A Proof Checker for HOL

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Wai Wong

The author is now in the Department of Computing Studies, Hong Kong Baptist University.
Email: `wwong@comp.hkbu.edu.hk`, URL: `http://www.comp.hkbu.edu.hk/~wwong/`

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Abstract

Formal proofs generated mechanically by theorem provers are often very large and shallow, and the theorem provers are themselves very complex. Therefore, in certain application areas, such as in safety-critical systems, it is necessary to have an independent means for ensuring the consistency of such formal proofs. This report describes an efficient proof checker for the HOL theorem prover. This proof checker has been tested with practical proofs consisting of thousands of inference steps. It was implemented in Standard ML of New Jersey.

The first part of the report gives an overview of the program. It describes

- the rationale of developing a proof checker,
- how to use the checker, and
- how the checker works.

The second part of the report describes the program in detail. The complete source code is included in the description.

Acknowledgement

The idea of recording inference steps and generating proof lines has been suggested by many people including Malcolm Newey and Keith Hanna. Mike Gordon implemented a prototype of the recording functions in HOL88. The translation of the formal HOL proof theory into ML functions used in the formal version was carried out by John Herbert. It was a great pleasure to work with Mike, Paul, Brian and others in the Hardware Verification Group in the Computer Laboratory in Cambridge. The work described here would not have been completed without their invaluable advice, help and company. A special thank is given to Paul for his efforts in proof reading a draft of the report. He corrected many mistakes and gave many suggestions.

Part I

An Overview

1 Introduction

Formal methods have been used in the development of many safety-critical systems in the form of formal specification and formal proof of correctness. Formal proofs are usually carried out using *theorem provers* or *proof assistants*. These systems are based on well-founded formal logic, and provide a programming environment within which the user can discover, construct and perform proofs. The result of this process is usually a set of theorems which can be stored in a disk file and used in subsequent proofs. HOL is one of the most popular theorem proving environments. The users interact with the system by writing and evaluating ML programs. The programs instruct the system how to perform proofs. A proof is a sequence of inferences. It is transient in the HOL system in the sense that there is no object that exists as a proof once a theorem has been derived.

In some safety-critical applications, computer systems are used to implement some of the highest risk category functions. The design of such a system is often formally verified. The verification usually produces a large proof consisting of tens of thousands, even up to several millions, of inferences. [9] describes a proof of correctness of an ALU consisting of a quarter of a million inference steps. In such situations, it is desirable to check the consistency of the sequence of inferences with an independent checker. The reasons for requiring independent checking are:

- the mechanically generated formal proofs are usually very long, often consists of thousands, even millions of inferences;
- the mechanically generated formal proofs are usually very shallow in the sense that they are not mathematically interesting;
- the theorem proving systems are usually very complex so that it is extremely difficult (if not impossible) to verify their correctness;
- the programs that a user develops while doing the proof are very often too complicated and do not have a simple mapping to the sequence of inferences performed by the system;
- the requirement of the certification bodies, for example, the U. K. Defence standard 00 – 55 calls for such an independent proof checker when the 'highest degree of assurance in the design' is required [7].

An independent proof checker can be much simpler than the theorem prover so that it is possible to be verified formally. The relation between the checker and the HOL system is shown in Figure 1.1. The top half shows the usual process of using the HOL system to perform proofs. The bottom half shows the process of checking proofs generated by the HOL system. These two process may be carried out at different times by different people in different places since the checker is a totally independent system.

Described in this document is a proof checker for the HOL theorem prover. The dominant requirement of this checker is that it is able to check large proofs generated from real applications. This means that the implementation should be fast and efficient, and should be able to perform reasonably well with limited resources, i.e., limited amount of memory. With its possible verification in mind, the checker fairly closely follows von Wright's formal theory[8] which is described briefly in Section 1.2.

