lemma *A_authenticates_B:*
```
"⟦ Outpts (Card A) A  ⦃Agent B, Nonce Nb, Key K, Certificate⦄ ∈ set evs;
   ¬illegalUse(Card B);
   evs ∈ srb ⟧
⟹ ∃ Na. Outpts (Card B) B ⦃Nonce Nb, Agent A, Key K,
            Crypt (pairK(A,B)) ⦃Nonce Na, Nonce Nb⦄,
            Crypt (pairK(A,B)) (Nonce Nb)⦄ ∈ set evs"
```
⟨*proof*⟩

lemma *A_authenticates_B_Gets:*
```
"⟦ Gets A ⦃Nonce Nb, Crypt (pairK(A,B)) ⦃Nonce Na, Nonce Nb⦄⦄
     ∈ set evs;
```

```
        ¬illegalUse(Card B);
        evs ∈ srb ⟧
   ⟹ Outpts (Card B) B {|Nonce Nb, Agent A, Key (sesK(Nb, pairK (A, B))),

                     Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|},
                     Crypt (pairK(A,B)) (Nonce Nb)|} ∈ set evs"
```

⟨proof⟩

lemma *A_authenticates_B_bis:*

```
     "⟦ Outpts (Card A) A  {|Agent B, Nonce Nb, Key K, Cert2|} ∈ set evs;
        ¬illegalUse(Card B);
        evs ∈ srb ⟧
 ⟹ ∃ Cert1. Outpts (Card B) B {|Nonce Nb, Agent A, Key K, Cert1, Cert2|}

               ∈ set evs"
```

⟨proof⟩

lemma *B_authenticates_A:*

```
     "⟦ Gets B (Crypt (pairK(A,B)) (Nonce Nb)) ∈ set evs;
         B ≠ Spy; ¬illegalUse(Card A); ¬illegalUse(Card B);
         evs ∈ srb ⟧
       ⟹ Outpts (Card A) A  {|Agent B, Nonce Nb,
         Key (sesK(Nb,pairK(A,B))), Crypt (pairK(A,B)) (Nonce Nb)|} ∈ set evs"
```

⟨proof⟩

lemma *B_authenticates_A_bis:*

```
     "⟦ Outpts (Card B) B {|Nonce Nb, Agent A, Key K, Cert1, Cert2|} ∈ set evs;
        Gets B (Cert2) ∈ set evs;
         B ≠ Spy; ¬illegalUse(Card A); ¬illegalUse(Card B);
         evs ∈ srb ⟧
       ⟹ Outpts (Card A) A  {|Agent B, Nonce Nb, Key K, Cert2|} ∈ set evs"
```

⟨proof⟩

lemma *Confidentiality_A:*

```
      "⟦ Outpts (Card A) A {|Agent B, Nonce Nb,
                        Key K, Certificate|} ∈ set evs;
         Notes Spy {|Key K, Nonce Nb, Agent A, Agent B|} ∉ set evs;
         A ≠ Spy; B ≠ Spy; ¬illegalUse(Card A); ¬illegalUse(Card B);
         evs ∈ srb ⟧
     ⟹ Key K ∉ analz (knows Spy evs)"
```

⟨proof⟩

lemma *Outpts_imp_knows_agents_secureM_srb:*

```
"⟦ Outpts (Card A) A X ∈ set evs; evs ∈ srb ⟧ ⟹ X ∈ knows A evs"
```

⟨*proof*⟩

lemma *A_keydist_to_B:*

```
"⟦ Outpts (Card A) A {|Agent B, Nonce Nb, Key K, Certificate|} ∈ set evs;

      ¬illegalUse(Card B);
      evs ∈ srb ⟧
   ⟹ Key K ∈ analz (knows B evs)"
```

⟨*proof*⟩

lemma *B_keydist_to_A:*

```
"⟦ Outpts (Card B) B {|Nonce Nb, Agent A, Key K, Cert1, Cert2|} ∈ set evs;
   Gets B (Cert2) ∈ set evs;
   B ≠ Spy; ¬illegalUse(Card A); ¬illegalUse(Card B);
   evs ∈ srb ⟧
 ⟹ Key K ∈ analz (knows A evs)"
```

⟨*proof*⟩

lemma *Nb_certificate_authentic_B:*

```
"⟦ Gets B (Crypt (pairK(A,B)) (Nonce Nb)) ∈ set evs;
   B ≠ Spy; ¬illegalUse(Card B);
   evs ∈ srb ⟧
⟹ ∃ Na.
      Outpts (Card B) B {|Nonce Nb, Agent A, Key (sesK(Nb,pairK(A,B))),

          Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|},
          Crypt (pairK(A,B)) (Nonce Nb)|} ∈ set evs"
```

⟨*proof*⟩

lemma *Pairkey_certificate_authentic_A_Card:*

```
"⟦ Inputs A (Card A)
       {|Agent B, Nonce Na, Nonce Nb, Nonce Pk,
         Crypt (shrK A) {|Nonce Pk, Agent B|},
         Cert2, Cert3|} ∈ set evs;
   A ≠ Spy; Card A ∉ cloned; evs ∈ srb ⟧
⟹ Pk = Pairkey(A,B) ∧
```

```
        Says Server A {|Nonce (Pairkey(A,B)),
                Crypt (shrK A) {|Nonce (Pairkey(A,B)), Agent B|}|}
          ∈ set evs "
```
⟨proof⟩

lemma *Na_Nb_certificate_authentic_A_Card:*
```
      "⟦ Inputs A (Card A)
             {|Agent B, Nonce Na, Nonce Nb, Nonce Pk,
         Cert1, Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|}, Cert3|} ∈ set evs;

      A ≠ Spy; ¬illegalUse(Card B); evs ∈ srb ⟧
   ⟹ Outpts (Card B) B {|Nonce Nb, Agent A, Key (sesK(Nb, pairK (A, B))),

                              Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|},
                              Crypt (pairK(A,B)) (Nonce Nb)|}
          ∈ set evs "
```
⟨proof⟩

lemma *Na_authentic_A_Card:*
```
     "⟦ Inputs A (Card A)
             {|Agent B, Nonce Na, Nonce Nb, Nonce Pk,
                Cert1, Cert2, Cert3|} ∈ set evs;
        A ≠ Spy; evs ∈ srb ⟧
     ⟹ Outpts (Card A) A {|Nonce Na, Cert3|}
          ∈ set evs"
```
⟨proof⟩

lemma *Inputs_A_Card_9_authentic:*
```
  "⟦ Inputs A (Card A)
             {|Agent B, Nonce Na, Nonce Nb, Nonce Pk,
               Crypt (shrK A) {|Nonce Pk, Agent B|},
               Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|}, Cert3|} ∈ set evs;

    A ≠ Spy; Card A ∉ cloned; ¬illegalUse(Card B); evs ∈ srb ⟧
    ⟹  Says Server A {|Nonce Pk, Crypt (shrK A) {|Nonce Pk, Agent B|}|}
          ∈ set evs  ∧
       Outpts (Card B) B {|Nonce Nb, Agent A,  Key (sesK(Nb, pairK (A, B))),

                              Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|},
                              Crypt (pairK(A,B)) (Nonce Nb)|}
          ∈ set evs  ∧
        Outpts (Card A) A {|Nonce Na, Cert3|}
          ∈ set evs"
```
⟨proof⟩

lemma *SR_U4_imp:*

```
"⟦ Outpts (Card A) A {|Nonce Na, Crypt (crdK (Card A)) (Nonce Na)|}
      ∈ set evs;
    A ≠ Spy; evs ∈ srb ⟧
⟹ ∃ Pk V. Gets A {|Pk, V|} ∈ set evs"
```

⟨*proof*⟩

lemma *SR_U7_imp:*

```
"⟦ Outpts (Card B) B {|Nonce Nb, Agent A, Key K,
                   Crypt (pairK(A,B)) {|Nonce Na, Nonce Nb|},
                   Cert2|} ∈ set evs;
    B ≠ Spy; evs ∈ srb ⟧
⟹ Gets B {|Agent A, Nonce Na|} ∈ set evs"
```

⟨*proof*⟩

lemma *SR_U10_imp:*

```
"⟦ Outpts (Card A) A {|Agent B, Nonce Nb,
                        Key K, Crypt (pairK(A,B)) (Nonce Nb)|}
    ∈ set evs;
   A ≠ Spy; evs ∈ srb ⟧
⟹ ∃ Cert1 Cert2.
               Gets A {|Nonce (Pairkey (A, B)), Cert1|} ∈ set evs ∧
               Gets A {|Nonce Nb, Cert2|} ∈ set evs"
```

⟨*proof*⟩

lemma *Outpts_Server_not_evs:*

```
 "evs ∈ srb ⟹ Outpts (Card Server) P X ∉ set evs"
```

⟨*proof*⟩

`step2_integrity` also is a reliability theorem

lemma *Says_Server_message_form:*

```
"⟦ Says Server A {|Pk, Certificate|} ∈ set evs;
    evs ∈ srb ⟧
⟹ ∃ B. Pk = Nonce (Pairkey(A,B)) ∧
    Certificate = Crypt (shrK A) {|Nonce (Pairkey(A,B)), Agent B|}"
```

⟨*proof*⟩

step4integrity is *Outpts_A_Card_form_4*

step7integrity is *Outpts_B_Card_form_7*

```
lemma step8_integrity:
     "⟦ Says B A ⦃Nonce Nb, Certificate⦄ ∈ set evs;
         B ≠ Server; B ≠ Spy; evs ∈ srb ⟧
     ⟹ ∃ Cert2 K.
    Outpts (Card B) B ⦃Nonce Nb, Agent A, Key K, Certificate, Cert2⦄ ∈ set
evs"
⟨proof⟩
```

step9integrity is *Inputs_A_Card_form_9* step10integrity is *Outpts_A_Card_form_10*.

```
lemma step11_integrity:
     "⟦ Says A B (Certificate) ∈ set evs;
         ∀ p q. Certificate ≠ ⦃p, q⦄;
         A ≠ Spy; evs ∈ srb ⟧
     ⟹ ∃ K Nb.
      Outpts (Card A) A ⦃Agent B, Nonce Nb, Key K, Certificate⦄ ∈ set evs"
⟨proof⟩

end
```

28 Smartcard protocols: rely on conventional Message and on new EventSC and Smartcard

```
theory Auth_Smartcard
imports
  ShoupRubin
  ShoupRubinBella
begin

end
```

29 Extensions to Standard Theories

```
theory Extensions
imports "../Event"
begin
```

29.1 Extensions to Theory *Set*

```
lemma eq: "⟦⋀x. x∈A ⟹ x∈B; ⋀x. x∈B ⟹ x∈A⟧ ⟹ A=B"
⟨proof⟩

lemma insert_Un: "P ({x} ∪ A) ⟹ P (insert x A)"
⟨proof⟩

lemma in_sub: "x∈A ⟹ {x}⊆A"
⟨proof⟩
```

29.2 Extensions to Theory *List*

29.2.1 "remove l x" erase the first element of "l" equal to "x"

primrec *remove :: "'a list => 'a => 'a list"* **where**
"remove [] y = []" |
"remove (x#xs) y = (if x=y then xs else x # remove xs y)"

lemma *set_remove: "set (remove l x) <= set l"*
⟨*proof*⟩

29.3 Extensions to Theory *Message*

29.3.1 declarations for tactics

declare *analz_subset_parts [THEN subsetD, dest]*
declare *parts_insert2 [simp]*
declare *analz_cut [dest]*
declare *if_split_asm [split]*
declare *analz_insertI [intro]*
declare *Un_Diff [simp]*

29.3.2 extract the agent number of an Agent message

primrec *agt_nb :: "msg => agent"* **where**
"agt_nb (Agent A) = A"

29.3.3 messages that are pairs

definition *is_MPair :: "msg => bool"* **where**
"is_MPair X == ∃ Y Z. X = ⦃Y,Z⦄"

declare *is_MPair_def [simp]*

lemma *MPair_is_MPair [iff]: "is_MPair ⦃X,Y⦄"*
⟨*proof*⟩

lemma *Agent_isnt_MPair [iff]: "~ is_MPair (Agent A)"*
⟨*proof*⟩

lemma *Number_isnt_MPair [iff]: "~ is_MPair (Number n)"*
⟨*proof*⟩

lemma *Key_isnt_MPair [iff]: "~ is_MPair (Key K)"*
⟨*proof*⟩

lemma *Nonce_isnt_MPair [iff]: "~ is_MPair (Nonce n)"*
⟨*proof*⟩

lemma *Hash_isnt_MPair [iff]: "~ is_MPair (Hash X)"*
⟨*proof*⟩

lemma *Crypt_isnt_MPair [iff]: "~ is_MPair (Crypt K X)"*
⟨*proof*⟩

abbreviation

not_MPair :: "msg => bool" **where**
"not_MPair X == ~ is_MPair X"

lemma *is_MPairE: "⟦is_MPair X ⟹ P; not_MPair X ⟹ P⟧ ⟹ P"*
⟨*proof*⟩

declare *is_MPair_def [simp del]*

definition *has_no_pair :: "msg set => bool"* **where**
"has_no_pair H == ∀X Y. ⦃X,Y⦄ ∉ H"

declare *has_no_pair_def [simp]*

29.3.4 well-foundedness of messages

lemma *wf_Crypt1 [iff]: "Crypt K X ~= X"*
⟨*proof*⟩

lemma *wf_Crypt2 [iff]: "X ~= Crypt K X"*
⟨*proof*⟩

lemma *parts_size: "X ∈ parts {Y} ⟹ X=Y ∨ size X < size Y"*
⟨*proof*⟩

lemma *wf_Crypt_parts [iff]: "Crypt K X ∉ parts {X}"*
⟨*proof*⟩

29.3.5 lemmas on keysFor

definition *usekeys :: "msg set => key set"* **where**
"usekeys G ≡ {K. ∃Y. Crypt K Y ∈ G}"

lemma *finite_keysFor [intro]: "finite G ⟹ finite (keysFor G)"*
⟨*proof*⟩

29.3.6 lemmas on parts

lemma *parts_sub: "⟦X ∈ parts G; G ⊆ H⟧ ⟹ X ∈ parts H"*
⟨*proof*⟩

lemma *parts_Diff [dest]: "X ∈ parts (G - H) ⟹ X ∈ parts G"*
⟨*proof*⟩

lemma *parts_Diff_notin: "⟦Y ∉ H; Nonce n ∉ parts (H - {Y})⟧*
⟹ Nonce n ∉ parts H"
⟨*proof*⟩

lemmas *parts_insert_substI = parts_insert [THEN ssubst]*
lemmas *parts_insert_substD = parts_insert [THEN sym, THEN ssubst]*

lemma *finite_parts_msg [iff]: "finite (parts {X})"*
⟨*proof*⟩

lemma *finite_parts [intro]: "finite H ⟹ finite (parts H)"*
⟨*proof*⟩

lemma *parts_parts: "⟦X ∈ parts {Y}; Y ∈ parts G⟧ ⟹ X ∈ parts G"*
⟨*proof*⟩

lemma *parts_parts_parts: "⟦X ∈ parts {Y}; Y ∈ parts {Z}; Z ∈ parts G⟧ ⟹ X ∈ parts G"*
⟨*proof*⟩

lemma *parts_parts_Crypt: "⟦Crypt K X ∈ parts G; Nonce n ∈ parts {X}⟧ ⟹ Nonce n ∈ parts G"*
⟨*proof*⟩

29.3.7 lemmas on synth

lemma *synth_sub: "⟦X ∈ synth G; G ⊆ H⟧ ⟹ X ∈ synth H"*
⟨*proof*⟩

lemma *Crypt_synth [rule_format]: "⟦X ∈ synth G; Key K ∉ G⟧ ⟹ Crypt K Y ∈ parts {X} ⟶ Crypt K Y ∈ parts G"*
⟨*proof*⟩

29.3.8 lemmas on analz

lemma *analz_UnI1 [intro]: "X ∈ analz G ⟹ X ∈ analz (G ∪ H)"*
⟨*proof*⟩

lemma *analz_sub: "⟦X ∈ analz G; G ⊆ H⟧ ⟹ X ∈ analz H"*
⟨*proof*⟩

lemma *analz_Diff [dest]: "X ∈ analz (G - H) ⟹ X ∈ analz G"*
⟨*proof*⟩

lemmas *in_analz_subset_cong = analz_subset_cong [THEN subsetD]*

lemma *analz_eq: "A=A' ⟹ analz A = analz A'"*
⟨*proof*⟩

lemmas *insert_commute_substI = insert_commute [THEN ssubst]*

lemma *analz_insertD:*
"⟦Crypt K Y ∈ H; Key (invKey K) ∈ H⟧ ⟹ analz (insert Y H) = analz H"
⟨*proof*⟩

lemma *must_decrypt [rule_format,dest]: "⟦X ∈ analz H; has_no_pair H⟧ ⟹ X ∉ H ⟶ (∃K Y. Crypt K Y ∈ H ∧ Key (invKey K) ∈ H)"*
⟨*proof*⟩

lemma *analz_needs_only_finite: "X ∈ analz H ⟹ ∃G. G ⊆ H ∧ finite G"*
⟨*proof*⟩

lemma *notin_analz_insert: "X ∉ analz (insert Y G) ⟹ X ∉ analz G"*
⟨*proof*⟩

29.3.9 lemmas on parts, synth and analz

lemma *parts_invKey [rule_format,dest]:"X ∈ parts {Y} ⟹
X ∈ analz (insert (Crypt K Y) H) ⟶ X ∉ analz H ⟶ Key (invKey K) ∈ analz H"*
⟨*proof*⟩

lemma *in_analz: "Y ∈ analz H ⟹ ∃X. X ∈ H ∧ Y ∈ parts {X}"*
⟨*proof*⟩

lemmas *in_analz_subset_parts = analz_subset_parts [THEN subsetD]*

lemma *Crypt_synth_insert: "⟦Crypt K X ∈ parts (insert Y H);
Y ∈ synth (analz H); Key K ∉ analz H⟧ ⟹ Crypt K X ∈ parts H"*
⟨*proof*⟩

29.3.10 greatest nonce used in a message

fun *greatest_msg :: "msg => nat"*
where
"greatest_msg (Nonce n) = n"
| "greatest_msg ⦃X,Y⦄ = max (greatest_msg X) (greatest_msg Y)"
| "greatest_msg (Crypt K X) = greatest_msg X"
| "greatest_msg other = 0"

lemma *greatest_msg_is_greatest: "Nonce n ∈ parts {X} ⟹ n ≤ greatest_msg X"*
⟨*proof*⟩

29.3.11 sets of keys

definition *keyset :: "msg set => bool"* **where**
"keyset G ≡ ∀X. X ∈ G ⟶ (∃K. X = Key K)"

lemma *keyset_in [dest]: "⟦keyset G; X ∈ G⟧ ⟹ ∃K. X = Key K"*
⟨*proof*⟩

lemma *MPair_notin_keyset [simp]: "keyset G ⟹ ⦃X,Y⦄ ∉ G"*
⟨*proof*⟩

lemma *Crypt_notin_keyset [simp]: "keyset G ⟹ Crypt K X ∉ G"*
⟨*proof*⟩

lemma *Nonce_notin_keyset [simp]: "keyset G ⟹ Nonce n ∉ G"*
⟨*proof*⟩

lemma *parts_keyset [simp]: "keyset G ⟹ parts G = G"*
⟨*proof*⟩

29.3.12 keys a priori necessary for decrypting the messages of G

definition *keysfor :: "msg set => msg set"* **where**
"keysfor G == Key ‘ keysFor (parts G)"

lemma *keyset_keysfor [iff]: "keyset (keysfor G)"*

⟨proof⟩

lemma *keyset_Diff_keysfor [simp]: "keyset H ⟹ keyset (H - keysfor G)"*
⟨proof⟩

lemma *keysfor_Crypt: "Crypt K X ∈ parts G ⟹ Key (invKey K) ∈ keysfor G"*
⟨proof⟩

lemma *no_key_no_Crypt: "Key K ∉ keysfor G ⟹ Crypt (invKey K) X ∉ parts G"*
⟨proof⟩

lemma *finite_keysfor [intro]: "finite G ⟹ finite (keysfor G)"*
⟨proof⟩

29.3.13 only the keys necessary for G are useful in analz

lemma *analz_keyset: "keyset H ⟹*
analz (G Un H) = H - keysfor G Un (analz (G Un (H Int keysfor G)))"
⟨proof⟩

lemmas *analz_keyset_substD = analz_keyset [THEN sym, THEN ssubst]*

29.4 Extensions to Theory Event

29.4.1 general protocol properties

definition *is_Says :: "event => bool"* **where**
"is_Says ev == (∃A B X. ev = Says A B X)"

lemma *is_Says_Says [iff]: "is_Says (Says A B X)"*
⟨proof⟩

definition *Gets_correct :: "event list set => bool"* **where**
"Gets_correct p == ∀evs B X. evs ∈ p ⟶ Gets B X ∈ set evs
⟶ (∃A. Says A B X ∈ set evs)"

lemma *Gets_correct_Says: "⟦Gets_correct p; Gets B X # evs ∈ p⟧*
⟹ ∃A. Says A B X ∈ set evs"
⟨proof⟩

definition *one_step :: "event list set => bool"* **where**
"one_step p == ∀evs ev. ev#evs ∈ p ⟶ evs ∈ p"

lemma *one_step_Cons [dest]: "⟦one_step p; ev#evs ∈ p⟧ ⟹ evs ∈ p"*
⟨proof⟩

lemma *one_step_app: "⟦evs@evs' ∈ p; one_step p; [] ∈ p⟧ ⟹ evs' ∈ p"*
⟨proof⟩

lemma *trunc: "⟦evs @ evs' ∈ p; one_step p⟧ ⟹ evs' ∈ p"*
⟨proof⟩

definition *has_only_Says :: "event list set => bool"* **where**

"has_only_Says p ≡ ∀ evs ev. evs ∈ p ⟶ ev ∈ set evs
⟶ (∃ A B X. ev = Says A B X)"

lemma *has_only_SaysD: "⟦ev ∈ set evs; evs ∈ p; has_only_Says p⟧*
⟹ ∃ A B X. ev = Says A B X"
⟨*proof*⟩

lemma *in_has_only_Says [dest]: "⟦has_only_Says p; evs ∈ p; ev ∈ set evs⟧*
⟹ ∃ A B X. ev = Says A B X"
⟨*proof*⟩

lemma *has_only_Says_imp_Gets_correct [simp]: "has_only_Says p*
⟹ Gets_correct p"
⟨*proof*⟩