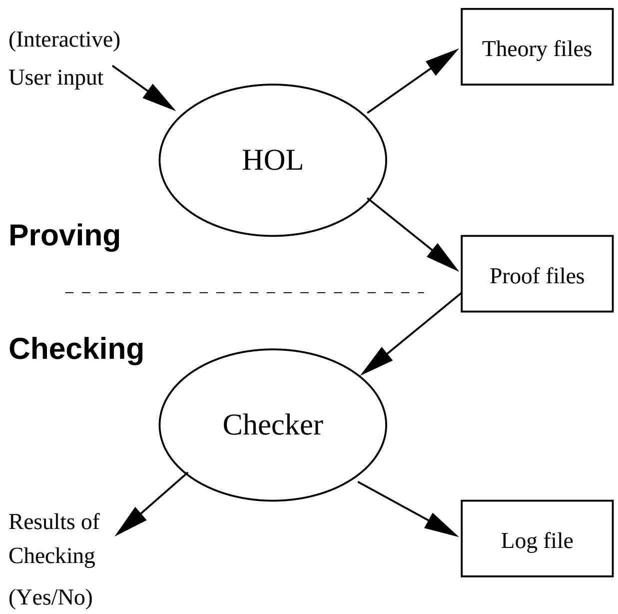


Figure 1.1: Relation between the checker and the HOL system

The proof checker is implemented in Standard ML of New Jersey (SML/NJ). Two versions of the checker have been implemented: the *core version* and the *efficient version*. The core version is able to check proofs consisting of only primitive inferences and its critical part is translated directly from the formal theory. The efficient version is able to check larger proofs. The remainder of PART I gives an overview of the checker, and PART II describes the entire program of the efficient version in detail.

1.1 Proofs in HOL

A proof is a sequence of logical inferences. All theorems in a logic can be derived from the axioms by applying the inference rules. The proof theory of the logic specifies what the valid inferences are.

In the HOL logic, there are five axioms and eight primitive inference rules. Derived inference rules are sequences of inferences grouped together. Both primitive and derived inference rules are implemented by ML functions in the HOL system. By calling these ML functions, a user can carry out simple proofs to derive theorems from existing theorems. This style is known as *forward proof*, and it is used mainly in simple proofs.

More often, users will do *goal-directed proofs* in HOL. In this style, a conjecture is set up as a goal. Tactics are then applied to reduce the goal to simpler subgoals recursively until the resulting subgoals are all resolved. The HOL system validates the tactics, assembles a derivation, and performs the inferences to derive the theorem.

Both proof styles in HOL can be modelled by Hilbert's proof style, i.e., a proof is a sequence of inference steps, each step is written on a single line with a line number, the theorem derived in the step and a justification, such as the simple example in Figure 1.2.

According to whether or not proofs are generated entirely in terms of primitive inferences of the logic, theorem provers can be classified as *fully-expansive* or *partially-expansive*[2]. Fully-expansive theorem provers perform every single primitive inference when deriving theorems.

1.	$\Gamma \vdash t_1 = t_2$	[Hypothesis]
2.	$\vdash t_1 = t_1$	[Reflexivity]
3.	$\Gamma \vdash t_2 = t_1$	[Substitution of 1 into 2]

Figure 1.2: A simple proof in Hilbert style

Because of the lengthy process, the performance of this type of theorem prover is often limited. Early versions of the HOL system were fully-expansive. To improve the performance, thirty-four frequently used derived inference rules are implemented directly in the current version of HOL. The primitive rules, directly implemented derived rules and several functions for introducing new constants and accessing stored theorems are collectively known as *basic* inference rules.

The proofs generated by the HOL system can be saved in a proof file which contains a textual representation of the sequences of inferences constituting the proofs. This file is the interface between the theorem prover and the proof checker.

1.2 Formal HOL proof theory

A formal theory of the HOL logic proof theory has been developed in HOL by von Wright [8]. This provides a theoretical base on which a formal specification of a proof checker may be developed and against which verification of the checker may be performed.

In von Wright's theory of the HOL logic, a type `Type` is defined to represent HOL types. Similarly, a type `Pterm` is defined to represent HOL terms. Theorems are represented by the type `Pseq`, standing for sequent.