29.4.2 lemma on knows

lemma *Says_imp_spies2: "Says A B ⦃X,Y⦄ ∈ set evs ⟹ Y ∈ parts (spies evs)"*
⟨*proof*⟩

lemma *Says_not_parts: "⟦Says A B X ∈ set evs; Y ∉ parts (spies evs)⟧*
⟹ Y ∉ parts {X}"
⟨*proof*⟩

29.4.3 knows without initState

```
primrec knows' :: "agent => event list => msg set" where
  knows'_Nil: "knows' A [] = {}" |
  knows'_Cons0:
 "knows' A (ev # evs) = (
   if A = Spy then (
     case ev of
       Says A' B X => insert X (knows' A evs)
     | Gets A' X => knows' A evs
     | Notes A' X => if A' ∈ bad then insert X (knows' A evs) else knows'
A evs
   ) else (
     case ev of
       Says A' B X => if A=A' then insert X (knows' A evs) else knows' A evs
     | Gets A' X => if A=A' then insert X (knows' A evs) else knows' A evs
     | Notes A' X => if A=A' then insert X (knows' A evs) else knows' A evs
   ))"
```

```
abbreviation
  spies' :: "event list => msg set" where
  "spies' == knows' Spy"
```

29.4.4 decomposition of knows into knows' and initState

lemma *knows_decomp: "knows A evs = knows' A evs Un (initState A)"*
⟨*proof*⟩

lemmas *knows_decomp_substI = knows_decomp [THEN ssubst]*
lemmas *knows_decomp_substD = knows_decomp [THEN sym, THEN ssubst]*

lemma *knows'_sub_knows: "knows' A evs <= knows A evs"*
⟨*proof*⟩

lemma *knows'_Cons: "knows' A (ev#evs) = knows' A [ev] Un knows' A evs"*
⟨*proof*⟩

lemmas *knows'_Cons_substI = knows'_Cons [THEN ssubst]*
lemmas *knows'_Cons_substD = knows'_Cons [THEN sym, THEN ssubst]*

lemma *knows_Cons: "knows A (ev#evs) = initState A Un knows' A [ev] Un knows A evs"*
⟨*proof*⟩

lemmas *knows_Cons_substI = knows_Cons [THEN ssubst]*
lemmas *knows_Cons_substD = knows_Cons [THEN sym, THEN ssubst]*

lemma *knows'_sub_spies': "⟦evs ∈ p; has_only_Says p; one_step p⟧ ⟹ knows' A evs ⊆ spies' evs"*
⟨*proof*⟩

29.4.5 knows' is finite

lemma *finite_knows' [iff]: "finite (knows' A evs)"*
⟨*proof*⟩

29.4.6 monotonicity of knows

lemma *knows_sub_Cons: "knows A evs <= knows A (ev#evs)"*
⟨*proof*⟩

lemma *knows_ConsI: "X ∈ knows A evs ⟹ X ∈ knows A (ev#evs)"*
⟨*proof*⟩

lemma *knows_sub_app: "knows A evs <= knows A (evs @ evs')"*
⟨*proof*⟩

29.4.7 maximum knowledge an agent can have includes messages sent to the agent

```
primrec knows_max' :: "agent => event list => msg set" where
knows_max'_def_Nil: "knows_max' A [] = {}" |
knows_max'_def_Cons: "knows_max' A (ev # evs) = (
  if A=Spy then (
    case ev of
      Says A' B X => insert X (knows_max' A evs)
    | Gets A' X => knows_max' A evs
    | Notes A' X =>
      if A' ∈ bad then insert X (knows_max' A evs) else knows_max' A evs
  ) else (
    case ev of
      Says A' B X =>
      if A=A' | A=B then insert X (knows_max' A evs) else knows_max' A evs
    | Gets A' X =>
      if A=A' then insert X (knows_max' A evs) else knows_max' A evs
```

```
    | Notes A' X =>
      if A=A' then insert X (knows_max' A evs) else knows_max' A evs
  ))"
```

definition *knows_max :: "agent => event list => msg set"* **where**
"knows_max A evs == knows_max' A evs Un initState A"

abbreviation
spies_max :: "event list => msg set" **where**
"spies_max evs == knows_max Spy evs"

29.4.8 basic facts about *knows_max*

lemma *spies_max_spies [iff]: "spies_max evs = spies evs"*
⟨*proof*⟩

lemma *knows_max'_Cons: "knows_max' A (ev#evs)*
= knows_max' A [ev] Un knows_max' A evs"
⟨*proof*⟩

lemmas *knows_max'_Cons_substI = knows_max'_Cons [THEN ssubst]*
lemmas *knows_max'_Cons_substD = knows_max'_Cons [THEN sym, THEN ssubst]*

lemma *knows_max_Cons: "knows_max A (ev#evs)*
= knows_max' A [ev] Un knows_max A evs"
⟨*proof*⟩

lemmas *knows_max_Cons_substI = knows_max_Cons [THEN ssubst]*
lemmas *knows_max_Cons_substD = knows_max_Cons [THEN sym, THEN ssubst]*

lemma *finite_knows_max' [iff]: "finite (knows_max' A evs)"*
⟨*proof*⟩

lemma *knows_max'_sub_spies': "⟦evs ∈ p; has_only_Says p; one_step p⟧*
⟹ knows_max' A evs ⊆ spies' evs"
⟨*proof*⟩

lemma *knows_max'_in_spies' [dest]: "⟦evs ∈ p; X ∈ knows_max' A evs;*
has_only_Says p; one_step p⟧ ⟹ X ∈ spies' evs"
⟨*proof*⟩

lemma *knows_max'_app: "knows_max' A (evs @ evs')*
= knows_max' A evs Un knows_max' A evs'"
⟨*proof*⟩

lemma *Says_to_knows_max': "Says A B X ∈ set evs ⟹ X ∈ knows_max' B evs"*
⟨*proof*⟩

lemma *Says_from_knows_max': "Says A B X ∈ set evs ⟹ X ∈ knows_max' A evs"*
⟨*proof*⟩

29.4.9 used without initState

primrec *used' :: "event list => msg set"* **where**
"used' [] = {}" |

```
"used' (ev # evs) = (
  case ev of
    Says A B X => parts {X} Un used' evs
    | Gets A X => used' evs
    | Notes A X => parts {X} Un used' evs
  )"
```

definition *init :: "msg set"* **where**
"init == used []"

lemma *used_decomp: "used evs = init Un used' evs"*
⟨*proof*⟩

lemma *used'_sub_app: "used' evs ⊆ used' (evs@evs')"*
⟨*proof*⟩

lemma *used'_parts [rule_format]: "X ∈ used' evs ⟹ Y ∈ parts {X} ⟶ Y ∈ used' evs"*
⟨*proof*⟩

29.4.10 monotonicity of used

lemma *used_sub_Cons: "used evs <= used (ev#evs)"*
⟨*proof*⟩

lemma *used_ConsI: "X ∈ used evs ⟹ X ∈ used (ev#evs)"*
⟨*proof*⟩

lemma *notin_used_ConsD: "X ∉ used (ev#evs) ⟹ X ∉ used evs"*
⟨*proof*⟩

lemma *used_appD [dest]: "X ∈ used (evs @ evs') ⟹ X ∈ used evs ∨ X ∈ used evs'"*
⟨*proof*⟩

lemma *used_ConsD: "X ∈ used (ev#evs) ⟹ X ∈ used [ev] ∨ X ∈ used evs"*
⟨*proof*⟩

lemma *used_sub_app: "used evs <= used (evs@evs')"*
⟨*proof*⟩

lemma *used_appIL: "X ∈ used evs ⟹ X ∈ used (evs' @ evs)"*
⟨*proof*⟩

lemma *used_appIR: "X ∈ used evs ⟹ X ∈ used (evs @ evs')"*
⟨*proof*⟩

lemma *used_parts: "⟦X ∈ parts {Y}; Y ∈ used evs⟧ ⟹ X ∈ used evs"*
⟨*proof*⟩

lemma *parts_Says_used: "⟦Says A B X ∈ set evs; Y ∈ parts {X}⟧ ⟹ Y ∈ used evs"*
⟨*proof*⟩

lemma *parts_used_app: "X ∈ parts {Y} ⟹ X ∈ used (evs @ Says A B Y # evs')"*
⟨*proof*⟩

29.4.11 lemmas on used and knows

lemma *initState_used: "X ∈ parts (initState A) ⟹ X ∈ used evs"*
⟨*proof*⟩

lemma *Says_imp_used: "Says A B X ∈ set evs ⟹ parts {X} ⊆ used evs"*
⟨*proof*⟩

lemma *not_used_not_spied: "X ∉ used evs ⟹ X ∉ parts (spies evs)"*
⟨*proof*⟩

lemma *not_used_not_parts: "⟦Y ∉ used evs; Says A B X ∈ set evs⟧*
⟹ Y ∉ parts {X}"
⟨*proof*⟩

lemma *not_used_parts_false: "⟦X ∉ used evs; Y ∈ parts (spies evs)⟧*
⟹ X ∉ parts {Y}"
⟨*proof*⟩

lemma *known_used [rule_format]: "⟦evs ∈ p; Gets_correct p; one_step p⟧*
⟹ X ∈ parts (knows A evs) ⟶ X ∈ used evs"
⟨*proof*⟩

lemma *known_max_used [rule_format]: "⟦evs ∈ p; Gets_correct p; one_step*
p⟧
⟹ X ∈ parts (knows_max A evs) ⟶ X ∈ used evs"
⟨*proof*⟩

lemma *not_used_not_known: "⟦evs ∈ p; X ∉ used evs;*
Gets_correct p; one_step p⟧ ⟹ X ∉ parts (knows A evs)"
⟨*proof*⟩

lemma *not_used_not_known_max: "⟦evs ∈ p; X ∉ used evs;*
Gets_correct p; one_step p⟧ ⟹ X ∉ parts (knows_max A evs)"
⟨*proof*⟩

29.4.12 a nonce or key in a message cannot equal a fresh nonce or key

lemma *Nonce_neq [dest]: "⟦Nonce n' ∉ used evs;*
Says A B X ∈ set evs; Nonce n ∈ parts {X}⟧ ⟹ n ≠ n'"
⟨*proof*⟩

lemma *Key_neq [dest]: "⟦Key n' ∉ used evs;*
Says A B X ∈ set evs; Key n ∈ parts {X}⟧ ⟹ n ~= n'"
⟨*proof*⟩

29.4.13 message of an event

primrec *msg :: "event => msg"*
where
  "msg (Says A B X) = X"

```
| "msg (Gets A X) = X"
| "msg (Notes A X) = X"
```

lemma *used_sub_parts_used: "X ∈ used (ev # evs) ⟹ X ∈ parts {msg ev} ∪ used evs"*
⟨*proof*⟩

end

30 Decomposition of Analz into two parts

theory *Analz* **imports** *Extensions* **begin**

decomposition of *analz* into two parts: *pparts* (for pairs) and analz of *kparts*

30.1 messages that do not contribute to analz

```
inductive_set
  pparts :: "msg set => msg set"
  for H :: "msg set"
where
  Inj [intro]: "⟦X ∈ H; is_MPair X⟧ ⟹ X ∈ pparts H"
| Fst [dest]: "⟦⦃X,Y⦄ ∈ pparts H; is_MPair X⟧ ⟹ X ∈ pparts H"
| Snd [dest]: "⟦⦃X,Y⦄ ∈ pparts H; is_MPair Y⟧ ⟹ Y ∈ pparts H"
```

30.2 basic facts about *pparts*

lemma *pparts_is_MPair [dest]: "X ∈ pparts H ⟹ is_MPair X"*
⟨*proof*⟩

lemma *Crypt_notin_pparts [iff]: "Crypt K X ∉ pparts H"*
⟨*proof*⟩

lemma *Key_notin_pparts [iff]: "Key K ∉ pparts H"*
⟨*proof*⟩

lemma *Nonce_notin_pparts [iff]: "Nonce n ∉ pparts H"*
⟨*proof*⟩

lemma *Number_notin_pparts [iff]: "Number n ∉ pparts H"*
⟨*proof*⟩

lemma *Agent_notin_pparts [iff]: "Agent A ∉ pparts H"*
⟨*proof*⟩

lemma *pparts_empty [iff]: "pparts {} = {}"*
⟨*proof*⟩

lemma *pparts_insertI [intro]: "X ∈ pparts H ⟹ X ∈ pparts (insert Y H)"*
⟨*proof*⟩

lemma *pparts_sub: "⟦X ∈ pparts G; G ⊆ H⟧ ⟹ X ∈ pparts H"*
⟨*proof*⟩

lemma *pparts_insert2 [iff]: "pparts (insert X (insert Y H))
= pparts {X} Un pparts {Y} Un pparts H"*
⟨*proof*⟩

lemma *pparts_insert_MPair [iff]: "pparts (insert {|X,Y|} H)
= insert {|X,Y|} (pparts ({X,Y} ∪ H))"*
⟨*proof*⟩

lemma *pparts_insert_Nonce [iff]: "pparts (insert (Nonce n) H) = pparts H"*
⟨*proof*⟩

lemma *pparts_insert_Crypt [iff]: "pparts (insert (Crypt K X) H) = pparts
H"*
⟨*proof*⟩

lemma *pparts_insert_Key [iff]: "pparts (insert (Key K) H) = pparts H"*
⟨*proof*⟩

lemma *pparts_insert_Agent [iff]: "pparts (insert (Agent A) H) = pparts H"*
⟨*proof*⟩

lemma *pparts_insert_Number [iff]: "pparts (insert (Number n) H) = pparts
H"*
⟨*proof*⟩

lemma *pparts_insert_Hash [iff]: "pparts (insert (Hash X) H) = pparts H"*
⟨*proof*⟩

lemma *pparts_insert: "X ∈ pparts (insert Y H) ⟹ X ∈ pparts {Y} ∪ pparts
H"*
⟨*proof*⟩

lemma *insert_pparts: "X ∈ pparts {Y} ∪ pparts H ⟹ X ∈ pparts (insert
Y H)"*
⟨*proof*⟩

lemma *pparts_Un [iff]: "pparts (G ∪ H) = pparts G ∪ pparts H"*
⟨*proof*⟩

lemma *pparts_pparts [iff]: "pparts (pparts H) = pparts H"*
⟨*proof*⟩

lemma *pparts_insert_eq: "pparts (insert X H) = pparts {X} Un pparts H"*
⟨*proof*⟩

lemmas *pparts_insert_substI = pparts_insert_eq [THEN ssubst]*

lemma *in_pparts: "Y ∈ pparts H ⟹ ∃X. X ∈ H ∧ Y ∈ pparts {X}"*
⟨*proof*⟩

30.3 facts about *pparts* and *parts*

lemma *pparts_no_Nonce [dest]: "⟦X ∈ pparts {Y}; Nonce n ∉ parts {Y}⟧*

⟹ *Nonce n ∉ parts {X}"*
⟨*proof*⟩

30.4 facts about *pparts* and *analz*

lemma *pparts_analz: "X ∈ pparts H ⟹ X ∈ analz H"*
⟨*proof*⟩

lemma *pparts_analz_sub: "⟦X ∈ pparts G; G ⊆ H⟧ ⟹ X ∈ analz H"*
⟨*proof*⟩

30.5 messages that contribute to analz

inductive_set
  kparts :: "msg set => msg set"
  for *H :: "msg set"*
where
  Inj [intro]: "⟦X ∈ H; not_MPair X⟧ ⟹ X ∈ kparts H"
| Fst [intro]: "⟦⦃X,Y⦄ ∈ pparts H; not_MPair X⟧ ⟹ X ∈ kparts H"
| Snd [intro]: "⟦⦃X,Y⦄ ∈ pparts H; not_MPair Y⟧ ⟹ Y ∈ kparts H"

30.6 basic facts about *kparts*

lemma *kparts_not_MPair [dest]: "X ∈ kparts H ⟹ not_MPair X"*
⟨*proof*⟩

lemma *kparts_empty [iff]: "kparts {} = {}"*
⟨*proof*⟩

lemma *kparts_insertI [intro]: "X ∈ kparts H ⟹ X ∈ kparts (insert Y H)"*
⟨*proof*⟩

lemma *kparts_insert2 [iff]: "kparts (insert X (insert Y H))*
= kparts {X} ∪ kparts {Y} ∪ kparts H"
⟨*proof*⟩

lemma *kparts_insert_MPair [iff]: "kparts (insert ⦃X,Y⦄ H)*
= kparts ({X,Y} ∪ H)"
⟨*proof*⟩

lemma *kparts_insert_Nonce [iff]: "kparts (insert (Nonce n) H)*
= insert (Nonce n) (kparts H)"
⟨*proof*⟩

lemma *kparts_insert_Crypt [iff]: "kparts (insert (Crypt K X) H)*
= insert (Crypt K X) (kparts H)"
⟨*proof*⟩

lemma *kparts_insert_Key [iff]: "kparts (insert (Key K) H)*
= insert (Key K) (kparts H)"
⟨*proof*⟩

lemma *kparts_insert_Agent [iff]: "kparts (insert (Agent A) H)*
= insert (Agent A) (kparts H)"

⟨*proof*⟩

lemma *kparts_insert_Number [iff]: "kparts (insert (Number n) H)
= insert (Number n) (kparts H)"*
⟨*proof*⟩

lemma *kparts_insert_Hash [iff]: "kparts (insert (Hash X) H)
= insert (Hash X) (kparts H)"*
⟨*proof*⟩

lemma *kparts_insert: "X ∈ kparts (insert X H) ⟹ X ∈ kparts {X} ∪ kparts
H"*
⟨*proof*⟩

lemma *kparts_insert_fst [rule_format,dest]: "X ∈ kparts (insert Z H) ⟹
X ∉ kparts H ⟶ X ∈ kparts {Z}"*
⟨*proof*⟩

lemma *kparts_sub: "⟦X ∈ kparts G; G ⊆ H⟧ ⟹ X ∈ kparts H"*
⟨*proof*⟩

lemma *kparts_Un [iff]: "kparts (G ∪ H) = kparts G ∪ kparts H"*
⟨*proof*⟩

lemma *pparts_kparts [iff]: "pparts (kparts H) = {}"*
⟨*proof*⟩

lemma *kparts_kparts [iff]: "kparts (kparts H) = kparts H"*
⟨*proof*⟩

lemma *kparts_insert_eq: "kparts (insert X H) = kparts {X} ∪ kparts H"*
⟨*proof*⟩

lemmas *kparts_insert_substI = kparts_insert_eq [THEN ssubst]*

lemma *in_kparts: "Y ∈ kparts H ⟹ ∃X. X ∈ H ∧ Y ∈ kparts {X}"*
⟨*proof*⟩

lemma *kparts_has_no_pair [iff]: "has_no_pair (kparts H)"*
⟨*proof*⟩

30.7 facts about *kparts* and *parts*

lemma *kparts_no_Nonce [dest]: "⟦X ∈ kparts {Y}; Nonce n ∉ parts {Y}⟧
⟹ Nonce n ∉ parts {X}"*
⟨*proof*⟩

lemma *kparts_parts: "X ∈ kparts H ⟹ X ∈ parts H"*
⟨*proof*⟩

lemma *parts_kparts: "X ∈ parts (kparts H) ⟹ X ∈ parts H"*
⟨*proof*⟩

lemma *Crypt_kparts_Nonce_parts [dest]: "⟦Crypt K Y ∈ kparts {Z};*

Nonce n ∈ parts {Y}⟧ ⟹ Nonce n ∈ parts {Z}"
⟨proof⟩

30.8 facts about kparts and analz

lemma *kparts_analz: "X ∈ kparts H ⟹ X ∈ analz H"*
⟨proof⟩

lemma *kparts_analz_sub: "⟦X ∈ kparts G; G ⊆ H⟧ ⟹ X ∈ analz H"*
⟨proof⟩

lemma *analz_kparts [rule_format,dest]: "X ∈ analz H ⟹*
Y ∈ kparts {X} ⟶ Y ∈ analz H"
⟨proof⟩

lemma *analz_kparts_analz: "X ∈ analz (kparts H) ⟹ X ∈ analz H"*
⟨proof⟩

lemma *analz_kparts_insert: "X ∈ analz (kparts (insert Z H)) ⟹ X ∈ analz*
(kparts {Z} ∪ kparts H)"
⟨proof⟩

lemma *Nonce_kparts_synth [rule_format]: "Y ∈ synth (analz G)*
⟹ Nonce n ∈ kparts {Y} ⟶ Nonce n ∈ analz G"
⟨proof⟩

lemma *kparts_insert_synth: "⟦Y ∈ parts (insert X G); X ∈ synth (analz G);*
Nonce n ∈ kparts {Y}; Nonce n ∉ analz G⟧ ⟹ Y ∈ parts G"
⟨proof⟩

lemma *Crypt_insert_synth:*
"⟦Crypt K Y ∈ parts (insert X G); X ∈ synth (analz G); Nonce n ∈ kparts
{Y}; Nonce n ∉ analz G⟧
⟹ Crypt K Y ∈ parts G"
⟨proof⟩

30.9 analz is pparts + analz of kparts

lemma *analz_pparts_kparts: "X ∈ analz H ⟹ X ∈ pparts H ∨ X ∈ analz (kparts*
H)"
⟨proof⟩

lemma *analz_pparts_kparts_eq: "analz H = pparts H Un analz (kparts H)"*
⟨proof⟩

lemmas *analz_pparts_kparts_substI = analz_pparts_kparts_eq [THEN ssubst]*
lemmas *analz_pparts_kparts_substD = analz_pparts_kparts_eq [THEN sym, THEN*
ssubst]

end

31 Protocol-Independent Confidentiality Theorem on Nonces

theory *Guard* **imports** *Analz Extensions* **begin**

inductive_set
  guard :: "nat ⇒ key set ⇒ msg set"
  for *n :: nat* **and** *Ks :: "key set"*
where
  No_Nonce [intro]: "Nonce n ∉ parts {X} ⟹ X ∈ guard n Ks"
| Guard_Nonce [intro]: "invKey K ∈ Ks ⟹ Crypt K X ∈ guard n Ks"
| Crypt [intro]: "X ∈ guard n Ks ⟹ Crypt K X ∈ guard n Ks"
| Pair [intro]: "⟦X ∈ guard n Ks; Y ∈ guard n Ks⟧ ⟹ ⦃X,Y⦄ ∈ guard n Ks"

31.1 basic facts about *guard*

lemma *Key_is_guard [iff]: "Key K ∈ guard n Ks"*
⟨*proof*⟩

lemma *Agent_is_guard [iff]: "Agent A ∈ guard n Ks"*
⟨*proof*⟩

lemma *Number_is_guard [iff]: "Number r ∈ guard n Ks"*
⟨*proof*⟩

lemma *Nonce_notin_guard: "X ∈ guard n Ks ⟹ X ≠ Nonce n"*
⟨*proof*⟩

lemma *Nonce_notin_guard_iff [iff]: "Nonce n ∉ guard n Ks"*
⟨*proof*⟩

lemma *guard_has_Crypt [rule_format]: "X ∈ guard n Ks ⟹ Nonce n ∈ parts {X}*
⟶ (∃K Y. Crypt K Y ∈ kparts {X} ∧ Nonce n ∈ parts {Y})"
⟨*proof*⟩

lemma *Nonce_notin_kparts_msg: "X ∈ guard n Ks ⟹ Nonce n ∉ kparts {X}"*
⟨*proof*⟩

lemma *Nonce_in_kparts_imp_no_guard: "Nonce n ∈ kparts H*
⟹ ∃X. X ∈ H ∧ X ∉ guard n Ks"
⟨*proof*⟩

lemma *guard_kparts [rule_format]: "X ∈ guard n Ks ⟹*
Y ∈ kparts {X} ⟶ Y ∈ guard n Ks"
⟨*proof*⟩

lemma *guard_Crypt: "⟦Crypt K Y ∈ guard n Ks; K ∉ invKey‘Ks⟧ ⟹ Y ∈ guard n Ks"*
  ⟨*proof*⟩

lemma *guard_MPair [iff]: "(⦃X,Y⦄ ∈ guard n Ks) = (X ∈ guard n Ks ∧ Y ∈*

guard n Ks)"
⟨*proof*⟩

lemma *guard_not_guard [rule_format]: "X ∈ guard n Ks ⟹
Crypt K Y ∈ kparts {X} ⟶ Nonce n ∈ kparts {Y} ⟶ Y ∉ guard n Ks"*
⟨*proof*⟩

lemma *guard_extand: "⟦X ∈ guard n Ks; Ks ⊆ Ks'⟧ ⟹ X ∈ guard n Ks'"*
⟨*proof*⟩

31.2 guarded sets

definition *Guard :: "nat ⇒ key set ⇒ msg set ⇒ bool"* **where**
"Guard n Ks H ≡ ∀X. X ∈ H ⟶ X ∈ guard n Ks"

31.3 basic facts about *Guard*

lemma *Guard_empty [iff]: "Guard n Ks {}"*
⟨*proof*⟩

lemma *notin_parts_Guard [intro]: "Nonce n ∉ parts G ⟹ Guard n Ks G"*
⟨*proof*⟩

lemma *Nonce_notin_kparts [simplified]: "Guard n Ks H ⟹ Nonce n ∉ kparts H"*
⟨*proof*⟩

lemma *Guard_must_decrypt: "⟦Guard n Ks H; Nonce n ∈ analz H⟧ ⟹
∃K Y. Crypt K Y ∈ kparts H ∧ Key (invKey K) ∈ kparts H"*
⟨*proof*⟩

lemma *Guard_kparts [intro]: "Guard n Ks H ⟹ Guard n Ks (kparts H)"*
⟨*proof*⟩

lemma *Guard_mono: "⟦Guard n Ks H; G <= H⟧ ⟹ Guard n Ks G"*
⟨*proof*⟩

lemma *Guard_insert [iff]: "Guard n Ks (insert X H)
= (Guard n Ks H ∧ X ∈ guard n Ks)"*
⟨*proof*⟩

lemma *Guard_Un [iff]: "Guard n Ks (G Un H) = (Guard n Ks G & Guard n Ks H)"*
⟨*proof*⟩

lemma *Guard_synth [intro]: "Guard n Ks G ⟹ Guard n Ks (synth G)"*
⟨*proof*⟩

lemma *Guard_analz [intro]: "⟦Guard n Ks G; ∀K. K ∈ Ks ⟶ Key K ∉ analz G⟧
⟹ Guard n Ks (analz G)"*
⟨*proof*⟩

lemma *in_Guard [dest]: "⟦X ∈ G; Guard n Ks G⟧ ⟹ X ∈ guard n Ks"*
⟨*proof*⟩

lemma *in_synth_Guard: "⟦X ∈ synth G; Guard n Ks G⟧ ⟹ X ∈ guard n Ks"*
⟨*proof*⟩

lemma *in_analz_Guard: "⟦X ∈ analz G; Guard n Ks G;*
∀K. K ∈ Ks ⟶ Key K ∉ analz G⟧ ⟹ X ∈ guard n Ks"
⟨*proof*⟩

lemma *Guard_keyset [simp]: "keyset G ⟹ Guard n Ks G"*
⟨*proof*⟩

lemma *Guard_Un_keyset: "⟦Guard n Ks G; keyset H⟧ ⟹ Guard n Ks (G ∪ H)"*
⟨*proof*⟩

lemma *in_Guard_kparts: "⟦X ∈ G; Guard n Ks G; Y ∈ kparts {X}⟧ ⟹ Y ∈ guard n Ks"*
⟨*proof*⟩

lemma *in_Guard_kparts_neq: "⟦X ∈ G; Guard n Ks G; Nonce n' ∈ kparts {X}⟧ ⟹ n ≠ n'"*
⟨*proof*⟩

lemma *in_Guard_kparts_Crypt: "⟦X ∈ G; Guard n Ks G; is_MPair X;*
Crypt K Y ∈ kparts {X}; Nonce n ∈ kparts {Y}⟧ ⟹ invKey K ∈ Ks"
⟨*proof*⟩

lemma *Guard_extand: "⟦Guard n Ks G; Ks ⊆ Ks'⟧ ⟹ Guard n Ks' G"*
⟨*proof*⟩

lemma *guard_invKey [rule_format]: "⟦X ∈ guard n Ks; Nonce n ∈ kparts {Y}⟧ ⟹*
Crypt K Y ∈ kparts {X} ⟶ invKey K ∈ Ks"
⟨*proof*⟩

lemma *Crypt_guard_invKey [rule_format]: "⟦Crypt K Y ∈ guard n Ks;*
Nonce n ∈ kparts {Y}⟧ ⟹ invKey K ∈ Ks"
⟨*proof*⟩

31.4 set obtained by decrypting a message

abbreviation *(input)*
  decrypt :: "msg set => key => msg => msg set" **where**
  "decrypt H K Y == insert Y (H - {Crypt K Y})"

lemma *analz_decrypt: "⟦Crypt K Y ∈ H; Key (invKey K) ∈ H; Nonce n ∈ analz H⟧*
⟹ Nonce n ∈ analz (decrypt H K Y)"
⟨*proof*⟩

lemma *parts_decrypt: "⟦Crypt K Y ∈ H; X ∈ parts (decrypt H K Y)⟧ ⟹ X ∈ parts H"*
⟨*proof*⟩

31.5 number of Crypt's in a message

fun *crypt_nb :: "msg => nat"*
where
"crypt_nb (Crypt K X) = Suc (crypt_nb X)"
| "crypt_nb {|X,Y|} = crypt_nb X + crypt_nb Y"
| "crypt_nb X = 0"

31.6 basic facts about *crypt_nb*

lemma *non_empty_crypt_msg: "Crypt K Y ∈ parts {X} ⟹ crypt_nb X ≠ 0"*
⟨*proof*⟩

31.7 number of Crypt's in a message list

primrec *cnb :: "msg list => nat"*
where
"cnb [] = 0"
| "cnb (X#l) = crypt_nb X + cnb l"

31.8 basic facts about *cnb*

lemma *cnb_app [simp]: "cnb (l @ l') = cnb l + cnb l'"*
⟨*proof*⟩

lemma *mem_cnb_minus: "x ∈ set l ⟹ cnb l = crypt_nb x + (cnb l - crypt_nb x)"*
⟨*proof*⟩

lemmas *mem_cnb_minus_substI = mem_cnb_minus [THEN ssubst]*

lemma *cnb_minus [simp]: "x ∈ set l ⟹ cnb (remove l x) = cnb l - crypt_nb x"*
⟨*proof*⟩

lemma *parts_cnb: "Z ∈ parts (set l) ⟹*
cnb l = (cnb l - crypt_nb Z) + crypt_nb Z"
⟨*proof*⟩

lemma *non_empty_crypt: "Crypt K Y ∈ parts (set l) ⟹ cnb l ≠ 0"*
⟨*proof*⟩

31.9 list of kparts

lemma *kparts_msg_set: "∃l. kparts {X} = set l ∧ cnb l = crypt_nb X"*
⟨*proof*⟩

lemma *kparts_set: "∃l'. kparts (set l) = set l' ∧ cnb l' = cnb l"*
⟨*proof*⟩

31.10 list corresponding to "decrypt"

definition *decrypt' :: "msg list => key => msg => msg list"* **where**
"decrypt' l K Y == Y # remove l (Crypt K Y)"

declare *decrypt'_def [simp]*

31.11 basic facts about *decrypt'*

lemma *decrypt_minus: "decrypt (set l) K Y <= set (decrypt' l K Y)"*
⟨*proof*⟩

31.12 if the analyse of a finite guarded set gives n then it must also gives one of the keys of Ks

lemma *Guard_invKey_by_list [rule_format]: "∀l. cnb l = p*
⟶ Guard n Ks (set l) ⟶ Nonce n ∈ analz (set l)
⟶ (∃K. K ∈ Ks ∧ Key K ∈ analz (set l))"
⟨*proof*⟩

lemma *Guard_invKey_finite: "⟦Nonce n ∈ analz G; Guard n Ks G; finite G⟧*
⟹ ∃K. K ∈ Ks ∧ Key K ∈ analz G"
⟨*proof*⟩

lemma *Guard_invKey: "⟦Nonce n ∈ analz G; Guard n Ks G⟧*
⟹ ∃K. K ∈ Ks ∧ Key K ∈ analz G"
⟨*proof*⟩

31.13 if the analyse of a finite guarded set and a (possibly infinite) set of keys gives n then it must also gives Ks

lemma *Guard_invKey_keyset: "⟦Nonce n ∈ analz (G ∪ H); Guard n Ks G; finite G;*
keyset H⟧ ⟹ ∃K. K ∈ Ks ∧ Key K ∈ analz (G ∪ H)"
⟨*proof*⟩

end

32 protocol-independent confidentiality theorem on keys

theory *GuardK*
imports *Analz Extensions*
begin

inductive_set
  guardK :: "nat => key set => msg set"
  for *n :: nat* **and** *Ks :: "key set"*
where
  No_Key [intro]: "Key n ∉ parts {X} ⟹ X ∈ guardK n Ks"
| Guard_Key [intro]: "invKey K ∈ Ks ⟹ Crypt K X ∈ guardK n Ks"
| Crypt [intro]: "X ∈ guardK n Ks ⟹ Crypt K X ∈ guardK n Ks"
| Pair [intro]: "⟦X ∈ guardK n Ks; Y ∈ guardK n Ks⟧ ⟹ ⦃X,Y⦄ ∈ guardK n Ks"

32.1 basic facts about *guardK*

lemma *Nonce_is_guardK [iff]: "Nonce p ∈ guardK n Ks"*
⟨*proof*⟩

lemma *Agent_is_guardK [iff]: "Agent A ∈ guardK n Ks"*
⟨*proof*⟩

lemma *Number_is_guardK [iff]: "Number r ∈ guardK n Ks"*
⟨*proof*⟩

lemma *Key_notin_guardK: "X ∈ guardK n Ks ⟹ X ≠ Key n"*
⟨*proof*⟩

lemma *Key_notin_guardK_iff [iff]: "Key n ∉ guardK n Ks"*
⟨*proof*⟩

lemma *guardK_has_Crypt [rule_format]: "X ∈ guardK n Ks ⟹ Key n ∈ parts {X}*
⟶ (∃K Y. Crypt K Y ∈ kparts {X} ∧ Key n ∈ parts {Y})"
⟨*proof*⟩

lemma *Key_notin_kparts_msg: "X ∈ guardK n Ks ⟹ Key n ∉ kparts {X}"*
⟨*proof*⟩

lemma *Key_in_kparts_imp_no_guardK: "Key n ∈ kparts H*
⟹ ∃X. X ∈ H ∧ X ∉ guardK n Ks"
⟨*proof*⟩

lemma *guardK_kparts [rule_format]: "X ∈ guardK n Ks ⟹*
Y ∈ kparts {X} ⟶ Y ∈ guardK n Ks"
⟨*proof*⟩

lemma *guardK_Crypt: "⟦Crypt K Y ∈ guardK n Ks; K ∉ invKey‘Ks⟧ ⟹ Y ∈ guardK n Ks"*
⟨*proof*⟩

lemma *guardK_MPair [iff]: "(⦃X,Y⦄ ∈ guardK n Ks)*
= (X ∈ guardK n Ks ∧ Y ∈ guardK n Ks)"
⟨*proof*⟩

lemma *guardK_not_guardK [rule_format]: "X ∈guardK n Ks ⟹*
Crypt K Y ∈ kparts {X} ⟶ Key n ∈ kparts {Y} ⟶ Y ∉ guardK n Ks"
⟨*proof*⟩

lemma *guardK_extand: "⟦X ∈ guardK n Ks; Ks ⊆ Ks’;*
⟦K ∈ Ks’; K ∉ Ks⟧ ⟹ Key K ∉ parts {X}⟧ ⟹ X ∈ guardK n Ks’"
⟨*proof*⟩

32.2 guarded sets

definition *GuardK :: "nat ⇒ key set ⇒ msg set ⇒ bool"* **where**
"GuardK n Ks H ≡ ∀X. X ∈ H ⟶ X ∈ guardK n Ks"

32.3 basic facts about *GuardK*

lemma *GuardK_empty [iff]: "GuardK n Ks {}"*
⟨*proof*⟩

lemma *Key_notin_kparts [simplified]: "GuardK n Ks H ⟹ Key n ∉ kparts H"*
⟨*proof*⟩

lemma *GuardK_must_decrypt: "⟦GuardK n Ks H; Key n ∈ analz H⟧ ⟹*
∃K Y. Crypt K Y ∈ kparts H ∧ Key (invKey K) ∈ kparts H"
⟨*proof*⟩

lemma *GuardK_kparts [intro]: "GuardK n Ks H ⟹ GuardK n Ks (kparts H)"*
⟨*proof*⟩

lemma *GuardK_mono: "⟦GuardK n Ks H; G ⊆ H⟧ ⟹ GuardK n Ks G"*
⟨*proof*⟩

lemma *GuardK_insert [iff]: "GuardK n Ks (insert X H)*
= (GuardK n Ks H ∧ X ∈ guardK n Ks)"
⟨*proof*⟩

lemma *GuardK_Un [iff]: "GuardK n Ks (G Un H) = (GuardK n Ks G & GuardK n Ks H)"*
⟨*proof*⟩

lemma *GuardK_synth [intro]: "GuardK n Ks G ⟹ GuardK n Ks (synth G)"*
⟨*proof*⟩

lemma *GuardK_analz [intro]: "⟦GuardK n Ks G; ∀K. K ∈ Ks ⟶ Key K ∉ analz G⟧*
⟹ GuardK n Ks (analz G)"
⟨*proof*⟩

lemma *in_GuardK [dest]: "⟦X ∈ G; GuardK n Ks G⟧ ⟹ X ∈ guardK n Ks"*
⟨*proof*⟩

lemma *in_synth_GuardK: "⟦X ∈ synth G; GuardK n Ks G⟧ ⟹ X ∈ guardK n Ks"*
⟨*proof*⟩

lemma *in_analz_GuardK: "⟦X ∈ analz G; GuardK n Ks G;*
∀K. K ∈ Ks ⟶ Key K ∉ analz G⟧ ⟹ X ∈ guardK n Ks"
⟨*proof*⟩

lemma *GuardK_keyset [simp]: "⟦keyset G; Key n ∉ G⟧ ⟹ GuardK n Ks G"*
⟨*proof*⟩

lemma *GuardK_Un_keyset: "⟦GuardK n Ks G; keyset H; Key n ∉ H⟧*
⟹ GuardK n Ks (G Un H)"
⟨*proof*⟩

lemma *in_GuardK_kparts: "⟦X ∈ G; GuardK n Ks G; Y ∈ kparts {X}⟧ ⟹ Y ∈ guardK n Ks"*
⟨*proof*⟩

lemma *in_GuardK_kparts_neq: "⟦X ∈ G; GuardK n Ks G; Key n' ∈ kparts {X}⟧ ⟹ n ≠ n'"*
⟨*proof*⟩

lemma *in_GuardK_kparts_Crypt: "⟦X ∈ G; GuardK n Ks G; is_MPair X; Crypt K Y ∈ kparts {X}; Key n ∈ kparts {Y}⟧ ⟹ invKey K ∈ Ks"*
⟨*proof*⟩

lemma *GuardK_extand: "⟦GuardK n Ks G; Ks ⊆ Ks'; ⟦K ∈ Ks'; K ∉ Ks⟧ ⟹ Key K ∉ parts G⟧ ⟹ GuardK n Ks' G"*
⟨*proof*⟩