HOL inference rules are written as

$$\frac{\Gamma_1 \vdash t_1 \quad \cdots \quad \Gamma_n \vdash t_n}{\Gamma \vdash t}\,\texttt{INF\_RULE}$$

where the sequents above the line are known as *hypotheses* and the sequent below the line is known as the *conclusion*. The name of the inference rule and the side conditions, if any, are written next to the line. Inference rules are implemented in the HOL system as ML functions which usually take the hypotheses as the arguments and returns the conclusion. In von Wright's theory, inference rules are presented by predicates on the hypotheses and conclusion. A theorem asserting the validity of an inference can be derived if the conclusion can be derived from the set of hypothesis sequents using the inference rule. For example, if $P_{infrule}$ is the predicate representing the hypothetical inference `INF_RULE` above, if the inference is valid, the following theorem can be derived.

$$P_{infrule}\{\Gamma_1 \vdash t_1 \quad \cdots \quad \Gamma_n \vdash t_n\} \quad \Gamma \vdash t$$

1.3 Proof file format

The proof accepted by the checker is in a proof file format `prf`. The format is described in detail in the Cambridge University Computer Laboratory technical report No. 306 [10]. The `prf` format is based on the Hilbert proof style model described in Section 1.1.

The proof file format `prf` has two levels: the *core* level, which allows proofs consisting of only primitive inference rules to be written into the file, and the *extended* level, which allows all basic inference rules.

A proof file may contain one or more proofs. Each proof has a name, a goal list and a list of proof lines. The name is a string used to identify the proof. The goal list may be empty or have one or more theorems. This is a mechanism for improving the efficiency of the checker. If the goal list is empty, the checker will check the entire proof. If all the theorems in the goal list have been found in the proof lines, the checker can conclude that the goals have been proved and stop processing the remainder of the proof. This is useful because certain automatic proof procedures may perform more inferences than necessary to derive the required theorem. This is based on some observations on the proofs generated by HOL. Its usefulness in practice is still not known. However, the checker can always ignore the goal list and check the entire proof. In fact, this is the default behaviour of the checker. Each proof line has three parts: the *line number*, the *justification*, which is the name of the inference rule and the arguments supplied to the rule, and the *derived theorem*.

The proof file format is primarily an interface between the HOL theorem prover and other independent proof checkers. Files in `prf` format are intended to be read by machines not humans. The concrete syntax of the `prf` format is similar to a LISP S-expression. For example, the proof file in `prf` format of the proof shown in Figure 1.2 is listed in Figure 1.3 below.

```
(VERSION PRF FORMAT 1.0 EXTENDED)
(ENV HOL[][])

(PROOF sym
[]
[
(LINE -1(Hypothesis)
 (THM [](A (A (C = (o fun[(c bool)(o fun[(c bool)(c bool)])]))
  (A (A (C /\ (o fun[(c bool)(o fun[(c bool)(c bool)])]))
  (V t1(c bool)))(V t2(c bool))))
  (A (A (C /\ (o fun[(c bool)(o fun[(c bool)(c bool)])]))
  (V t2(c bool)))(V t1(c bool)))))
)
(LINE 1(Refl (A (A (C /\ (o fun[(c bool)(o fun[(c bool)(c bool)])]))
  (V t1(c bool)))(V t2(c bool))))
 (THM [](A (A (C = (o fun[(c bool)(o fun[(c bool)(c bool)])]))
  (A (A (C /\ (o fun[(c bool)(o fun[(c bool)(c bool)])]))
  (V t1(c bool)))(V t2(c bool))))(A (A (C /\ (o fun[(c bool)
   (o fun[(c bool)(c bool)])]))(V t1(c bool)))(V t2(c bool)))))
)
(LINE 2(Subst [{-1(V GEN%VAR%509(c bool))}]
  (A (A (C = (o fun[(c bool)(o fun[(c bool)(c bool)])]))
   (V GEN%VAR%509(c bool)))(A (A (C /\ (o fun[(c bool)
    (o fun[(c bool)(c bool)])]))(V t1(c bool)))(V t2(c bool))))1)
 (THM [](A (A (C = (o fun[(c bool)(o fun[(c bool)(c bool)])]))
  (A (A (C /\ (o fun[(c bool)(o fun[(c bool)(c bool)])]))
  (V t2(c bool)))(V t1(c bool))))(A (A (C /\ (o fun[(c bool)
  (o fun[(c bool)(c bool)])]))(V t1(c bool)))(V t2(c bool)))))
)
])
```