32.4 set obtained by decrypting a message

abbreviation *(input)*
decrypt :: "msg set ⇒ key ⇒ msg ⇒ msg set" **where**
"decrypt H K Y ≡ insert Y (H - {Crypt K Y})"

lemma *analz_decrypt: "⟦Crypt K Y ∈ H; Key (invKey K) ∈ H; Key n ∈ analz H⟧ ⟹ Key n ∈ analz (decrypt H K Y)"*
⟨*proof*⟩

lemma *parts_decrypt: "⟦Crypt K Y ∈ H; X ∈ parts (decrypt H K Y)⟧ ⟹ X ∈ parts H"*
⟨*proof*⟩

32.5 number of Crypt's in a message

fun *crypt_nb :: "msg => nat"* **where**
"crypt_nb (Crypt K X) = Suc (crypt_nb X)" |
"crypt_nb {|X,Y|} = crypt_nb X + crypt_nb Y" |
"crypt_nb X = 0"

32.6 basic facts about *crypt_nb*

lemma *non_empty_crypt_msg: "Crypt K Y ∈ parts {X} ⟹ crypt_nb X ≠ 0"*
⟨*proof*⟩

32.7 number of Crypt's in a message list

primrec *cnb :: "msg list => nat"* **where**
"cnb [] = 0" |
"cnb (X#l) = crypt_nb X + cnb l"

32.8 basic facts about *cnb*

lemma *cnb_app [simp]: "cnb (l @ l') = cnb l + cnb l'"*
⟨*proof*⟩

lemma *mem_cnb_minus: "x ∈ set l ⟹ cnb l = crypt_nb x + (cnb l - crypt_nb x)"*
⟨*proof*⟩

lemmas *mem_cnb_minus_substI = mem_cnb_minus [THEN ssubst]*

lemma *cnb_minus [simp]: "x ∈ set l ⟹ cnb (remove l x) = cnb l - crypt_nb x"*
⟨*proof*⟩

lemma *parts_cnb: "Z ∈ parts (set l) ⟹
cnb l = (cnb l - crypt_nb Z) + crypt_nb Z"*
⟨*proof*⟩

lemma *non_empty_crypt: "Crypt K Y ∈ parts (set l) ⟹ cnb l ≠ 0"*
⟨*proof*⟩

32.9 list of kparts

lemma *kparts_msg_set: "∃l. kparts {X} = set l ∧ cnb l = crypt_nb X"*
⟨*proof*⟩

lemma *kparts_set: "∃l'. kparts (set l) = set l' & cnb l' = cnb l"*
⟨*proof*⟩

32.10 list corresponding to "decrypt"

definition *decrypt' :: "msg list => key => msg => msg list"* **where**
"decrypt' l K Y == Y # remove l (Crypt K Y)"

declare *decrypt'_def [simp]*

32.11 basic facts about *decrypt'*

lemma *decrypt_minus: "decrypt (set l) K Y <= set (decrypt' l K Y)"*
⟨*proof*⟩

if the analysis of a finite guarded set gives n then it must also give one of the keys of Ks

lemma *GuardK_invKey_by_list [rule_format]: "∀l. cnb l = p
⟶ GuardK n Ks (set l) ⟶ Key n ∈ analz (set l)
⟶ (∃K. K ∈ Ks ∧ Key K ∈ analz (set l))"*
⟨*proof*⟩

lemma *GuardK_invKey_finite: "⟦Key n ∈ analz G; GuardK n Ks G; finite G⟧
⟹ ∃K. K ∈ Ks ∧ Key K ∈ analz G"*
⟨*proof*⟩

lemma *GuardK_invKey: "⟦Key n ∈ analz G; GuardK n Ks G⟧
⟹ ∃K. K ∈ Ks ∧ Key K ∈ analz G"*
⟨*proof*⟩

if the analyse of a finite guarded set and a (possibly infinite) set of keys gives n then it must also gives Ks

lemma *GuardK_invKey_keyset: "⟦Key n ∈ analz (G ∪ H); GuardK n Ks G; finite G;
keyset H; Key n ∉ H⟧ ⟹ ∃K. K ∈ Ks ∧ Key K ∈ analz (G ∪ H)"*

⟨proof⟩

end

theory *Shared*
imports *Event All_Symmetric*
begin

consts
```
  shrK    :: "agent ⇒ key"
```

specification *(shrK)*
```
  inj_shrK: "inj shrK"
```
— No two agents have the same long-term key
⟨proof⟩

Server knows all long-term keys; other agents know only their own

overloading
```
  initState ≡ initState
```
begin

primrec *initState* **where**
```
  initState_Server:  "initState Server     = Key ‘ range shrK"
| initState_Friend:  "initState (Friend i) = {Key (shrK (Friend i))}"
| initState_Spy:     "initState Spy        = Key‘shrK‘bad"
```

end

32.12 Basic properties of shrK

lemmas *shrK_injective = inj_shrK [THEN inj_eq]*
declare *shrK_injective [iff]*

lemma *invKey_K [simp]: "invKey K = K"*
⟨proof⟩

lemma *analz_Decrypt' [dest]:*
```
     "⟦Crypt K X ∈ analz H;  Key K  ∈ analz H⟧ ⟹ X ∈ analz H"
```
⟨proof⟩

Now cancel the *dest* attribute given to *analz.Decrypt* in its declaration.

declare *analz.Decrypt [rule del]*

Rewrites should not refer to *initState (Friend i)* because that expression is not in normal form.

lemma *keysFor_parts_initState [simp]: "keysFor (parts (initState C)) = {}"*
⟨proof⟩

lemma *keysFor_parts_insert:*
```
     "⟦K ∈ keysFor (parts (insert X G));  X ∈ synth (analz H)⟧
```

    ⟹ K ∈ keysFor (parts (G ∪ H)) | Key K ∈ parts H"
⟨proof⟩

lemma *Crypt_imp_keysFor: "Crypt K X ∈ H ⟹ K ∈ keysFor H"*
⟨proof⟩

32.13 Function "knows"

lemma *Spy_knows_Spy_bad [intro!]: "A ∈ bad ⟹ Key (shrK A) ∈ knows Spy evs"*
⟨proof⟩

lemma *Crypt_Spy_analz_bad: "⟦Crypt (shrK A) X ∈ analz (knows Spy evs); A ∈ bad⟧ ⟹ X ∈ analz (knows Spy evs)"*
⟨proof⟩

lemma *shrK_in_initState [iff]: "Key (shrK A) ∈ initState A"*
⟨proof⟩

lemma *shrK_in_used [iff]: "Key (shrK A) ∈ used evs"*
⟨proof⟩

lemma *Key_not_used [simp]: "Key K ∉ used evs ⟹ K ∉ range shrK"*
⟨proof⟩

lemma *shrK_neq [simp]: "Key K ∉ used evs ⟹ shrK B ≠ K"*
⟨proof⟩

lemmas *shrK_sym_neq = shrK_neq [THEN not_sym]*
declare *shrK_sym_neq [simp]*

32.14 Fresh nonces

lemma *Nonce_notin_initState [iff]: "Nonce N ∉ parts (initState B)"*
⟨proof⟩

lemma *Nonce_notin_used_empty [simp]: "Nonce N ∉ used []"*
⟨proof⟩

32.15 Supply fresh nonces for possibility theorems.

lemma *Nonce_supply_lemma: "∃N. ∀n. N ≤ n ⟶ Nonce n ∉ used evs"*
⟨proof⟩

lemma *Nonce_supply1: "∃N. Nonce N ∉ used evs"*
⟨proof⟩

lemma *Nonce_supply2: "∃N N'. Nonce N ∉ used evs ∧ Nonce N' ∉ used evs' ∧ N ≠ N'"*
⟨*proof*⟩

lemma *Nonce_supply3: "∃N N' N''. Nonce N ∉ used evs ∧ Nonce N' ∉ used evs' ∧
Nonce N'' ∉ used evs'' ∧ N ≠ N' ∧ N' ≠ N'' ∧ N ≠ N''"*
⟨*proof*⟩

lemma *Nonce_supply: "Nonce (SOME N. Nonce N ∉ used evs) ∉ used evs"*
⟨*proof*⟩

Unlike the corresponding property of nonces, we cannot prove *finite KK ⟹ ∃K. K ∉ KK ∧ Key K ∉ used evs*. We have infinitely many agents and there is nothing to stop their long-term keys from exhausting all the natural numbers. Instead, possibility theorems must assume the existence of a few keys.

32.16 Specialized Rewriting for Theorems About *analz* and Image

lemma *subset_Compl_range: "A ⊆ - (range shrK) ⟹ shrK x ∉ A"*
⟨*proof*⟩

lemma *insert_Key_singleton: "insert (Key K) H = Key ` {K} ∪ H"*
⟨*proof*⟩

lemma *insert_Key_image: "insert (Key K) (Key`KK ∪ C) = Key`(insert K KK) ∪ C"*
⟨*proof*⟩

lemmas *analz_image_freshK_simps =*
simp_thms mem_simps — these two allow its use with *only:*
disj_comms
image_insert [THEN sym] image_Un [THEN sym] empty_subsetI insert_subset
analz_insert_eq Un_upper2 [THEN analz_mono, THEN [2] rev_subsetD]
insert_Key_singleton subset_Compl_range
Key_not_used insert_Key_image Un_assoc [THEN sym]

lemma *analz_image_freshK_lemma:*
*"(Key K ∈ analz (Key`nE ∪ H)) ⟶ (K ∈ nE | Key K ∈ analz H) ⟹
(Key K ∈ analz (Key`nE ∪ H)) = (K ∈ nE | Key K ∈ analz H)"*
⟨*proof*⟩

32.17 Tactics for possibility theorems

⟨*ML*⟩

lemma *invKey_shrK_iff [iff]:*
    "(Key (invKey K) ∈ X) = (Key K ∈ X)"
⟨*proof*⟩

⟨*ML*⟩

lemma *knows_subset_knows_Cons: "knows A evs ⊆ knows A (e # evs)"*
⟨*proof*⟩

end

33 lemmas on guarded messages for protocols with symmetric keys

theory *Guard_Shared* **imports** *Guard GuardK "../Shared"* **begin**

33.1 Extensions to Theory *Shared*

declare *initState.simps [simp del]*

33.1.1 a little abbreviation

abbreviation
  Ciph :: "agent => msg => msg" **where**
  "Ciph A X == Crypt (shrK A) X"

33.1.2 agent associated to a key

definition *agt :: "key => agent"* **where**
"agt K == SOME A. K = shrK A"

lemma *agt_shrK [simp]: "agt (shrK A) = A"*
⟨*proof*⟩

33.1.3 basic facts about *initState*

lemma *no_Crypt_in_parts_init [simp]: "Crypt K X ∉ parts (initState A)"*
⟨*proof*⟩

lemma *no_Crypt_in_analz_init [simp]: "Crypt K X ∉ analz (initState A)"*
⟨*proof*⟩

lemma *no_shrK_in_analz_init [simp]: "A ∉ bad*
⟹ Key (shrK A) ∉ analz (initState Spy)"
⟨*proof*⟩

lemma *shrK_notin_initState_Friend [simp]: "A ≠ Friend C*
⟹ Key (shrK A) ∉ parts (initState (Friend C))"
⟨*proof*⟩

lemma *keyset_init [iff]: "keyset (initState A)"*
⟨*proof*⟩

33.1.4 sets of symmetric keys

definition *shrK_set :: "key set => bool"* **where**
"shrK_set Ks ≡ ∀K. K ∈ Ks ⟶ (∃A. K = shrK A)"

lemma *in_shrK_set: "⟦shrK_set Ks; K ∈ Ks⟧ ⟹ ∃A. K = shrK A"*
⟨*proof*⟩

lemma *shrK_set1 [iff]: "shrK_set {shrK A}"*
⟨*proof*⟩

lemma *shrK_set2 [iff]: "shrK_set {shrK A, shrK B}"*
⟨*proof*⟩

33.1.5 sets of good keys

definition *good :: "key set ⇒ bool"* **where**
"good Ks ≡ ∀K. K ∈ Ks ⟶ agt K ∉ bad"

lemma *in_good: "⟦good Ks; K ∈ Ks⟧ ⟹ agt K ∉ bad"*
⟨*proof*⟩

lemma *good1 [simp]: "A ∉ bad ⟹ good {shrK A}"*
⟨*proof*⟩

lemma *good2 [simp]: "⟦A ∉ bad; B ∉ bad⟧ ⟹ good {shrK A, shrK B}"*
⟨*proof*⟩

33.2 Proofs About Guarded Messages

33.2.1 small hack

lemma *shrK_is_invKey_shrK: "shrK A = invKey (shrK A)"*
⟨*proof*⟩

lemmas *shrK_is_invKey_shrK_substI = shrK_is_invKey_shrK [THEN ssubst]*

lemmas *invKey_invKey_substI = invKey [THEN ssubst]*

lemma *"Nonce n ∈ parts {X} ⟹ Crypt (shrK A) X ∈ guard n {shrK A}"*
⟨*proof*⟩

33.2.2 guardedness results on nonces

lemma *guard_ciph [simp]: "shrK A ∈ Ks ⟹ Ciph A X ∈ guard n Ks"*
⟨*proof*⟩

lemma *guardK_ciph [simp]: "shrK A ∈ Ks ⟹ Ciph A X ∈ guardK n Ks"*
⟨*proof*⟩

lemma *Guard_init [iff]: "Guard n Ks (initState B)"*
⟨*proof*⟩

lemma *Guard_knows_max': "Guard n Ks (knows_max' C evs)*
⟹ Guard n Ks (knows_max C evs)"

⟨*proof*⟩

lemma *Nonce_not_used_Guard_spies [dest]: "Nonce n ∉ used evs ⟹ Guard n Ks (spies evs)"*
⟨*proof*⟩

lemma *Nonce_not_used_Guard [dest]: "⟦evs ∈ p; Nonce n ∉ used evs; Gets_correct p; one_step p⟧ ⟹ Guard n Ks (knows (Friend C) evs)"*
⟨*proof*⟩

lemma *Nonce_not_used_Guard_max [dest]: "⟦evs ∈ p; Nonce n ∉ used evs; Gets_correct p; one_step p⟧ ⟹ Guard n Ks (knows_max (Friend C) evs)"*
⟨*proof*⟩

lemma *Nonce_not_used_Guard_max' [dest]: "⟦evs ∈ p; Nonce n ∉ used evs; Gets_correct p; one_step p⟧ ⟹ Guard n Ks (knows_max' (Friend C) evs)"*
⟨*proof*⟩

33.2.3 guardedness results on keys

lemma *GuardK_init [simp]: "n ∉ range shrK ⟹ GuardK n Ks (initState B)"*
⟨*proof*⟩

lemma *GuardK_knows_max': "⟦GuardK n A (knows_max' C evs); n ∉ range shrK⟧ ⟹ GuardK n A (knows_max C evs)"*
⟨*proof*⟩

lemma *Key_not_used_GuardK_spies [dest]: "Key n ∉ used evs ⟹ GuardK n A (spies evs)"*
⟨*proof*⟩

lemma *Key_not_used_GuardK [dest]: "⟦evs ∈ p; Key n ∉ used evs; Gets_correct p; one_step p⟧ ⟹ GuardK n A (knows (Friend C) evs)"*
⟨*proof*⟩

lemma *Key_not_used_GuardK_max [dest]: "⟦evs ∈ p; Key n ∉ used evs; Gets_correct p; one_step p⟧ ⟹ GuardK n A (knows_max (Friend C) evs)"*
⟨*proof*⟩

lemma *Key_not_used_GuardK_max' [dest]: "⟦evs ∈ p; Key n ∉ used evs; Gets_correct p; one_step p⟧ ⟹ GuardK n A (knows_max' (Friend C) evs)"*
⟨*proof*⟩

33.2.4 regular protocols

definition *regular :: "event list set => bool"* **where**
"regular p ≡ ∀evs A. evs ∈ p ⟶ (Key (shrK A) ∈ parts (spies evs)) = (A ∈ bad)"

lemma *shrK_parts_iff_bad [simp]: "⟦evs ∈ p; regular p⟧ ⟹ (Key (shrK A) ∈ parts (spies evs)) = (A ∈ bad)"*
⟨*proof*⟩

lemma *shrK_analz_iff_bad [simp]: "⟦evs ∈ p; regular p⟧ ⟹ (Key (shrK A) ∈ analz (spies evs)) = (A ∈ bad)"*

⟨proof⟩

```
lemma Guard_Nonce_analz: "⟦Guard n Ks (spies evs); evs ∈ p;
shrK_set Ks; good Ks; regular p⟧ ⟹ Nonce n ∉ analz (spies evs)"
⟨proof⟩

lemma GuardK_Key_analz:
  assumes "GuardK n Ks (spies evs)" "evs ∈ p" "shrK_set Ks"
    "good Ks" "regular p" "n ∉ range shrK"
  shows "Key n ∉ analz (spies evs)"
⟨proof⟩

end
```

34 Otway-Rees Protocol

```
theory Guard_OtwayRees imports Guard_Shared begin
```

34.1 messages used in the protocol

```
abbreviation
  nil :: "msg" where
  "nil == Number 0"

abbreviation
  or1 :: "agent => agent => nat => event" where
  "or1 A B NA ==
    Says A B {|Nonce NA, Agent A, Agent B, Ciph A {|Nonce NA, Agent A, Agent
B|}|}"

abbreviation
  or1' :: "agent => agent => agent => nat => msg => event" where
  "or1' A' A B NA X == Says A' B {|Nonce NA, Agent A, Agent B, X|}"

abbreviation
  or2 :: "agent => agent => nat => nat => msg => event" where
  "or2 A B NA NB X ==
    Says B Server {|Nonce NA, Agent A, Agent B, X,
                    Ciph B {|Nonce NA, Nonce NB, Agent A, Agent B|}|}"

abbreviation
  or2' :: "agent => agent => agent => nat => nat => event" where
  "or2' B' A B NA NB ==
    Says B' Server {|Nonce NA, Agent A, Agent B,
                     Ciph A {|Nonce NA, Agent A, Agent B|},
                     Ciph B {|Nonce NA, Nonce NB, Agent A, Agent B|}|}"

abbreviation
  or3 :: "agent => agent => nat => nat => key => event" where
  "or3 A B NA NB K ==
    Says Server B {|Nonce NA, Ciph A {|Nonce NA, Key K|},
                    Ciph B {|Nonce NB, Key K|}|}"
```

abbreviation
or3':: "agent => msg => agent => agent => nat => nat => key => event" **where**
"or3' S Y A B NA NB K ==
Says S B {|Nonce NA, Y, Ciph B {|Nonce NB, Key K|}|}"

abbreviation
or4 :: "agent => agent => nat => msg => event" **where**
"or4 A B NA X == Says B A {|Nonce NA, X, nil|}"

abbreviation
or4' :: "agent => agent => nat => key => event" **where**
"or4' B' A NA K == Says B' A {|Nonce NA, Ciph A {|Nonce NA, Key K|}, nil|}"

34.2 definition of the protocol

inductive_set *or :: "event list set"*
where

Nil: "[] ∈ or"

| Fake: "⟦evs ∈ or; X ∈ synth (analz (spies evs))⟧ ⟹ Says Spy B X # evs ∈ or"

| OR1: "⟦evs1 ∈ or; Nonce NA ∉ used evs1⟧ ⟹ or1 A B NA # evs1 ∈ or"

| OR2: "⟦evs2 ∈ or; or1' A' A B NA X ∈ set evs2; Nonce NB ∉ used evs2⟧
⟹ or2 A B NA NB X # evs2 ∈ or"

| OR3: "⟦evs3 ∈ or; or2' B' A B NA NB ∈ set evs3; Key K ∉ used evs3⟧
⟹ or3 A B NA NB K # evs3 ∈ or"

| OR4: "⟦evs4 ∈ or; or2 A B NA NB X ∈ set evs4; or3' S Y A B NA NB K ∈ set evs4⟧
⟹ or4 A B NA X # evs4 ∈ or"

34.3 declarations for tactics

declare *knows_Spy_partsEs [elim]*
declare *Fake_parts_insert [THEN subsetD, dest]*
declare *initState.simps [simp del]*

34.4 general properties of or

lemma *or_has_no_Gets: "evs ∈ or ⟹ ∀A X. Gets A X ∉ set evs"*
⟨*proof*⟩

lemma *or_is_Gets_correct [iff]: "Gets_correct or"*
⟨*proof*⟩

lemma *or_is_one_step [iff]: "one_step or"*
⟨*proof*⟩

lemma *or_has_only_Says' [rule_format]: "evs ∈ or ⟹*
ev ∈ set evs ⟶ (∃A B X. ev=Says A B X)"

⟨proof⟩

lemma *or_has_only_Says [iff]: "has_only_Says or"*
⟨proof⟩

34.5 or is regular

lemma *or1'_parts_spies [dest]: "or1' A' A B NA X ∈ set evs*
⟹ X ∈ parts (spies evs)"
⟨proof⟩

lemma *or2_parts_spies [dest]: "or2 A B NA NB X ∈ set evs*
⟹ X ∈ parts (spies evs)"
⟨proof⟩

lemma *or3_parts_spies [dest]: "Says S B {|NA, Y, Ciph B {|NB, K|}|} ∈ set evs*
⟹ K ∈ parts (spies evs)"
⟨proof⟩

lemma *or_is_regular [iff]: "regular or"*
⟨proof⟩

34.6 guardedness of KAB

lemma *Guard_KAB [rule_format]: "⟦evs ∈ or; A ∉ bad; B ∉ bad⟧ ⟹*
or3 A B NA NB K ∈ set evs ⟶ GuardK K {shrK A,shrK B} (spies evs)"
⟨proof⟩

34.7 guardedness of NB

lemma *Guard_NB [rule_format]: "⟦evs ∈ or; B ∉ bad⟧ ⟹*
or2 A B NA NB X ∈ set evs ⟶ Guard NB {shrK B} (spies evs)"
⟨proof⟩

end

35 Yahalom Protocol

theory *Guard_Yahalom* **imports** *"../Shared" Guard_Shared* **begin**

35.1 messages used in the protocol

abbreviation *(input)*
  ya1 :: "agent => agent => nat => event" **where**
  "ya1 A B NA == Says A B {|Agent A, Nonce NA|}"

abbreviation *(input)*
  ya1' :: "agent => agent => agent => nat => event" **where**
  "ya1' A' A B NA == Says A' B {|Agent A, Nonce NA|}"

abbreviation *(input)*
  ya2 :: "agent => agent => nat => nat => event" **where**
  "ya2 A B NA NB == Says B Server {|Agent B, Ciph B {|Agent A, Nonce NA, Nonce NB|}|}"

```
abbreviation (input)
  ya2' :: "agent => agent => agent => nat => nat => event" where
  "ya2' B' A B NA NB == Says B' Server {|Agent B, Ciph B {|Agent A, Nonce NA,
Nonce NB|}|}"

abbreviation (input)
  ya3 :: "agent => agent => nat => nat => key => event" where
  "ya3 A B NA NB K ==
    Says Server A {|Ciph A {|Agent B, Key K, Nonce NA, Nonce NB|},
                    Ciph B {|Agent A, Key K|}|}"

abbreviation (input)
  ya3':: "agent => msg => agent => agent => nat => nat => key => event" where
  "ya3' S Y A B NA NB K ==
    Says S A {|Ciph A {|Agent B, Key K, Nonce NA, Nonce NB|}, Y|}"

abbreviation (input)
  ya4 :: "agent => agent => nat => nat => msg => event" where
  "ya4 A B K NB Y == Says A B {|Y, Crypt K (Nonce NB)|}"

abbreviation (input)
  ya4' :: "agent => agent => nat => nat => msg => event" where
  "ya4' A' B K NB Y == Says A' B {|Y, Crypt K (Nonce NB)|}"
```

35.2 definition of the protocol

```
inductive_set ya :: "event list set"
where

  Nil: "[] ∈ ya"

| Fake: "[|evs ∈ ya; X ∈ synth (analz (spies evs))|] ⟹ Says Spy B X # evs
∈ ya"

| YA1: "[|evs1 ∈ ya; Nonce NA ∉ used evs1|] ⟹ ya1 A B NA # evs1 ∈ ya"

| YA2: "[|evs2 ∈ ya; ya1' A' A B NA ∈ set evs2; Nonce NB ∉ used evs2|]
  ⟹ ya2 A B NA NB # evs2 ∈ ya"

| YA3: "[|evs3 ∈ ya; ya2' B' A B NA NB ∈ set evs3; Key K ∉ used evs3|]
  ⟹ ya3 A B NA NB K # evs3 ∈ ya"

| YA4: "[|evs4 ∈ ya; ya1 A B NA ∈ set evs4; ya3' S Y A B NA NB K ∈ set evs4|]
  ⟹ ya4 A B K NB Y # evs4 ∈ ya"
```

35.3 declarations for tactics

```
declare knows_Spy_partsEs [elim]
declare Fake_parts_insert [THEN subsetD, dest]
declare initState.simps [simp del]
```

35.4 general properties of ya

lemma *ya_has_no_Gets: "evs ∈ ya ⟹ ∀A X. Gets A X ∉ set evs"*
⟨*proof*⟩

lemma *ya_is_Gets_correct [iff]: "Gets_correct ya"*
⟨*proof*⟩

lemma *ya_is_one_step [iff]: "one_step ya"*
⟨*proof*⟩

lemma *ya_has_only_Says' [rule_format]: "evs ∈ ya ⟹*
ev ∈ set evs ⟶ (∃A B X. ev=Says A B X)"
⟨*proof*⟩

lemma *ya_has_only_Says [iff]: "has_only_Says ya"*
⟨*proof*⟩

lemma *ya_is_regular [iff]: "regular ya"*
⟨*proof*⟩

35.5 guardedness of KAB

lemma *Guard_KAB [rule_format]: "⟦evs ∈ ya; A ∉ bad; B ∉ bad⟧ ⟹*
ya3 A B NA NB K ∈ set evs ⟶ GuardK K {shrK A,shrK B} (spies evs)"
⟨*proof*⟩

35.6 session keys are not symmetric keys

lemma *KAB_isnt_shrK [rule_format]: "evs ∈ ya ⟹*
ya3 A B NA NB K ∈ set evs ⟶ K ∉ range shrK"
⟨*proof*⟩

lemma *ya3_shrK: "evs ∈ ya ⟹ ya3 A B NA NB (shrK C) ∉ set evs"*
⟨*proof*⟩

35.7 ya2' implies ya1'

lemma *ya2'_parts_imp_ya1'_parts [rule_format]:*
"⟦evs ∈ ya; B ∉ bad⟧ ⟹
Ciph B ⦃Agent A, Nonce NA, Nonce NB⦄ ∈ parts (spies evs) ⟶
⦃Agent A, Nonce NA⦄ ∈ spies evs"
⟨*proof*⟩

lemma *ya2'_imp_ya1'_parts: "⟦ya2' B' A B NA NB ∈ set evs; evs ∈ ya; B ∉ bad⟧*
⟹ ⦃Agent A, Nonce NA⦄ ∈ spies evs"
⟨*proof*⟩

35.8 uniqueness of NB

lemma *NB_is_uniq_in_ya2'_parts [rule_format]: "⟦evs ∈ ya; B ∉ bad; B' ∉ bad⟧ ⟹*
Ciph B ⦃Agent A, Nonce NA, Nonce NB⦄ ∈ parts (spies evs) ⟶
Ciph B' ⦃Agent A', Nonce NA', Nonce NB⦄ ∈ parts (spies evs) ⟶

A=A' ∧ B=B' ∧ NA=NA'"
⟨*proof*⟩

lemma *NB_is_uniq_in_ya2': "⟦ya2' C A B NA NB ∈ set evs;*
ya2' C' A' B' NA' NB ∈ set evs; evs ∈ ya; B ∉ bad; B' ∉ bad⟧
⟹ A=A' ∧ B=B' ∧ NA=NA'"
⟨*proof*⟩

35.9 ya3' implies ya2'

lemma *ya3'_parts_imp_ya2'_parts [rule_format]: "⟦evs ∈ ya; A ∉ bad⟧ ⟹*
Ciph A ⦃Agent B, Key K, Nonce NA, Nonce NB⦄ ∈ parts (spies evs)
⟶ Ciph B ⦃Agent A, Nonce NA, Nonce NB⦄ ∈ parts (spies evs)"
⟨*proof*⟩

lemma *ya3'_parts_imp_ya2' [rule_format]: "⟦evs ∈ ya; A ∉ bad⟧ ⟹*
Ciph A ⦃Agent B, Key K, Nonce NA, Nonce NB⦄ ∈ parts (spies evs)
⟶ (∃B'. ya2' B' A B NA NB ∈ set evs)"
⟨*proof*⟩

lemma *ya3'_imp_ya2': "⟦ya3' S Y A B NA NB K ∈ set evs; evs ∈ ya; A ∉ bad⟧*
⟹ (∃B'. ya2' B' A B NA NB ∈ set evs)"
⟨*proof*⟩

35.10 ya3' implies ya3

lemma *ya3'_parts_imp_ya3 [rule_format]: "⟦evs ∈ ya; A ∉ bad⟧ ⟹*
Ciph A ⦃Agent B, Key K, Nonce NA, Nonce NB⦄ ∈ parts(spies evs)
⟶ ya3 A B NA NB K ∈ set evs"
⟨*proof*⟩

lemma *ya3'_imp_ya3: "⟦ya3' S Y A B NA NB K ∈ set evs; evs ∈ ya; A ∉ bad⟧*
⟹ ya3 A B NA NB K ∈ set evs"
⟨*proof*⟩

35.11 guardedness of NB

definition *ya_keys :: "agent ⇒ agent ⇒ nat ⇒ nat ⇒ event list ⇒ key set"*
where
"ya_keys A B NA NB evs ≡ {shrK A,shrK B} ∪ {K. ya3 A B NA NB K ∈ set evs}"

lemma *Guard_NB [rule_format]: "⟦evs ∈ ya; A ∉ bad; B ∉ bad⟧ ⟹*
ya2 A B NA NB ∈ set evs ⟶ Guard NB (ya_keys A B NA NB evs) (spies evs)"
⟨*proof*⟩

end

36 Blanqui's "guard" concept: protocol-independent secrecy

theory *Auth_Guard_Shared*
imports
Guard_OtwayRees

```
  Guard_Yahalom
begin

end
```

theory *Guard_Public* **imports** *Guard "../Public" Extensions* **begin**

36.1 Extensions to Theory Public

declare *initState.simps [simp del]*

36.1.1 signature

definition *sign :: "agent => msg => msg"* **where**
"sign A X == {|Agent A, X, Crypt (priK A) (Hash X)|}"

lemma *sign_inj [iff]: "(sign A X = sign A' X') = (A=A' & X=X')"*
⟨*proof*⟩

36.1.2 agent associated to a key

definition *agt :: "key => agent"* **where**
"agt K == SOME A. K = priK A | K = pubK A"

lemma *agt_priK [simp]: "agt (priK A) = A"*
⟨*proof*⟩

lemma *agt_pubK [simp]: "agt (pubK A) = A"*
⟨*proof*⟩

36.1.3 basic facts about initState

lemma *no_Crypt_in_parts_init [simp]: "Crypt K X ∉ parts (initState A)"*
⟨*proof*⟩

lemma *no_Crypt_in_analz_init [simp]: "Crypt K X ∉ analz (initState A)"*
⟨*proof*⟩

lemma *no_priK_in_analz_init [simp]: "A ∉ bad*
⟹ Key (priK A) ∉ analz (initState Spy)"
⟨*proof*⟩

lemma *priK_notin_initState_Friend [simp]: "A ≠ Friend C*
⟹ Key (priK A) ∉ parts (initState (Friend C))"
⟨*proof*⟩

lemma *keyset_init [iff]: "keyset (initState A)"*
⟨*proof*⟩

36.1.4 sets of private keys

definition *priK_set :: "key set => bool"* **where**
"priK_set Ks ≡ ∀K. K ∈ Ks ⟶ (∃A. K = priK A)"

lemma *in_priK_set: "⟦priK_set Ks; K ∈ Ks⟧ ⟹ ∃A. K = priK A"*
⟨*proof*⟩

lemma *priK_set1 [iff]: "priK_set {priK A}"*
⟨*proof*⟩

lemma *priK_set2 [iff]: "priK_set {priK A, priK B}"*
⟨*proof*⟩

36.1.5 sets of good keys

definition *good :: "key set => bool"* **where**
"good Ks == ∀K. K ∈ Ks ⟶ agt K ∉ bad"

lemma *in_good: "⟦good Ks; K ∈ Ks⟧ ⟹ agt K ∉ bad"*
⟨*proof*⟩

lemma *good1 [simp]: "A ∉ bad ⟹ good {priK A}"*
⟨*proof*⟩

lemma *good2 [simp]: "⟦A ∉ bad; B ∉ bad⟧ ⟹ good {priK A, priK B}"*
⟨*proof*⟩

36.1.6 greatest nonce used in a trace, 0 if there is no nonce

primrec *greatest :: "event list => nat"*
where
  "greatest [] = 0"
| "greatest (ev # evs) = max (greatest_msg (msg ev)) (greatest evs)"

lemma *greatest_is_greatest: "Nonce n ∈ used evs ⟹ n ≤ greatest evs"*
⟨*proof*⟩

36.1.7 function giving a new nonce

definition *new :: "event list ⇒ nat"* **where**
"new evs ≡ Suc (greatest evs)"

lemma *new_isnt_used [iff]: "Nonce (new evs) ∉ used evs"*
⟨*proof*⟩

36.2 Proofs About Guarded Messages

36.2.1 small hack necessary because priK is defined as the inverse of pubK

lemma *pubK_is_invKey_priK: "pubK A = invKey (priK A)"*
⟨*proof*⟩

lemmas *pubK_is_invKey_priK_substI = pubK_is_invKey_priK [THEN ssubst]*

lemmas *invKey_invKey_substI = invKey [THEN ssubst]*

lemma *"Nonce n ∈ parts {X} ⟹ Crypt (pubK A) X ∈ guard n {priK A}"*
⟨*proof*⟩

36.2.2 guardedness results

lemma *sign_guard [intro]: "X ∈ guard n Ks ⟹ sign A X ∈ guard n Ks"*
⟨*proof*⟩

lemma *Guard_init [iff]: "Guard n Ks (initState B)"*
⟨*proof*⟩

lemma *Guard_knows_max': "Guard n Ks (knows_max' C evs)*
⟹ Guard n Ks (knows_max C evs)"
⟨*proof*⟩

lemma *Nonce_not_used_Guard_spies [dest]: "Nonce n ∉ used evs*
⟹ Guard n Ks (spies evs)"
⟨*proof*⟩

lemma *Nonce_not_used_Guard [dest]: "⟦evs ∈ p; Nonce n ∉ used evs;*
Gets_correct p; one_step p⟧ ⟹ Guard n Ks (knows (Friend C) evs)"
⟨*proof*⟩

lemma *Nonce_not_used_Guard_max [dest]: "⟦evs ∈ p; Nonce n ∉ used evs;*
Gets_correct p; one_step p⟧ ⟹ Guard n Ks (knows_max (Friend C) evs)"
⟨*proof*⟩

lemma *Nonce_not_used_Guard_max' [dest]: "⟦evs ∈ p; Nonce n ∉ used evs;*
Gets_correct p; one_step p⟧ ⟹ Guard n Ks (knows_max' (Friend C) evs)"
⟨*proof*⟩

36.2.3 regular protocols

definition *regular :: "event list set ⇒ bool"* **where**
"regular p ≡ ∀evs A. evs ∈ p ⟶ (Key (priK A) ∈ parts (spies evs)) = (A
∈ bad)"

lemma *priK_parts_iff_bad [simp]: "⟦evs ∈ p; regular p⟧ ⟹*
(Key (priK A) ∈ parts (spies evs)) = (A ∈ bad)"
⟨*proof*⟩

lemma *priK_analz_iff_bad [simp]: "⟦evs ∈ p; regular p⟧ ⟹*
(Key (priK A) ∈ analz (spies evs)) = (A ∈ bad)"
⟨*proof*⟩

lemma *Guard_Nonce_analz: "⟦Guard n Ks (spies evs); evs ∈ p;*
priK_set Ks; good Ks; regular p⟧ ⟹ Nonce n ∉ analz (spies evs)"
⟨*proof*⟩

end

37 Lists of Messages and Lists of Agents

theory *List_Msg* **imports** *Extensions* **begin**

37.1 Implementation of Lists by Messages

37.1.1 nil is represented by any message which is not a pair

abbreviation *(input)*
  cons :: "msg => msg => msg" **where**
  "cons x l == {|x,l|}"

37.1.2 induction principle

lemma *lmsg_induct: "[|!!x. not_MPair x ⟹ P x; !!x l. P l ⟹ P (cons x l)|]*
⟹ P l"
⟨*proof*⟩

37.1.3 head

primrec *head :: "msg => msg"* **where**
"head (cons x l) = x"

37.1.4 tail

primrec *tail :: "msg => msg"* **where**
"tail (cons x l) = l"

37.1.5 length

fun *len :: "msg => nat"* **where**
"len (cons x l) = Suc (len l)" |
"len other = 0"

lemma *len_not_empty: "n < len l ⟹ ∃x l'. l = cons x l'"*
⟨*proof*⟩

37.1.6 membership

fun *isin :: "msg * msg => bool"* **where**
"isin (x, cons y l) = (x=y | isin (x,l))" |
"isin (x, other) = False"

37.1.7 delete an element

fun *del :: "msg * msg => msg"* **where**
"del (x, cons y l) = (if x=y then l else cons y (del (x,l)))" |
"del (x, other) = other"

lemma *notin_del [simp]: "~ isin (x,l) ⟹ del (x,l) = l"*
⟨*proof*⟩

lemma *isin_del [rule_format]: "isin (y, del (x,l)) --> isin (y,l)"*
⟨*proof*⟩

37.1.8 concatenation

fun *app :: "msg * msg => msg"* **where**
"app (cons x l, l') = cons x (app (l,l'))" |
"app (other, l') = l'"

lemma *isin_app [iff]: "isin (x, app(l,l')) = (isin (x,l) | isin (x,l'))"*
⟨*proof*⟩

37.1.9 replacement

fun *repl :: "msg * nat * msg => msg"* **where**
"repl (cons x l, Suc i, x') = cons x (repl (l,i,x'))" |
"repl (cons x l, 0, x') = cons x' l" |
"repl (other, i, M') = other"

37.1.10 ith element

fun *ith :: "msg * nat => msg"* **where**
"ith (cons x l, Suc i) = ith (l,i)" |
"ith (cons x l, 0) = x" |
"ith (other, i) = other"

lemma *ith_head: "0 < len l ⟹ ith (l,0) = head l"*
⟨*proof*⟩

37.1.11 insertion

fun *ins :: "msg * nat * msg => msg"* **where**
"ins (cons x l, Suc i, y) = cons x (ins (l,i,y))" |
"ins (l, 0, y) = cons y l"

lemma *ins_head [simp]: "ins (l,0,y) = cons y l"*
⟨*proof*⟩

37.1.12 truncation

fun *trunc :: "msg * nat => msg"* **where**
"trunc (l,0) = l" |
"trunc (cons x l, Suc i) = trunc (l,i)"

lemma *trunc_zero [simp]: "trunc (l,0) = l"*
⟨*proof*⟩

37.2 Agent Lists

37.2.1 set of well-formed agent-list messages

abbreviation
  nil :: msg **where**
  "nil == Number 0"

inductive_set *agl :: "msg set"*
where
  Nil[intro]: "nil ∈ agl"
| Cons[intro]: "⟦A ∈ agent; I ∈ agl⟧ ⟹ cons (Agent A) I ∈ agl"