Figure 1.3: Proof file in `prf` format of a simple proof

2 The Checker

2.1 Checking HOL Proofs

To check a HOL proof is to make sure that every inference step in the proof is valid. An inference step is valid if and only if

- all hypotheses are in the correct form as specified by the inference rule;
- all hypotheses are either axioms or theorems derived in previous inference steps;
- the conclusion is in the correct form as specified by the inference rule; and
- all the types and terms appeared in the hypotheses and conclusion are well formed in the current signature, i.e., the types and constants known at that point.

Since the HOL logic has only eight primitive inference rules, a checker for proofs consisting of only primitive inferences will be relatively simple, so it may possibly be verified formally. This corresponds to the core level of the **prf** proof file format. A version of the checker, the *core checker*, accepting only core level proof files was developed first. With the ultimate goal of formal verification in mind, the core checker follows von Wright's formal HOL proof theory very closely.

However, the current version (HOL88 version 2.2) of the HOL system is not fully-expansive. The proofs generated by HOL consist of all the basic inferences. The proof files are in an extended level and cannot be accepted by the core checker.

There are basically two different approaches to implementing an extended level checker. The first approach is to write a program to expand the inference steps involving derived rules into a sequence of primitive steps before being sent to the core checker. This approach has the advantage of utilising the core checker which may be formally verified, therefore, achieving higher confidence in the consistency of the proof. However, this approach can increase the number of inference steps considerably so the amount of time required to check the proof will take much longer.[1] The second approach is to check all basic inference rules directly. This approach can result in a more efficient checker since the basic derived rules are relatively simple to check.

Since one of the requirements of this project is to demonstrate the feasibility of proof checking for real practical proofs, which consist of thousands or tens of thousands of inference steps, the checker should be fast and efficient, and should be able to perform reasonably well with limited resources, i.e., limited amount of physical memory and disk space.

Described in Part II of this report is an implementation of the checker using the second approach. This version is known as the *efficient checker*. It defers from the core level checker mainly in the internal representation of the terms, the handling of the derived theorems. Other parts of the checker, such as the file I/O, the proof file parser and the error and exception handling are identical.

For internal representation of terms, the efficient checker uses de Bruijn's nameless representation. This makes the α-equivalence test and substitution simpler.

[1] By examining the derivations of the derived rules, one can see that each derived rule may be expanded into five to twenty primitive rules. A large proportion of inferences in normal proofs are non-primitive.

No matter which approach is used to implement a checker, its memory requirements are very large for large proofs because all theorems derived in the sequence have to be kept in memory. This is because a theorem derived in an earlier step may be referred to by the very last step. Logically, many modern systems are able to address many gigabytes, even up to terabytes of virtual memory, but physical memory is still limited. When large numbers of theorems are kept in memory, thrashing occurs, thus slowing down the process. This problem has been solved in the efficient checker by processing the proof file in two passes (see Section 2.3).

2.2 Using the Proof Checker

To a user, the checker is a program which reads a proof file, checks the proofs in it and reports back with either a success which means the proofs are correct or a failure which means the opposite (see Figure 1.1). It creates a log file containing information of what hypotheses and stored theorems have been used and the resulting theorems of the proofs. The log file is in a format similar to the proof file. It is mainly for use by other programs.