37.2.2 basic facts about agent lists

lemma *del_in_agl [intro]: "I ∈ agl ⟹ del (a,I) ∈ agl"*
⟨*proof*⟩

lemma *app_in_agl [intro]: "⟦I ∈ agl; J ∈ agl⟧ ⟹ app (I,J) ∈ agl"*
⟨*proof*⟩

lemma *no_Key_in_agl: "I ∈ agl ⟹ Key K ∉ parts {I}"*
⟨*proof*⟩

lemma *no_Nonce_in_agl: "I ∈ agl ⟹ Nonce n ∉ parts {I}"*
⟨*proof*⟩

lemma *no_Key_in_appdel: "⟦I ∈ agl; J ∈ agl⟧ ⟹
Key K ∉ parts {app (J, del (Agent B, I))}"*
⟨*proof*⟩

lemma *no_Nonce_in_appdel: "⟦I ∈ agl; J ∈ agl⟧ ⟹
Nonce n ∉ parts {app (J, del (Agent B, I))}"*
⟨*proof*⟩

lemma *no_Crypt_in_agl: "I ∈ agl ⟹ Crypt K X ∉ parts {I}"*
⟨*proof*⟩

lemma *no_Crypt_in_appdel: "⟦I ∈ agl; J ∈ agl⟧ ⟹
Crypt K X ∉ parts {app (J, del (Agent B,I))}"*
⟨*proof*⟩

end

38 Protocol P1

theory *P1* **imports** *"../Public" Guard_Public List_Msg* **begin**

38.1 Protocol Definition

38.1.1 offer chaining: B chains his offer for A with the head offer of L for sending it to C

definition *chain :: "agent => nat => agent => msg => agent => msg"* **where**
*"chain B ofr A L C ==
let m1= Crypt (pubK A) (Nonce ofr) in
let m2= Hash ⦃head L, Agent C⦄ in
sign B ⦃m1,m2⦄"*

declare *Let_def [simp]*

lemma *chain_inj [iff]: "(chain B ofr A L C = chain B' ofr' A' L' C')
= (B=B' & ofr=ofr' & A=A' & head L = head L' & C=C')"*
⟨*proof*⟩

lemma *Nonce_in_chain [iff]: "Nonce ofr ∈ parts {chain B ofr A L C}"*
⟨*proof*⟩

38.1.2 agent whose key is used to sign an offer

fun *shop :: "msg => msg"* **where**

"shop {|B,X,Crypt K H|} = Agent (agt K)"

lemma *shop_chain [simp]: "shop (chain B ofr A L C) = Agent B"*
⟨*proof*⟩

38.1.3 nonce used in an offer

fun *nonce :: "msg => msg"* **where**
"nonce {|B,{|Crypt K ofr,m2|},CryptH|} = ofr"

lemma *nonce_chain [simp]: "nonce (chain B ofr A L C) = Nonce ofr"*
⟨*proof*⟩

38.1.4 next shop

fun *next_shop :: "msg => agent"* **where**
"next_shop {|B,{|m1,Hash{|headL,Agent C|}|},CryptH|} = C"

lemma *next_shop_chain [iff]: "next_shop (chain B ofr A L C) = C"*
⟨*proof*⟩

38.1.5 anchor of the offer list

definition *anchor :: "agent => nat => agent => msg"* **where**
"anchor A n B == chain A n A (cons nil nil) B"

lemma *anchor_inj [iff]: "(anchor A n B = anchor A' n' B')*
= (A=A' & n=n' & B=B')"
⟨*proof*⟩

lemma *Nonce_in_anchor [iff]: "Nonce n ∈ parts {anchor A n B}"*
⟨*proof*⟩

lemma *shop_anchor [simp]: "shop (anchor A n B) = Agent A"*
⟨*proof*⟩

lemma *nonce_anchor [simp]: "nonce (anchor A n B) = Nonce n"*
⟨*proof*⟩

lemma *next_shop_anchor [iff]: "next_shop (anchor A n B) = B"*
⟨*proof*⟩

38.1.6 request event

definition *reqm :: "agent => nat => nat => msg => agent => msg"* **where**
"reqm A r n I B == {|Agent A, Number r, cons (Agent A) (cons (Agent B) I),
cons (anchor A n B) nil|}"

lemma *reqm_inj [iff]: "(reqm A r n I B = reqm A' r' n' I' B')*
= (A=A' & r=r' & n=n' & I=I' & B=B')"
⟨*proof*⟩

lemma *Nonce_in_reqm [iff]: "Nonce n ∈ parts {reqm A r n I B}"*
⟨*proof*⟩

definition *req :: "agent => nat => nat => msg => agent => event"* **where**
"req A r n I B == Says A B (reqm A r n I B)"

lemma *req_inj [iff]: "(req A r n I B = req A' r' n' I' B')*
= (A=A' & r=r' & n=n' & I=I' & B=B')"
⟨*proof*⟩

38.1.7 propose event

definition *prom :: "agent => nat => agent => nat => msg => msg =>*
msg => agent => msg" **where**
"prom B ofr A r I L J C == ⦃Agent A, Number r,
app (J, del (Agent B, I)), cons (chain B ofr A L C) L⦄"

lemma *prom_inj [dest]: "prom B ofr A r I L J C*
= prom B' ofr' A' r' I' L' J' C'
⟹ B=B' & ofr=ofr' & A=A' & r=r' & L=L' & C=C'"
⟨*proof*⟩

lemma *Nonce_in_prom [iff]: "Nonce ofr ∈ parts {prom B ofr A r I L J C}"*
⟨*proof*⟩

definition *pro :: "agent => nat => agent => nat => msg => msg =>*
msg => agent => event" **where**
"pro B ofr A r I L J C == Says B C (prom B ofr A r I L J C)"

lemma *pro_inj [dest]: "pro B ofr A r I L J C = pro B' ofr' A' r' I' L' J'*
C'
⟹ B=B' & ofr=ofr' & A=A' & r=r' & L=L' & C=C'"
⟨*proof*⟩

38.1.8 protocol

inductive_set *p1 :: "event list set"*
where

Nil: "[] ∈ p1"

| Fake: "⟦evsf ∈ p1; X ∈ synth (analz (spies evsf))⟧ ⟹ Says Spy B X # evsf
∈ p1"

| Request: "⟦evsr ∈ p1; Nonce n ∉ used evsr; I ∈ agl⟧ ⟹ req A r n I B #
evsr ∈ p1"

| Propose: "⟦evsp ∈ p1; Says A' B ⦃Agent A,Number r,I,cons M L⦄ ∈ set evsp;
I ∈ agl; J ∈ agl; isin (Agent C, app (J, del (Agent B, I)));
Nonce ofr ∉ used evsp⟧ ⟹ pro B ofr A r I (cons M L) J C # evsp ∈ p1"

38.1.9 Composition of Traces

lemma *"evs' ∈ p1 ⟹*
evs ∈ p1 ∧ (∀n. Nonce n ∈ used evs' ⟶ Nonce n ∉ used evs) ⟶
evs' @ evs ∈ p1"
⟨*proof*⟩

38.1.10 Valid Offer Lists

inductive_set
  valid :: "agent ⇒ nat ⇒ agent ⇒ msg set"
  for *A :: agent* **and** *n :: nat* **and** *B :: agent*
where
  Request [intro]: "cons (anchor A n B) nil ∈ valid A n B"

| Propose [intro]: "L ∈ valid A n B
⟹ cons (chain (next_shop (head L)) ofr A L C) L ∈ valid A n B"

38.1.11 basic properties of valid

lemma *valid_not_empty: "L ∈ valid A n B ⟹ ∃M L'. L = cons M L'"*
⟨*proof*⟩

lemma *valid_pos_len: "L ∈ valid A n B ⟹ 0 < len L"*
⟨*proof*⟩

38.1.12 offers of an offer list

definition *offer_nonces :: "msg ⇒ msg set"* **where**
"offer_nonces L ≡ {X. X ∈ parts {L} ∧ (∃n. X = Nonce n)}"

38.1.13 the originator can get the offers

lemma *"L ∈ valid A n B ⟹ offer_nonces L ⊆ analz (insert L (initState A))"*
⟨*proof*⟩

38.1.14 list of offers

fun *offers :: "msg => msg"* **where**
"offers (cons M L) = cons ⦃shop M, nonce M⦄ (offers L)" |
"offers other = nil"

38.1.15 list of agents whose keys are used to sign a list of offers

fun *shops :: "msg => msg"* **where**
"shops (cons M L) = cons (shop M) (shops L)" |
"shops other = other"

lemma *shops_in_agl: "L ∈ valid A n B ⟹ shops L ∈ agl"*
⟨*proof*⟩

38.1.16 builds a trace from an itinerary

fun *offer_list :: "agent × nat × agent × msg × nat ⇒ msg"* **where**
"offer_list (A,n,B,nil,ofr) = cons (anchor A n B) nil" |
"offer_list (A,n,B,cons (Agent C) I,ofr) = (
let L = offer_list (A,n,B,I,Suc ofr) in
cons (chain (next_shop (head L)) ofr A L C) L)"

lemma *"I ∈ agl ⟹ ∀ofr. offer_list (A,n,B,I,ofr) ∈ valid A n B"*
⟨*proof*⟩

```
fun trace :: "agent × nat × agent × nat × msg × msg × msg
⇒ event list" where
"trace (B,ofr,A,r,I,L,nil) = []" |
"trace (B,ofr,A,r,I,L,cons (Agent D) K) = (
let C = (if K=nil then B else agt_nb (head K)) in
let I' = (if K=nil then cons (Agent A) (cons (Agent B) I)
          else cons (Agent A) (app (I, cons (head K) nil))) in
let I'' = app (I, cons (head K) nil) in
pro C (Suc ofr) A r I' L nil D
# trace (B,Suc ofr,A,r,I'',tail L,K))"
```

```
definition trace' :: "agent ⇒ nat ⇒ nat ⇒ msg ⇒ agent ⇒ nat ⇒ event
list" where
"trace' A r n I B ofr ≡ (
let AI = cons (Agent A) I in
let L = offer_list (A,n,B,AI,ofr) in
trace (B,ofr,A,r,nil,L,AI))"
```

```
declare trace'_def [simp]
```

38.1.17 there is a trace in which the originator receives a valid answer

```
lemma p1_not_empty: "evs ∈ p1 ⟹ req A r n I B ∈ set evs ⟶
(∃evs'. evs' @ evs ∈ p1 ∧ pro B' ofr A r I' L J A ∈ set evs' ∧ L ∈ valid
A n B)"
⟨proof⟩
```

38.2 properties of protocol P1

publicly verifiable forward integrity: anyone can verify the validity of an offer list

38.2.1 strong forward integrity: except the last one, no offer can be modified

```
lemma strong_forward_integrity: "∀L. Suc i < len L
⟶ L ∈ valid A n B ∧ repl (L,Suc i,M) ∈ valid A n B ⟶ M = ith (L,Suc i)"
⟨proof⟩
```

38.2.2 insertion resilience: except at the beginning, no offer can be inserted

```
lemma chain_isnt_head [simp]: "L ∈ valid A n B ⟹
head L ≠ chain (next_shop (head L)) ofr A L C"
⟨proof⟩
```

```
lemma insertion_resilience: "∀L. L ∈ valid A n B ⟶ Suc i < len L
⟶ ins (L,Suc i,M) ∉ valid A n B"
⟨proof⟩
```

38.2.3 truncation resilience: only shop i can truncate at offer i

```
lemma truncation_resilience: "∀L. L ∈ valid A n B ⟶ Suc i < len L
```

⟶ cons M (trunc (L,Suc i)) ∈ valid A n B ⟶ shop M = shop (ith (L,i))"
⟨*proof*⟩

38.2.4 declarations for tactics

declare *knows_Spy_partsEs [elim]*
declare *Fake_parts_insert [THEN subsetD, dest]*
declare *initState.simps [simp del]*

38.2.5 get components of a message

lemma *get_ML [dest]: "Says A' B {|A,r,I,M,L|} ∈ set evs ⟹*
M ∈ parts (spies evs) ∧ L ∈ parts (spies evs)"
⟨*proof*⟩

38.2.6 general properties of p1

lemma *reqm_neq_prom [iff]:*
"reqm A r n I B ≠ prom B' ofr A' r' I' (cons M L) J C"
⟨*proof*⟩

lemma *prom_neq_reqm [iff]:*
"prom B' ofr A' r' I' (cons M L) J C ≠ reqm A r n I B"
⟨*proof*⟩

lemma *req_neq_pro [iff]: "req A r n I B ≠ pro B' ofr A' r' I' (cons M L) J C"*
⟨*proof*⟩

lemma *pro_neq_req [iff]: "pro B' ofr A' r' I' (cons M L) J C ≠ req A r n I B"*
⟨*proof*⟩

lemma *p1_has_no_Gets: "evs ∈ p1 ⟹ ∀A X. Gets A X ∉ set evs"*
⟨*proof*⟩

lemma *p1_is_Gets_correct [iff]: "Gets_correct p1"*
⟨*proof*⟩

lemma *p1_is_one_step [iff]: "one_step p1"*
⟨*proof*⟩

lemma *p1_has_only_Says' [rule_format]: "evs ∈ p1 ⟹*
ev ∈ set evs ⟶ (∃A B X. ev=Says A B X)"
⟨*proof*⟩

lemma *p1_has_only_Says [iff]: "has_only_Says p1"*
⟨*proof*⟩

lemma *p1_is_regular [iff]: "regular p1"*
⟨*proof*⟩

38.2.7 private keys are safe

lemma *priK_parts_Friend_imp_bad [rule_format,dest]:*

"⟦evs ∈ p1; Friend B ≠ A⟧
⟹ (Key (priK A) ∈ parts (knows (Friend B) evs)) ⟶ (A ∈ bad)"
⟨proof⟩

lemma *priK_analz_Friend_imp_bad [rule_format,dest]:*
"⟦evs ∈ p1; Friend B ≠ A⟧
⟹ (Key (priK A) ∈ analz (knows (Friend B) evs)) ⟶ (A ∈ bad)"
⟨proof⟩

lemma *priK_notin_knows_max_Friend:* "⟦evs ∈ p1; A ∉ bad; A ≠ Friend C⟧
⟹ Key (priK A) ∉ analz (knows_max (Friend C) evs)"
⟨proof⟩

38.2.8 general guardedness properties

lemma *agl_guard [intro]:* "I ∈ agl ⟹ I ∈ guard n Ks"
⟨proof⟩

lemma *Says_to_knows_max'_guard:* "⟦Says A' C ⦃A'',r,I,L⦄ ∈ set evs;
Guard n Ks (knows_max' C evs)⟧ ⟹ L ∈ guard n Ks"
⟨proof⟩

lemma *Says_from_knows_max'_guard:* "⟦Says C A' ⦃A'',r,I,L⦄ ∈ set evs;
Guard n Ks (knows_max' C evs)⟧ ⟹ L ∈ guard n Ks"
⟨proof⟩

lemma *Says_Nonce_not_used_guard:* "⟦Says A' B ⦃A'',r,I,L⦄ ∈ set evs;
Nonce n ∉ used evs⟧ ⟹ L ∈ guard n Ks"
⟨proof⟩

38.2.9 guardedness of messages

lemma *chain_guard [iff]:* "chain B ofr A L C ∈ guard n {priK A}"
⟨proof⟩

lemma *chain_guard_Nonce_neq [intro]:* "n ≠ ofr
⟹ chain B ofr A' L C ∈ guard n {priK A}"
⟨proof⟩

lemma *anchor_guard [iff]:* "anchor A n' B ∈ guard n {priK A}"
⟨proof⟩

lemma *anchor_guard_Nonce_neq [intro]:* "n ≠ n'
⟹ anchor A' n' B ∈ guard n {priK A}"
⟨proof⟩

lemma *reqm_guard [intro]:* "I ∈ agl ⟹ reqm A r n' I B ∈ guard n {priK A}"
⟨proof⟩

lemma *reqm_guard_Nonce_neq [intro]:* "⟦n ≠ n'; I ∈ agl⟧
⟹ reqm A' r n' I B ∈ guard n {priK A}"
⟨proof⟩

lemma *prom_guard [intro]:* "⟦I ∈ agl; J ∈ agl; L ∈ guard n {priK A}⟧
⟹ prom B ofr A r I L J C ∈ guard n {priK A}"

⟨*proof*⟩

lemma *prom_guard_Nonce_neq [intro]: "⟦n ≠ ofr; I ∈ agl; J ∈ agl;
L ∈ guard n {priK A}⟧ ⟹ prom B ofr A' r I L J C ∈ guard n {priK A}"*
⟨*proof*⟩

38.2.10 Nonce uniqueness

lemma *uniq_Nonce_in_chain [dest]: "Nonce k ∈ parts {chain B ofr A L C} ⟹
k=ofr"*
⟨*proof*⟩

lemma *uniq_Nonce_in_anchor [dest]: "Nonce k ∈ parts {anchor A n B} ⟹ k=n"*
⟨*proof*⟩

lemma *uniq_Nonce_in_reqm [dest]: "⟦Nonce k ∈ parts {reqm A r n I B};
I ∈ agl⟧ ⟹ k=n"*
⟨*proof*⟩

lemma *uniq_Nonce_in_prom [dest]: "⟦Nonce k ∈ parts {prom B ofr A r I L J
C};
I ∈ agl; J ∈ agl; Nonce k ∉ parts {L}⟧ ⟹ k=ofr"*
⟨*proof*⟩

38.2.11 requests are guarded

lemma *req_imp_Guard [rule_format]: "⟦evs ∈ p1; A ∉ bad⟧ ⟹
req A r n I B ∈ set evs ⟶ Guard n {priK A} (spies evs)"*
⟨*proof*⟩

lemma *req_imp_Guard_Friend: "⟦evs ∈ p1; A ∉ bad; req A r n I B ∈ set evs⟧
⟹ Guard n {priK A} (knows_max (Friend C) evs)"*
⟨*proof*⟩

38.2.12 propositions are guarded

lemma *pro_imp_Guard [rule_format]: "⟦evs ∈ p1; B ∉ bad; A ∉ bad⟧ ⟹
pro B ofr A r I (cons M L) J C ∈ set evs ⟶ Guard ofr {priK A} (spies evs)"*
⟨*proof*⟩

lemma *pro_imp_Guard_Friend: "⟦evs ∈ p1; B ∉ bad; A ∉ bad;
pro B ofr A r I (cons M L) J C ∈ set evs⟧
⟹ Guard ofr {priK A} (knows_max (Friend D) evs)"*
⟨*proof*⟩

38.2.13 data confidentiality: no one other than the originator can decrypt the offers

lemma *Nonce_req_notin_spies: "⟦evs ∈ p1; req A r n I B ∈ set evs; A ∉ bad⟧
⟹ Nonce n ∉ analz (spies evs)"*
⟨*proof*⟩

lemma *Nonce_req_notin_knows_max_Friend: "⟦evs ∈ p1; req A r n I B ∈ set
evs;
A ∉ bad; A ≠ Friend C⟧ ⟹ Nonce n ∉ analz (knows_max (Friend C) evs)"*

⟨*proof*⟩

lemma *Nonce_pro_notin_spies: "⟦evs ∈ p1; B ∉ bad; A ∉ bad;*
pro B ofr A r I (cons M L) J C ∈ set evs⟧ ⟹ Nonce ofr ∉ analz (spies evs)"
⟨*proof*⟩

lemma *Nonce_pro_notin_knows_max_Friend: "⟦evs ∈ p1; B ∉ bad; A ∉ bad;*
A ≠ Friend D; pro B ofr A r I (cons M L) J C ∈ set evs⟧
⟹ Nonce ofr ∉ analz (knows_max (Friend D) evs)"
⟨*proof*⟩

38.2.14 non repudiability: an offer signed by B has been sent by B

lemma *Crypt_reqm: "⟦Crypt (priK A) X ∈ parts {reqm A' r n I B}; I ∈ agl⟧*
⟹ A=A'"
⟨*proof*⟩

lemma *Crypt_prom: "⟦Crypt (priK A) X ∈ parts {prom B ofr A' r I L J C};*
I ∈ agl; J ∈ agl⟧ ⟹ A=B ∨ Crypt (priK A) X ∈ parts {L}"
⟨*proof*⟩

lemma *Crypt_safeness: "⟦evs ∈ p1; A ∉ bad⟧ ⟹ Crypt (priK A) X ∈ parts*
(spies evs)
⟶ (∃B Y. Says A B Y ∈ set evs ∧ Crypt (priK A) X ∈ parts {Y})"
⟨*proof*⟩

lemma *Crypt_Hash_imp_sign: "⟦evs ∈ p1; A ∉ bad⟧ ⟹*
Crypt (priK A) (Hash X) ∈ parts (spies evs)
⟶ (∃B Y. Says A B Y ∈ set evs ∧ sign A X ∈ parts {Y})"
⟨*proof*⟩

lemma *sign_safeness: "⟦evs ∈ p1; A ∉ bad⟧ ⟹ sign A X ∈ parts (spies evs)*
⟶ (∃B Y. Says A B Y ∈ set evs ∧ sign A X ∈ parts {Y})"
⟨*proof*⟩

end

39 Protocol P2

theory *P2* **imports** *Guard_Public List_Msg* **begin**

39.1 Protocol Definition

Like P1 except the definitions of *chain*, *shop*, *next_shop* and *nonce*

39.1.1 offer chaining: B chains his offer for A with the head offer of L for sending it to C

definition *chain :: "agent => nat => agent => msg => agent => msg"* **where**
"chain B ofr A L C ==
let m1= sign B (Nonce ofr) in
let m2= Hash ⦃head L, Agent C⦄ in
⦃Crypt (pubK A) m1, m2⦄"

declare *Let_def [simp]*

lemma *chain_inj [iff]: "(chain B ofr A L C = chain B' ofr' A' L' C')
= (B=B' & ofr=ofr' & A=A' & head L = head L' & C=C')"*
⟨*proof*⟩

lemma *Nonce_in_chain [iff]: "Nonce ofr ∈ parts {chain B ofr A L C}"*
⟨*proof*⟩

39.1.2 agent whose key is used to sign an offer

fun *shop :: "msg => msg"* **where**
"shop ⦃Crypt K ⦃B,ofr,Crypt K' H⦄,m2⦄ = Agent (agt K')"

lemma *shop_chain [simp]: "shop (chain B ofr A L C) = Agent B"*
⟨*proof*⟩

39.1.3 nonce used in an offer

fun *nonce :: "msg => msg"* **where**
"nonce ⦃Crypt K ⦃B,ofr,CryptH⦄,m2⦄ = ofr"

lemma *nonce_chain [simp]: "nonce (chain B ofr A L C) = Nonce ofr"*
⟨*proof*⟩

39.1.4 next shop

fun *next_shop :: "msg => agent"* **where**
"next_shop ⦃m1,Hash ⦃headL,Agent C⦄⦄ = C"

lemma *"next_shop (chain B ofr A L C) = C"*
⟨*proof*⟩

39.1.5 anchor of the offer list

definition *anchor :: "agent => nat => agent => msg"* **where**
"anchor A n B == chain A n A (cons nil nil) B"

lemma *anchor_inj [iff]:
"(anchor A n B = anchor A' n' B') = (A=A' ∧ n=n' ∧ B=B')"*
⟨*proof*⟩

lemma *Nonce_in_anchor [iff]: "Nonce n ∈ parts {anchor A n B}"*
⟨*proof*⟩

lemma *shop_anchor [simp]: "shop (anchor A n B) = Agent A"*
⟨*proof*⟩

39.1.6 request event

definition *reqm :: "agent => nat => nat => msg => agent => msg"* **where**
*"reqm A r n I B == ⦃Agent A, Number r, cons (Agent A) (cons (Agent B) I),
cons (anchor A n B) nil⦄"*

lemma *reqm_inj [iff]: "(reqm A r n I B = reqm A' r' n' I' B')*
= (A=A' & r=r' & n=n' & I=I' & B=B')"
⟨*proof*⟩

lemma *Nonce_in_reqm [iff]: "Nonce n ∈ parts {reqm A r n I B}"*
⟨*proof*⟩

definition *req :: "agent => nat => nat => msg => agent => event"* **where**
"req A r n I B == Says A B (reqm A r n I B)"

lemma *req_inj [iff]: "(req A r n I B = req A' r' n' I' B')*
= (A=A' & r=r' & n=n' & I=I' & B=B')"
⟨*proof*⟩