2.2.1 Loading the checker

Currently, the checker program modules have to be loaded into SML by evaluating the expression

```
use "join1.sml";
```

This will compile and link the modules to form the checker. A top-level function, namely `check_proof`, will then be defined as the entry point of the checker. When the program becomes stable, it will be possible to save an executable image of the checker. Then, it can be invoked as a shell command.

2.2.2 Invoking the checker

After loading the checker, it can be invoked by evaluating the function `check_proof` which takes a string as its sole argument. The string is the proof file name which, by convention, has the suffix `.prf` but the checker accepts any name. If the filename has a suffix `.gz`, the checker will assume it is a compressed file. It will run a decompresser automatically, and the log file will also be stored in a compressed form. The default compression/decompression utilities are the GNU `gzip/gunzip` programs. Below is a sample session of using the checker to check a compressed proof file named `MUL_FUN_CURRY` in the directory `proofs` parallel to the current directory. (Some of the output produced by the checker are omitted for brevity.)

```
                                                                      1
- check_proof "../proofs/MULT_FUN_CURRY.prf.gz";

Current environment: MULT_FUN_CURRY

Proof: MULT_FUN_CURRY
Proof MULT_FUN_CURRY has been checked

Proof: MULT_FUN_CURRY_THM
Proof MULT_FUN_CURRY_THM has been checked

Using the following hypotheses:
<-8>   |- T :bool

   { ...  some theorems omitted ...}

Proof: MULT_FUN
Proof MULT_FUN has been checked

Proof: MULT_FUN_DEF
Proof MULT_FUN_DEF has been checked

Using the following hypotheses:

   { ...  some theorems omitted ...}

val it = () : unit
-
```

The name of the log file is derived from the input file name. If the input file name has the suffix `.prf`, it is replaced by the suffix `.clg` (stands for *c*hecker *log* file). If the input file name has no suffix, the log file suffix is appended.

2.3 Operations of the checker

When the checker is invoked, it creates a decompress process running as a filter in the background. The communication between this process and the checker is via a socket. In the case where the file is not compressed, no decompression process is needed, but the communication is still via a socket. This arrangement simplifies the checker as its input routine always reads from the input socket. Similarly, an output socket is created with a compression process running to compress the output to the log file on the fly. Figure 2.1 illustrates this arrangement.

The checker processes the input file in *two* passes. In the first pass, it builds up a table of theorem references. Each inference step (also known as proof line) may refer to theorems derived in any previous steps. The table has an entry for each line. The entry contains the largest line number that that proof line is referenced by. As the references only occur in some justifications, most of the text can be quickly skipped over.

In the second pass, the checker analyses and checks every proof line according to the specification of the inference rules. After checking each line, the derived theorem is saved in a dictionary keyed by the line numbers if it is referred to by later lines. When a theorem is fetched from the dictionary for the last time, it is removed. Thus, a minimum amount of memory is required.