39.1.7 propose event

definition *prom :: "agent => nat => agent => nat => msg => msg => msg => agent => msg"* **where**
"prom B ofr A r I L J C == {|Agent A, Number r,
app (J, del (Agent B, I)), cons (chain B ofr A L C) L|}"

lemma *prom_inj [dest]: "prom B ofr A r I L J C = prom B' ofr' A' r' I' L' J' C'*
⟹ B=B' & ofr=ofr' & A=A' & r=r' & L=L' & C=C'"
⟨*proof*⟩

lemma *Nonce_in_prom [iff]: "Nonce ofr ∈ parts {prom B ofr A r I L J C}"*
⟨*proof*⟩

definition *pro :: "agent => nat => agent => nat => msg => msg =>*
msg => agent => event" **where**
"pro B ofr A r I L J C == Says B C (prom B ofr A r I L J C)"

lemma *pro_inj [dest]: "pro B ofr A r I L J C = pro B' ofr' A' r' I' L' J' C'*
⟹ B=B' & ofr=ofr' & A=A' & r=r' & L=L' & C=C'"
⟨*proof*⟩

39.1.8 protocol

inductive_set *p2 :: "event list set"*
where

Nil: "[] ∈ p2"

| Fake: "⟦evsf ∈ p2; X ∈ synth (analz (spies evsf))⟧ ⟹ Says Spy B X # evsf ∈ p2"

| Request: "⟦evsr ∈ p2; Nonce n ∉ used evsr; I ∈ agl⟧ ⟹ req A r n I B # evsr ∈ p2"

| Propose: "⟦evsp ∈ p2; Says A' B {|Agent A,Number r,I,cons M L|} ∈ set evsp;
I ∈ agl; J ∈ agl; isin (Agent C, app (J, del (Agent B, I)));
Nonce ofr ∉ used evsp⟧ ⟹ pro B ofr A r I (cons M L) J C # evsp ∈ p2"

39.1.9 valid offer lists

```
inductive_set
  valid :: "agent ⇒ nat ⇒ agent ⇒ msg set"
  for A :: agent and  n :: nat and B :: agent
where
  Request [intro]: "cons (anchor A n B) nil ∈ valid A n B"

| Propose [intro]: "L ∈ valid A n B
  ⟹ cons (chain (next_shop (head L)) ofr A L C) L ∈ valid A n B"
```

39.1.10 basic properties of valid

lemma *valid_not_empty: "L ∈ valid A n B ⟹ ∃M L'. L = cons M L'"*
⟨*proof*⟩

lemma *valid_pos_len: "L ∈ valid A n B ⟹ 0 < len L"*
⟨*proof*⟩

39.1.11 list of offers

```
fun offers :: "msg ⇒ msg"
where
  "offers (cons M L) = cons {|shop M, nonce M|} (offers L)"
| "offers other = nil"
```

39.2 Properties of Protocol P2

same as *P1_Prop* except that publicly verifiable forward integrity is replaced by forward privacy

39.3 strong forward integrity: except the last one, no offer can be modified

lemma *strong_forward_integrity: "∀L. Suc i < len L
⟶ L ∈ valid A n B ⟶ repl (L,Suc i,M) ∈ valid A n B ⟶ M = ith (L,Suc i)"*
⟨*proof*⟩

39.4 insertion resilience: except at the beginning, no offer can be inserted

lemma *chain_isnt_head [simp]: "L ∈ valid A n B ⟹
head L ≠ chain (next_shop (head L)) ofr A L C"*
⟨*proof*⟩

lemma *insertion_resilience: "∀L. L ∈ valid A n B ⟶ Suc i < len L
⟶ ins (L,Suc i,M) ∉ valid A n B"*
⟨*proof*⟩

39.5 truncation resilience: only shop i can truncate at offer i

lemma *truncation_resilience: "∀L. L ∈ valid A n B ⟶ Suc i < len L*

⟶ cons M (trunc (L,Suc i)) ∈ valid A n B ⟶ shop M = shop (ith (L,i))"
⟨*proof*⟩

39.6 declarations for tactics

declare *knows_Spy_partsEs [elim]*
declare *Fake_parts_insert [THEN subsetD, dest]*
declare *initState.simps [simp del]*

39.7 get components of a message

lemma *get_ML [dest]: "Says A' B ⦃A,R,I,M,L⦄ ∈ set evs ⟹*
M ∈ parts (spies evs) ∧ L ∈ parts (spies evs)"
⟨*proof*⟩

39.8 general properties of p2

lemma *reqm_neq_prom [iff]:*
"reqm A r n I B ≠ prom B' ofr A' r' I' (cons M L) J C"
⟨*proof*⟩

lemma *prom_neq_reqm [iff]:*
"prom B' ofr A' r' I' (cons M L) J C ≠ reqm A r n I B"
⟨*proof*⟩

lemma *req_neq_pro [iff]: "req A r n I B ≠ pro B' ofr A' r' I' (cons M L) J C"*
⟨*proof*⟩

lemma *pro_neq_req [iff]: "pro B' ofr A' r' I' (cons M L) J C ≠ req A r n I B"*
⟨*proof*⟩

lemma *p2_has_no_Gets: "evs ∈ p2 ⟹ ∀A X. Gets A X ∉ set evs"*
⟨*proof*⟩

lemma *p2_is_Gets_correct [iff]: "Gets_correct p2"*
⟨*proof*⟩

lemma *p2_is_one_step [iff]: "one_step p2"*
⟨*proof*⟩

lemma *p2_has_only_Says' [rule_format]: "evs ∈ p2 ⟹*
ev ∈ set evs ⟶ (∃A B X. ev=Says A B X)"
⟨*proof*⟩

lemma *p2_has_only_Says [iff]: "has_only_Says p2"*
⟨*proof*⟩

lemma *p2_is_regular [iff]: "regular p2"*
⟨*proof*⟩

39.9 private keys are safe

lemma *priK_parts_Friend_imp_bad [rule_format,dest]:*
"⟦evs ∈ p2; Friend B ≠ A⟧
⟹ (Key (priK A) ∈ parts (knows (Friend B) evs)) ⟶ (A ∈ bad)"
⟨*proof*⟩

lemma *priK_analz_Friend_imp_bad [rule_format,dest]:*
"⟦evs ∈ p2; Friend B ≠ A⟧
⟹ (Key (priK A) ∈ analz (knows (Friend B) evs)) ⟶ (A ∈ bad)"
⟨*proof*⟩

lemma *priK_notin_knows_max_Friend:*
"⟦evs ∈ p2; A ∉ bad; A ≠ Friend C⟧
⟹ Key (priK A) ∉ analz (knows_max (Friend C) evs)"
⟨*proof*⟩

39.10 general guardedness properties

lemma *agl_guard [intro]: "I ∈ agl ⟹ I ∈ guard n Ks"*
⟨*proof*⟩

lemma *Says_to_knows_max'_guard: "⟦Says A' C ⦃A'',r,I,L⦄ ∈ set evs;*
Guard n Ks (knows_max' C evs)⟧ ⟹ L ∈ guard n Ks"
⟨*proof*⟩

lemma *Says_from_knows_max'_guard: "⟦Says C A' ⦃A'',r,I,L⦄ ∈ set evs;*
Guard n Ks (knows_max' C evs)⟧ ⟹ L ∈ guard n Ks"
⟨*proof*⟩

lemma *Says_Nonce_not_used_guard: "⟦Says A' B ⦃A'',r,I,L⦄ ∈ set evs;*
Nonce n ∉ used evs⟧ ⟹ L ∈ guard n Ks"
⟨*proof*⟩

39.11 guardedness of messages

lemma *chain_guard [iff]: "chain B ofr A L C ∈ guard n {priK A}"*
⟨*proof*⟩

lemma *chain_guard_Nonce_neq [intro]: "n ≠ ofr*
⟹ chain B ofr A' L C ∈ guard n {priK A}"
⟨*proof*⟩

lemma *anchor_guard [iff]: "anchor A n' B ∈ guard n {priK A}"*
⟨*proof*⟩

lemma *anchor_guard_Nonce_neq [intro]: "n ≠ n'*
⟹ anchor A' n' B ∈ guard n {priK A}"
⟨*proof*⟩

lemma *reqm_guard [intro]: "I ∈ agl ⟹ reqm A r n' I B ∈ guard n {priK A}"*
⟨*proof*⟩

lemma *reqm_guard_Nonce_neq [intro]: "⟦n ≠ n'; I ∈ agl⟧*
⟹ reqm A' r n' I B ∈ guard n {priK A}"

⟨*proof*⟩

lemma *prom_guard [intro]: "⟦I ∈ agl; J ∈ agl; L ∈ guard n {priK A}⟧
⟹ prom B ofr A r I L J C ∈ guard n {priK A}"*
⟨*proof*⟩

lemma *prom_guard_Nonce_neq [intro]: "⟦n ≠ ofr; I ∈ agl; J ∈ agl;
L ∈ guard n {priK A}⟧ ⟹ prom B ofr A' r I L J C ∈ guard n {priK A}"*
⟨*proof*⟩

39.12 Nonce uniqueness

lemma *uniq_Nonce_in_chain [dest]: "Nonce k ∈ parts {chain B ofr A L C} ⟹
k=ofr"*
⟨*proof*⟩

lemma *uniq_Nonce_in_anchor [dest]: "Nonce k ∈ parts {anchor A n B} ⟹ k=n"*
⟨*proof*⟩

lemma *uniq_Nonce_in_reqm [dest]: "⟦Nonce k ∈ parts {reqm A r n I B};
I ∈ agl⟧ ⟹ k=n"*
⟨*proof*⟩

lemma *uniq_Nonce_in_prom [dest]: "⟦Nonce k ∈ parts {prom B ofr A r I L J
C};
I ∈ agl; J ∈ agl; Nonce k ∉ parts {L}⟧ ⟹ k=ofr"*
⟨*proof*⟩

39.13 requests are guarded

lemma *req_imp_Guard [rule_format]: "⟦evs ∈ p2; A ∉ bad⟧ ⟹
req A r n I B ∈ set evs ⟶ Guard n {priK A} (spies evs)"*
⟨*proof*⟩

lemma *req_imp_Guard_Friend: "⟦evs ∈ p2; A ∉ bad; req A r n I B ∈ set evs⟧
⟹ Guard n {priK A} (knows_max (Friend C) evs)"*
⟨*proof*⟩

39.14 propositions are guarded

lemma *pro_imp_Guard [rule_format]: "⟦evs ∈ p2; B ∉ bad; A ∉ bad⟧ ⟹
pro B ofr A r I (cons M L) J C ∈ set evs ⟶ Guard ofr {priK A} (spies evs)"*
⟨*proof*⟩

lemma *pro_imp_Guard_Friend: "⟦evs ∈ p2; B ∉ bad; A ∉ bad;
pro B ofr A r I (cons M L) J C ∈ set evs⟧
⟹ Guard ofr {priK A} (knows_max (Friend D) evs)"*
⟨*proof*⟩

39.15 data confidentiality: no one other than the originator can decrypt the offers

lemma *Nonce_req_notin_spies: "⟦evs ∈ p2; req A r n I B ∈ set evs; A ∉ bad⟧
⟹ Nonce n ∉ analz (spies evs)"*

⟨proof⟩

lemma *Nonce_req_notin_knows_max_Friend: "⟦evs ∈ p2; req A r n I B ∈ set evs;
A ∉ bad; A ≠ Friend C⟧ ⟹ Nonce n ∉ analz (knows_max (Friend C) evs)"*
⟨proof⟩

lemma *Nonce_pro_notin_spies: "⟦evs ∈ p2; B ∉ bad; A ∉ bad;
pro B ofr A r I (cons M L) J C ∈ set evs⟧ ⟹ Nonce ofr ∉ analz (spies evs)"*
⟨proof⟩

lemma *Nonce_pro_notin_knows_max_Friend: "⟦evs ∈ p2; B ∉ bad; A ∉ bad;
A ≠ Friend D; pro B ofr A r I (cons M L) J C ∈ set evs⟧
⟹ Nonce ofr ∉ analz (knows_max (Friend D) evs)"*
⟨proof⟩

39.16 forward privacy: only the originator can know the identity of the shops

lemma *forward_privacy_Spy: "⟦evs ∈ p2; B ∉ bad; A ∉ bad;
pro B ofr A r I (cons M L) J C ∈ set evs⟧
⟹ sign B (Nonce ofr) ∉ analz (spies evs)"*
⟨proof⟩

lemma *forward_privacy_Friend: "⟦evs ∈ p2; B ∉ bad; A ∉ bad; A ≠ Friend D;
pro B ofr A r I (cons M L) J C ∈ set evs⟧
⟹ sign B (Nonce ofr) ∉ analz (knows_max (Friend D) evs)"*
⟨proof⟩

39.17 non repudiability: an offer signed by B has been sent by B

lemma *Crypt_reqm: "⟦Crypt (priK A) X ∈ parts {reqm A' r n I B}; I ∈ agl⟧
⟹ A=A'"*
⟨proof⟩

lemma *Crypt_prom: "⟦Crypt (priK A) X ∈ parts {prom B ofr A' r I L J C};
I ∈ agl; J ∈ agl⟧ ⟹ A=B | Crypt (priK A) X ∈ parts {L}"*
⟨proof⟩

lemma *Crypt_safeness: "⟦evs ∈ p2; A ∉ bad⟧ ⟹ Crypt (priK A) X ∈ parts (spies evs)
⟶ (∃B Y. Says A B Y ∈ set evs & Crypt (priK A) X ∈ parts {Y})"*
⟨proof⟩

lemma *Crypt_Hash_imp_sign: "⟦evs ∈ p2; A ∉ bad⟧ ⟹
Crypt (priK A) (Hash X) ∈ parts (spies evs)
⟶ (∃B Y. Says A B Y ∈ set evs ∧ sign A X ∈ parts {Y})"*
⟨proof⟩

lemma *sign_safeness: "⟦evs ∈ p2; A ∉ bad⟧ ⟹ sign A X ∈ parts (spies evs)
⟶ (∃B Y. Says A B Y ∈ set evs ∧ sign A X ∈ parts {Y})"*
⟨proof⟩

end

40 Needham-Schroeder-Lowe Public-Key Protocol

theory *Guard_NS_Public* **imports** *Guard_Public* **begin**

40.1 messages used in the protocol

```
abbreviation (input)
  ns1 :: "agent => agent => nat => event" where
  "ns1 A B NA == Says A B (Crypt (pubK B) {|Nonce NA, Agent A|})"

abbreviation (input)
  ns1' :: "agent => agent => agent => nat => event" where
  "ns1' A' A B NA == Says A' B (Crypt (pubK B) {|Nonce NA, Agent A|})"

abbreviation (input)
  ns2 :: "agent => agent => nat => nat => event" where
  "ns2 B A NA NB == Says B A (Crypt (pubK A) {|Nonce NA, Nonce NB, Agent B|})"

abbreviation (input)
  ns2' :: "agent => agent => agent => nat => nat => event" where
  "ns2' B' B A NA NB == Says B' A (Crypt (pubK A) {|Nonce NA, Nonce NB, Agent
B|})"

abbreviation (input)
  ns3 :: "agent => agent => nat => event" where
  "ns3 A B NB == Says A B (Crypt (pubK B) (Nonce NB))"
```

40.2 definition of the protocol

```
inductive_set nsp :: "event list set"
where

  Nil: "[] ∈ nsp"

| Fake: "⟦evs ∈ nsp; X ∈ synth (analz (spies evs))⟧ ⟹ Says Spy B X # evs
∈ nsp"

| NS1: "⟦evs1 ∈ nsp; Nonce NA ∉ used evs1⟧ ⟹ ns1 A B NA # evs1 ∈ nsp"

| NS2: "⟦evs2 ∈ nsp; Nonce NB ∉ used evs2; ns1' A' A B NA ∈ set evs2⟧ ⟹
  ns2 B A NA NB # evs2 ∈ nsp"

| NS3: "⋀A B B' NA NB evs3. ⟦evs3 ∈ nsp; ns1 A B NA ∈ set evs3; ns2' B' B
A NA NB ∈ set evs3⟧ ⟹
  ns3 A B NB # evs3 ∈ nsp"
```

40.3 declarations for tactics

declare *knows_Spy_partsEs [elim]*

declare *Fake_parts_insert [THEN subsetD, dest]*
declare *initState.simps [simp del]*

40.4 general properties of nsp

lemma *nsp_has_no_Gets: "evs ∈ nsp ⟹ ∀A X. Gets A X ∉ set evs"*
⟨*proof*⟩

lemma *nsp_is_Gets_correct [iff]: "Gets_correct nsp"*
⟨*proof*⟩

lemma *nsp_is_one_step [iff]: "one_step nsp"*
⟨*proof*⟩

lemma *nsp_has_only_Says' [rule_format]: "evs ∈ nsp ⟹*
ev ∈ set evs ⟶ (∃A B X. ev=Says A B X)"
⟨*proof*⟩

lemma *nsp_has_only_Says [iff]: "has_only_Says nsp"*
⟨*proof*⟩

lemma *nsp_is_regular [iff]: "regular nsp"*
⟨*proof*⟩

40.5 nonce are used only once

lemma *NA_is_uniq [rule_format]: "evs ∈ nsp ⟹*
Crypt (pubK B) ⦃Nonce NA, Agent A⦄ ∈ parts (spies evs)
⟶ Crypt (pubK B') ⦃Nonce NA, Agent A'⦄ ∈ parts (spies evs)
⟶ Nonce NA ∉ analz (spies evs) ⟶ A=A' ∧ B=B'"
⟨*proof*⟩

lemma *no_Nonce_NS1_NS2 [rule_format]: "evs ∈ nsp ⟹*
Crypt (pubK B') ⦃Nonce NA', Nonce NA, Agent A'⦄ ∈ parts (spies evs)
⟶ Crypt (pubK B) ⦃Nonce NA, Agent A⦄ ∈ parts (spies evs)
⟶ Nonce NA ∈ analz (spies evs)"
⟨*proof*⟩

lemma *no_Nonce_NS1_NS2' [rule_format]:*
"⟦Crypt (pubK B') ⦃Nonce NA', Nonce NA, Agent A'⦄ ∈ parts (spies evs);
Crypt (pubK B) ⦃Nonce NA, Agent A⦄ ∈ parts (spies evs); evs ∈ nsp⟧
⟹ Nonce NA ∈ analz (spies evs)"
⟨*proof*⟩

lemma *NB_is_uniq [rule_format]: "evs ∈ nsp ⟹*
Crypt (pubK A) ⦃Nonce NA, Nonce NB, Agent B⦄ ∈ parts (spies evs)
⟶ Crypt (pubK A') ⦃Nonce NA', Nonce NB, Agent B'⦄ ∈ parts (spies evs)
⟶ Nonce NB ∉ analz (spies evs) ⟶ A=A' ∧ B=B' ∧ NA=NA'"
⟨*proof*⟩