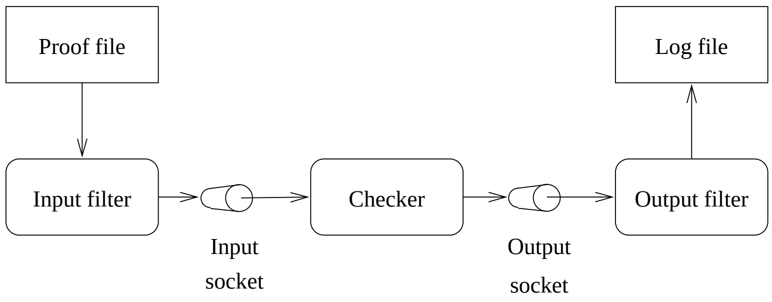


Figure 2.1: Checker input/output arrangement

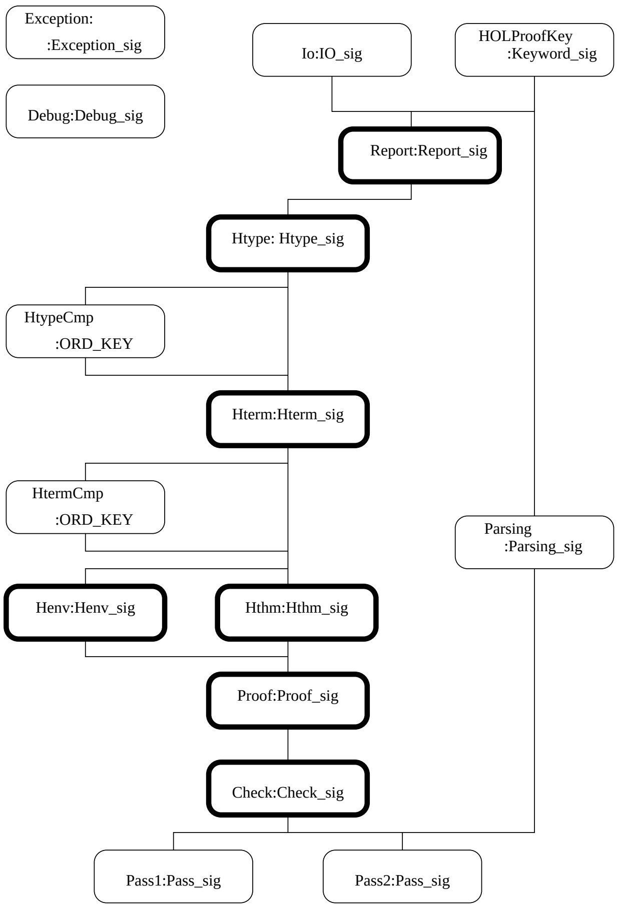


Figure 2.2: Organisation of modules

2.4 Organisation of modules

The checker is structured into a number of modules as shown in Fig. 2.2. The modules can be divided into two groups: the core group and the auxiliary group. Modules in the core group are shown in the figure with a thick border, whereas other modules are shown with a thin border.

$$\overline{t \vdash t}$$

a) The inference rule

```
|- !Typl Conl as t tm.
 PASSUME Typl Conl (Pseq as t) tm =



  Pwell_typed Typl Conl tm /\
  Pboolean tm /\
  (t = tm) /\
  (as = {tm})
```

b) The formal definition

```
fun chk_Assume (term, thm) =
  let val [ass] =
        HtermSet.listItems (hyp thm)
      val conc = concl thm
  in
    is_bool_ty term andalso
    (conc = term) andalso
    (conc = ass)
  end;
```

c) The checking function

Figure 2.3: Checking function and formal definition of the primitive rule `ASSUME`

2.4.1 The Core Modules

The core modules implement the internal representation of HOL types, terms, theorems, proofs, and proof environment. Each kind of objects is represented by an ML type. The type and associated operations on the objects of the type are implemented in a module. The module names are generally descriptive so it is very easy to identify the module for a particular kind of object. The `Check` module contains functions for checking the consistency of all basic inference rules. All checking functions for the primitive rules are the same as the formal version except for very minor changes to take care of the slightly different representation of HOL types and terms. Figure 2.3 shows the primitive rule `ASSUME` , its formal definition in the HOL proof theory and the checking function. The SML function and the HOL definition are very close.

The functions for checking other basic rules are derived from specification of these rules found in [5]. Fig. 2.4 shows the basic inference rule `SYM` and its checking function.

2.4.2 The two passes

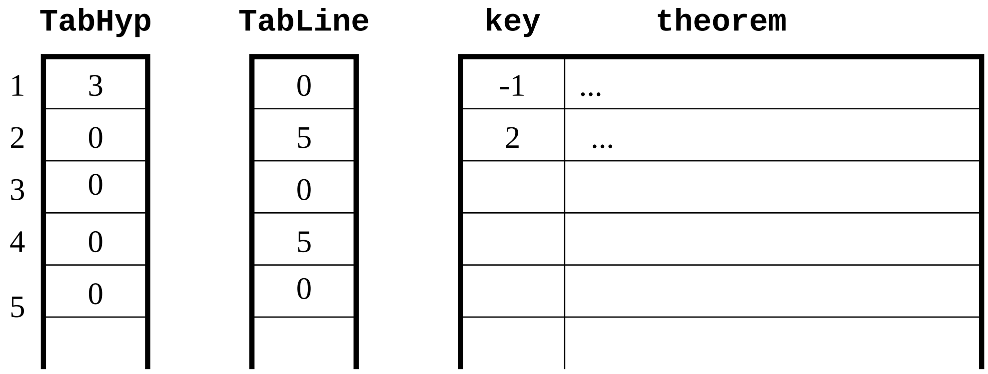
In the first pass, the checker builds a theorem reference table. This table consists of two dynamic arrays whose elements are integers as shown in Fig. 2.5a. Each element represents a proof line. The indices to the elements are the proof line numbers. Since the proof lines are numbered with both positive and negative numbers, but only non-negative numbers are allowed in indexing the array, two arrays are used. The `TabHyp` array is for the hypothesis lines whose line numbers are negative, and the `TabLine` array is for proof lines whose numbers are positive. These arrays

$$\frac{\Gamma \vdash t_1 = t_2}{\Gamma \vdash t_2 = t_1}$$

```
fun chk_Sym(line, n, thm) =
 let val thm1 = get_thm(line, n)
     val (left,right) = dest_eq (concl thm1)
 in
    ((right,left) = dest_eq (concl thm)) andalso
     (HtermSet.equal((hyp thm), (hyp thm1)))
 end
```

Figure 2.4: Basic rule `SYM` and its checking function

are created using the `DynamicArray` module in the SML/NJ library. The use of dynamic arrays instead of static ones removes the upper limit of the number of lines in the proof.



a) Theorem reference table

b) Theorem table

Figure 2.5: Data structures for theorem references

In the first pass, the checker looks at the justification part of the proof lines. When it encounters a reference to a theorem in a previous proof line, it enters the current line number into the element corresponding to the referred line in the table. For example, when the checker is at Line 3, it finds that this line refers to the theorem in hypothesis Line 1. It enters 3 into the first element of `TabHyp`. To speed up the Pass 1 process, the checker skips over other parts of the proof line quickly. This is done by scanning the input and looking for matching parentheses only. At the end of this pass, each element of the theorem reference table will contain the highest line number which is the latest line referring to the theorem. In the table shown in Fig. 2.5a, Line 5 is the last line referring to the theorem derived in Line 2 and Line 4.

In the second pass, the checker stores theorems referred to by other proof lines in a theorem table. This table is implemented by a dictionary in the `Dict` module of the SML/NJ library. The key of each entry is the line number. Since the dictionary is represented by a balanced splay tree, searching for a theorem is fast. After checking a proof line, the checker examines the theorem reference table, if the value of the current element is greater than the current line number, i.e., it will be referred to later, the theorem is saved in the theorem table. Fig. 2.5b illustrates the situation in which the checker has just stored the theorem derived in Line 2. When the checker retrieves a theorem, it also examines the theorem reference table. If the current line is the last one to refer to the theorem, i.e., the current line number is equal to the value in the table, the theorem is removed from the dictionary. Continuing the scenario in Fig. 2.5b, the next line is Line 3, which refers to hypotheses Line 1. Since this is the last line referring to the theorem, the checker removes it from the table. This arrangement minimises the number of theorems stored in the table, thus reducing the memory requirement.

2.4.3 Auxiliary Modules

The `HOLProofKey` module defines the concrete syntax, i.e., the tags, of the proof files. The `Parsing` module consists of several higher order parsing functions. The parser proper is in the modules `Pass1` and `Pass2`. It is a recursive descent parser.

The `Exception` and `Debug` modules are responsible for handling errors. The `Debug` module maintains a debug flag for each module. The values of these flags are non-negative integers. The higher the value, the more information will be displayed while checking a proof. The `Report` module is for formatting the output to the log file.