40.6 guardedness of NA

lemma *ns1_imp_Guard [rule_format]: "⟦evs ∈ nsp; A ∉ bad; B ∉ bad⟧ ⟹*
ns1 A B NA ∈ set evs ⟶ Guard NA {priK A,priK B} (spies evs)"

⟨proof⟩

40.7 guardedness of NB

lemma *ns2_imp_Guard [rule_format]: "⟦evs ∈ nsp; A ∉ bad; B ∉ bad⟧ ⟹ ns2 B A NA NB ∈ set evs ⟶ Guard NB {priK A,priK B} (spies evs)"*
⟨proof⟩

40.8 Agents' Authentication

lemma *B_trusts_NS1: "⟦evs ∈ nsp; A ∉ bad; B ∉ bad⟧ ⟹ Crypt (pubK B) ⦃Nonce NA, Agent A⦄ ∈ parts (spies evs) ⟶ Nonce NA ∉ analz (spies evs) ⟶ ns1 A B NA ∈ set evs"*
⟨proof⟩

lemma *A_trusts_NS2: "⟦evs ∈ nsp; A ∉ bad; B ∉ bad⟧ ⟹ ns1 A B NA ∈ set evs ⟶ Crypt (pubK A) ⦃Nonce NA, Nonce NB, Agent B⦄ ∈ parts (spies evs) ⟶ ns2 B A NA NB ∈ set evs"*
⟨proof⟩

lemma *B_trusts_NS3: "⟦evs ∈ nsp; A ∉ bad; B ∉ bad⟧ ⟹ ns2 B A NA NB ∈ set evs ⟶ Crypt (pubK B) (Nonce NB) ∈ parts (spies evs) ⟶ ns3 A B NB ∈ set evs"*
⟨proof⟩

end

41 Other Protocol-Independent Results

theory *Proto* **imports** *Guard_Public* **begin**

41.1 protocols

type_synonym *rule = "event set * event"*

abbreviation
  msg' :: "rule => msg" **where**
  "msg' R == msg (snd R)"

type_synonym *proto = "rule set"*

definition *wdef :: "proto => bool"* **where**
"wdef p ≡ ∀R k. R ∈ p ⟶ Number k ∈ parts {msg' R} ⟶ Number k ∈ parts (msg'‘(fst R))"

41.2 substitutions

record *subs =*
  agent :: "agent => agent"
  nonce :: "nat => nat"
  nb :: "nat => msg"
  key :: "key => key"

```
primrec apm :: "subs => msg => msg" where
  "apm s (Agent A) = Agent (agent s A)"
| "apm s (Nonce n) = Nonce (nonce s n)"
| "apm s (Number n) = nb s n"
| "apm s (Key K) = Key (key s K)"
| "apm s (Hash X) = Hash (apm s X)"
| "apm s (Crypt K X) = (
if (∃A. K = pubK A) then Crypt (pubK (agent s (agt K))) (apm s X)
else if (∃A. K = priK A) then Crypt (priK (agent s (agt K))) (apm s X)
else Crypt (key s K) (apm s X))"
| "apm s {|X,Y|} = {|apm s X, apm s Y|}"

lemma apm_parts: "X ∈ parts {Y} ⟹ apm s X ∈ parts {apm s Y}"
⟨proof⟩

lemma Nonce_apm [rule_format]: "Nonce n ∈ parts {apm s X} ⟹
(∀k. Number k ∈ parts {X} ⟶ Nonce n ∉ parts {nb s k}) ⟶
(∃k. Nonce k ∈ parts {X} ∧ nonce s k = n)"
⟨proof⟩

lemma wdef_Nonce: "⟦Nonce n ∈ parts {apm s X}; R ∈ p; msg' R = X; wdef p;
Nonce n ∉ parts (apm s `(msg `(fst R)))⟧ ⟹
(∃k. Nonce k ∈ parts {X} ∧ nonce s k = n)"
⟨proof⟩

primrec ap :: "subs ⇒ event ⇒ event" where
  "ap s (Says A B X) = Says (agent s A) (agent s B) (apm s X)"
| "ap s (Gets A X) = Gets (agent s A) (apm s X)"
| "ap s (Notes A X) = Notes (agent s A) (apm s X)"

abbreviation
  ap' :: "subs ⇒ rule ⇒ event" where
  "ap' s R ≡ ap s (snd R)"

abbreviation
  apm' :: "subs ⇒ rule ⇒ msg" where
  "apm' s R ≡ apm s (msg' R)"

abbreviation
  priK' :: "subs ⇒ agent ⇒ key" where
  "priK' s A ≡ priK (agent s A)"

abbreviation
  pubK' :: "subs ⇒ agent ⇒ key" where
  "pubK' s A ≡ pubK (agent s A)"
```

41.3 nonces generated by a rule

```
definition newn :: "rule ⇒ nat set" where
"newn R ≡ {n. Nonce n ∈ parts {msg (snd R)} ∧ Nonce n ∉ parts (msg`(fst
R))}"

lemma newn_parts: "n ∈ newn R ⟹ Nonce (nonce s n) ∈ parts {apm' s R}"
⟨proof⟩
```

41.4 traces generated by a protocol

definition *ok :: "event list ⇒ rule ⇒ subs ⇒ bool"* **where**
"ok evs R s ≡ ((∀x. x ∈ fst R ⟶ ap s x ∈ set evs)
∧ (∀n. n ∈ newn R ⟶ Nonce (nonce s n) ∉ used evs))"

inductive_set
tr :: "proto => event list set"
for *p :: proto*
where

Nil [intro]: "[] ∈ tr p"

| Fake [intro]: "⟦evsf ∈ tr p; X ∈ synth (analz (spies evsf))⟧
⟹ Says Spy B X # evsf ∈ tr p"

| Proto [intro]: "⟦evs ∈ tr p; R ∈ p; ok evs R s⟧ ⟹ ap' s R # evs ∈ tr p"

41.5 general properties

lemma *one_step_tr [iff]: "one_step (tr p)"*
⟨*proof*⟩

definition *has_only_Says' :: "proto => bool"* **where**
"has_only_Says' p ≡ ∀R. R ∈ p ⟶ is_Says (snd R)"

lemma *has_only_Says'D: "⟦R ∈ p; has_only_Says' p⟧*
⟹ (∃A B X. snd R = Says A B X)"
⟨*proof*⟩

lemma *has_only_Says_tr [simp]: "has_only_Says' p ⟹ has_only_Says (tr p)"*
⟨*proof*⟩

lemma *has_only_Says'_in_trD: "⟦has_only_Says' p; list @ ev # evs1 ∈ tr p⟧*
⟹ (∃A B X. ev = Says A B X)"
⟨*proof*⟩

lemma *ok_not_used: "⟦Nonce n ∉ used evs; ok evs R s;*
∀x. x ∈ fst R ⟶ is_Says x⟧ ⟹ Nonce n ∉ parts (apm s ‘(msg ‘(fst R)))"
⟨*proof*⟩

lemma *ok_is_Says: "⟦evs' @ ev # evs ∈ tr p; ok evs R s; has_only_Says' p;*
R ∈ p; x ∈ fst R⟧ ⟹ is_Says x"
⟨*proof*⟩

41.6 types

type_synonym *keyfun = "rule ⇒ subs ⇒ nat ⇒ event list ⇒ key set"*

type_synonym *secfun = "rule ⇒ nat ⇒ subs ⇒ key set ⇒ msg"*

41.7 introduction of a fresh guarded nonce

definition *fresh :: "proto ⇒ rule ⇒ subs ⇒ nat ⇒ key set ⇒ event list*

⇒ *bool"* **where**
"fresh p R s n Ks evs ≡ (∃evs1 evs2. evs = evs2 @ ap' s R # evs1
∧ Nonce n ∉ used evs1 ∧ R ∈ p ∧ ok evs1 R s ∧ Nonce n ∈ parts {apm' s R}
∧ apm' s R ∈ guard n Ks)"

lemma *freshD: "fresh p R s n Ks evs ⟹ (∃evs1 evs2.*
evs = evs2 @ ap' s R # evs1 ∧ Nonce n ∉ used evs1 ∧ R ∈ p ∧ ok evs1 R s
∧ Nonce n ∈ parts {apm' s R} ∧ apm' s R ∈ guard n Ks)"
⟨*proof*⟩

lemma *freshI [intro]: "⟦Nonce n ∉ used evs1; R ∈ p; Nonce n ∈ parts {apm' s R};*
ok evs1 R s; apm' s R ∈ guard n Ks⟧
⟹ fresh p R s n Ks (list @ ap' s R # evs1)"
⟨*proof*⟩

lemma *freshI': "⟦Nonce n ∉ used evs1; (l,r) ∈ p;*
Nonce n ∈ parts {apm s (msg r)}; ok evs1 (l,r) s; apm s (msg r) ∈ guard n Ks⟧
⟹ fresh p (l,r) s n Ks (evs2 @ ap s r # evs1)"
⟨*proof*⟩

lemma *fresh_used: "⟦fresh p R' s' n Ks evs; has_only_Says' p⟧*
⟹ Nonce n ∈ used evs"
⟨*proof*⟩

lemma *fresh_newn: "⟦evs' @ ap' s R # evs ∈ tr p; wdef p; has_only_Says' p;*
Nonce n ∉ used evs; R ∈ p; ok evs R s; Nonce n ∈ parts {apm' s R}⟧
⟹ ∃k. k ∈ newn R ∧ nonce s k = n"
⟨*proof*⟩

lemma *fresh_rule: "⟦evs' @ ev # evs ∈ tr p; wdef p; Nonce n ∉ used evs;*
Nonce n ∈ parts {msg ev}⟧ ⟹ ∃R s. R ∈ p ∧ ap' s R = ev"
⟨*proof*⟩

lemma *fresh_ruleD: "⟦fresh p R' s' n Ks evs; keys R' s' n evs ⊆ Ks; wdef p;*
has_only_Says' p; evs ∈ tr p; ∀R k s. nonce s k = n ⟶ Nonce n ∈ used evs ⟶
R ∈ p ⟶ k ∈ newn R ⟶ Nonce n ∈ parts {apm' s R} ⟶ apm' s R ∈ guard n Ks ⟶
apm' s R ∈ parts (spies evs) ⟶ keys R s n evs ⊆ Ks ⟶ P⟧ ⟹ P"
⟨*proof*⟩

41.8 safe keys

definition *safe :: "key set ⇒ msg set ⇒ bool"* **where**
"safe Ks G ≡ ∀K. K ∈ Ks ⟶ Key K ∉ analz G"

lemma *safeD [dest]: "⟦safe Ks G; K ∈ Ks⟧ ⟹ Key K ∉ analz G"*
⟨*proof*⟩

lemma *safe_insert: "safe Ks (insert X G) ⟹ safe Ks G"*

⟨*proof*⟩

lemma *Guard_safe: "⟦Guard n Ks G; safe Ks G⟧ ⟹ Nonce n ∉ analz G"*
⟨*proof*⟩

41.9 guardedness preservation

definition *preserv :: "proto ⇒ keyfun ⇒ nat ⇒ key set ⇒ bool"* **where**
"preserv p keys n Ks ≡ (∀evs R' s' R s. evs ∈ tr p ⟶
Guard n Ks (spies evs) ⟶ safe Ks (spies evs) ⟶ fresh p R' s' n Ks evs
⟶
keys R' s' n evs ⊆ Ks ⟶ R ∈ p ⟶ ok evs R s ⟶ apm' s R ∈ guard n Ks)"

lemma *preservD: "⟦preserv p keys n Ks; evs ∈ tr p; Guard n Ks (spies evs);*
safe Ks (spies evs); fresh p R' s' n Ks evs; R ∈ p; ok evs R s;
keys R' s' n evs ⊆ Ks⟧ ⟹ apm' s R ∈ guard n Ks"
⟨*proof*⟩

lemma *preservD': "⟦preserv p keys n Ks; evs ∈ tr p; Guard n Ks (spies evs);*
safe Ks (spies evs); fresh p R' s' n Ks evs; (l,Says A B X) ∈ p;
ok evs (l,Says A B X) s; keys R' s' n evs ⊆ Ks⟧ ⟹ apm s X ∈ guard n Ks"
⟨*proof*⟩

41.10 monotonic keyfun

definition *monoton :: "proto => keyfun => bool"* **where**
"monoton p keys ≡ ∀R' s' n ev evs. ev # evs ∈ tr p ⟶
keys R' s' n evs ⊆ keys R' s' n (ev # evs)"

lemma *monotonD [dest]: "⟦keys R' s' n (ev # evs) ⊆ Ks; monoton p keys;*
ev # evs ∈ tr p⟧ ⟹ keys R' s' n evs ⊆ Ks"
⟨*proof*⟩

41.11 guardedness theorem

lemma *Guard_tr [rule_format]: "⟦evs ∈ tr p; has_only_Says' p;*
preserv p keys n Ks; monoton p keys; Guard n Ks (initState Spy)⟧ ⟹
safe Ks (spies evs) ⟶ fresh p R' s' n Ks evs ⟶ keys R' s' n evs ⊆ Ks
⟶
Guard n Ks (spies evs)"
⟨*proof*⟩

41.12 useful properties for guardedness

lemma *newn_neq_used: "⟦Nonce n ∈ used evs; ok evs R s; k ∈ newn R⟧*
⟹ n ≠ nonce s k"
⟨*proof*⟩

lemma *ok_Guard: "⟦ok evs R s; Guard n Ks (spies evs); x ∈ fst R; is_Says*
x⟧
⟹ apm s (msg x) ∈ parts (spies evs) ∧ apm s (msg x) ∈ guard n Ks"
⟨*proof*⟩

lemma *ok_parts_not_new: "⟦Y ∈ parts (spies evs); Nonce (nonce s n) ∈ parts {Y};
ok evs R s⟧ ⟹ n ∉ newn R"*
⟨*proof*⟩

41.13 unicity

definition *uniq :: "proto ⇒ secfun ⇒ bool"* **where**
*"uniq p secret ≡ ∀evs R R' n n' Ks s s'. R ∈ p ⟶ R' ∈ p ⟶
n ∈ newn R ⟶ n' ∈ newn R' ⟶ nonce s n = nonce s' n' ⟶
Nonce (nonce s n) ∈ parts {apm' s R} ⟶ Nonce (nonce s n) ∈ parts {apm' s' R'} ⟶
apm' s R ∈ guard (nonce s n) Ks ⟶ apm' s' R' ∈ guard (nonce s n) Ks ⟶
evs ∈ tr p ⟶ Nonce (nonce s n) ∉ analz (spies evs) ⟶
secret R n s Ks ∈ parts (spies evs) ⟶ secret R' n' s' Ks ∈ parts (spies evs) ⟶
secret R n s Ks = secret R' n' s' Ks"*

lemma *uniqD: "⟦uniq p secret; evs ∈ tr p; R ∈ p; R' ∈ p; n ∈ newn R; n' ∈ newn R';
nonce s n = nonce s' n'; Nonce (nonce s n) ∉ analz (spies evs);
Nonce (nonce s n) ∈ parts {apm' s R}; Nonce (nonce s n) ∈ parts {apm' s' R'};
secret R n s Ks ∈ parts (spies evs); secret R' n' s' Ks ∈ parts (spies evs);
apm' s R ∈ guard (nonce s n) Ks; apm' s' R' ∈ guard (nonce s n) Ks⟧ ⟹
secret R n s Ks = secret R' n' s' Ks"*
⟨*proof*⟩

definition *ord :: "proto ⇒ (rule ⇒ rule ⇒ bool) ⇒ bool"* **where**
"ord p inff ≡ ∀R R'. R ∈ p ⟶ R' ∈ p ⟶ ¬ inff R R' ⟶ inff R' R"

lemma *ordD: "⟦ord p inff; ¬ inff R R'; R ∈ p; R' ∈ p⟧ ⟹ inff R' R"*
⟨*proof*⟩

definition *uniq' :: "proto ⇒ (rule ⇒ rule ⇒ bool) ⇒ secfun ⇒ bool"* **where**
*"uniq' p inff secret ≡ ∀evs R R' n n' Ks s s'. R ∈ p ⟶ R' ∈ p ⟶
inff R R' ⟶ n ∈ newn R ⟶ n' ∈ newn R' ⟶ nonce s n = nonce s' n' ⟶
Nonce (nonce s n) ∈ parts {apm' s R} ⟶ Nonce (nonce s n) ∈ parts {apm' s' R'} ⟶
apm' s R ∈ guard (nonce s n) Ks ⟶ apm' s' R' ∈ guard (nonce s n) Ks ⟶
evs ∈ tr p ⟶ Nonce (nonce s n) ∉ analz (spies evs) ⟶
secret R n s Ks ∈ parts (spies evs) ⟶ secret R' n' s' Ks ∈ parts (spies evs) ⟶
secret R n s Ks = secret R' n' s' Ks"*

lemma *uniq'D: "⟦uniq' p inff secret; evs ∈ tr p; inff R R'; R ∈ p; R' ∈ p; n ∈ newn R;
n' ∈ newn R'; nonce s n = nonce s' n'; Nonce (nonce s n) ∉ analz (spies evs);
Nonce (nonce s n) ∈ parts {apm' s R}; Nonce (nonce s n) ∈ parts {apm' s' R'};
secret R n s Ks ∈ parts (spies evs); secret R' n' s' Ks ∈ parts (spies evs);
apm' s R ∈ guard (nonce s n) Ks; apm' s' R' ∈ guard (nonce s n) Ks⟧ ⟹
secret R n s Ks = secret R' n' s' Ks"*
⟨*proof*⟩

lemma *uniq'_imp_uniq: "⟦uniq' p inff secret; ord p inff⟧ ⟹ uniq p secret"*

⟨proof⟩

41.14 Needham-Schroeder-Lowe

```
definition a :: agent where "a == Friend 0"
definition b :: agent where "b == Friend 1"
definition a' :: agent where "a' == Friend 2"
definition b' :: agent where "b' == Friend 3"
definition Na :: nat where "Na == 0"
definition Nb :: nat where "Nb == 1"

abbreviation
  ns1 :: rule where
  "ns1 == ({}, Says a b (Crypt (pubK b) {|Nonce Na, Agent a|}))"

abbreviation
  ns2 :: rule where
  "ns2 == ({Says a' b (Crypt (pubK b) {|Nonce Na, Agent a|})},
    Says b a (Crypt (pubK a) {|Nonce Na, Nonce Nb, Agent b|}))"

abbreviation
  ns3 :: rule where
  "ns3 == ({Says a b (Crypt (pubK b) {|Nonce Na, Agent a|}),
    Says b' a (Crypt (pubK a) {|Nonce Na, Nonce Nb, Agent b|})},
    Says a b (Crypt (pubK b) (Nonce Nb)))"

inductive_set ns :: proto where
  [iff]: "ns1 ∈ ns"
| [iff]: "ns2 ∈ ns"
| [iff]: "ns3 ∈ ns"

abbreviation (input)
  ns3a :: event where
  "ns3a == Says a b (Crypt (pubK b) {|Nonce Na, Agent a|})"

abbreviation (input)
  ns3b :: event where
  "ns3b == Says b' a (Crypt (pubK a) {|Nonce Na, Nonce Nb, Agent b|})"

definition keys :: "keyfun" where
"keys R' s' n evs == {priK' s' a, priK' s' b}"

lemma "monoton ns keys"
⟨proof⟩

definition secret :: "secfun" where
"secret R n s Ks ==
(if R=ns1 then apm s (Crypt (pubK b) {|Nonce Na, Agent a|})
else if R=ns2 then apm s (Crypt (pubK a) {|Nonce Na, Nonce Nb, Agent b|})
else Number 0)"

definition inf :: "rule => rule => bool" where
"inf R R' == (R=ns1 | (R=ns2 & R'~=ns1) | (R=ns3 & R'=ns3))"
```

lemma *inf_is_ord [iff]: "ord ns inf"*
⟨*proof*⟩

41.15 general properties

lemma *ns_has_only_Says' [iff]: "has_only_Says' ns"*
⟨*proof*⟩

lemma *newn_ns1 [iff]: "newn ns1 = {Na}"*
⟨*proof*⟩

lemma *newn_ns2 [iff]: "newn ns2 = {Nb}"*
⟨*proof*⟩

lemma *newn_ns3 [iff]: "newn ns3 = {}"*
⟨*proof*⟩

lemma *ns_wdef [iff]: "wdef ns"*
⟨*proof*⟩

41.16 guardedness for NSL

lemma *"uniq ns secret ⟹ preserv ns keys n Ks"*
⟨*proof*⟩

41.17 unicity for NSL

lemma *"uniq' ns inf secret"*
⟨*proof*⟩

end

42 Blanqui's "guard" concept: protocol-independent secrecy

theory *Auth_Guard_Public*
imports
P1
P2
Guard_NS_Public
Proto
begin

end