The `Io` module handles all file input and output. When the checker is invoked, it creates a decompression process running as a filter in the background. The communication between this process and the checker is via a UNIX domain socket. If the file is uncompressed, no decompression is needed, but a dummy `cat` process is created, and the communication is still via a socket. This arrangement simplifies the checker as its input routine always reads from the input socket. Similarly, an output socket is created with a compression process to compress the output to the log file on the fly. This arrangement is illustrated in Fig. 2.1.

3 Benchmarking

The proof checker has been tested with many small proof files generated by the HOL system, including all derivations of basic derived rules. The largest proof that has been checked by the checker is a proof of correctness of a simple multiplier described in [4]. This is a medium size proof which generates 14500 intermediate theorems. This proof has been used as a benchmark for many versions of the HOL system.

The multiplier proof consists of four ML files. A proof file is generated for each ML file. It contains all the sub-proofs in the corresponding ML file. [10] describes how to generate these proof files and lists the time required and the files sizes.

Table 3.1: Benchmark for checking the multiplier proof (Time in seconds)

Proof	**No. of**	**Time**			
File	**Thm.**	**Run**	**System**	**GC**	**Real**
`mk_NEXT`	2972	139.3	15.3	3.7	170.0
`MULT_FUN_CURRY`	670	77.3	9.6	2.6	100.4
`MULT_FUN`	6943	406.3	44.6	15.4	488.8
`HOL_MULT`	3946	1472.1	152.0	98.5	1783.3
Total	14531	2095.0	138.8	120.2	2542.5

The multiplier proof files were successfully checked by the checker. No error was found from the proof files. Table 3.1 lists the time taken to check the proof files. This test ran on a SUN SparcStation 20. The time is in the same order of magnitude as the recording. One important observation is that the process size is relatively small when performing the checking. The process size of the checker when it is just loaded is 14 Mbytes. The maximum size when performing the checking is only 16 Mbtyes. This shows that the implementation does keep the memory usage very small.

Part II

The Program

In this part, the checker program is described in detail. Each module of the checker is described in its own chapter. There are two sections within most chapters: the first describes the specification of the module, and the second the implementation. The specification describes the interface between the module and its user. The specifications of the modules are SML signatures. The modules are implemented as SML structures. The order of the chapters follows roughly the reversed sequence in Figure 2.2, i.e., the chapter of the bottom module appears first.

The checker program is written in the `MlWeb` format, a simple literate programming tool[1]. In the literate programming paradigm[6], the master file contains both the program source and its documentation. This encourages people to write more human readable programs and helps to keep the program and its document consistent. Literate programming tools extract the program source and the documentation from the master file separately. The former will be in a format acceptable to a compiler so that an executable program can be produced. The latter will be processed by a text formatter to produce a printed document. The `MlWeb` format and tools allow a single master file to contain programs and other machine readable text, such as specifications, in more than one language.

4 The first pass

The primary task of the first pass is to build a table of theorem references. Therefore, it only looks at those justifications which refer to theorems. Any irrelevant items can be skipped over in high speed.

4.1 The specification

The only exported function is `parse_file`. It takes a file name as its sole argument and processes it. If the file name ends with the suffix `.gz`, it is taken as a compressed file. The default decompresser `gzcat` is passed to the IO module for preprocessing the file.

```
signature Pass_sig =
  sig
    val parse_file : string -> unit
  end
```

4.2 The implementation

The Pass 1 parser is implemented as the functor `Pass1FUN` with the signature `Pass_sig`.

```
functor Pass1FUN (structure Parsing :Parsing_sig and Henv: Henv_sig
                  and Proof: Proof_sig and Hthm: Hthm_sig
                  and Hterm: Hterm_sig and Htype: Htype_sig
                  sharing Hterm = Hthm.Hterm = Proof.Hterm
                  and Htype = Hterm.Htype = Hthm.Hterm.Htype =
                           Proof.Htype = Henv.Htype
                  and Hthm = Proof.Hthm) : Pass_sig =
  struct
    structure Parsing = Parsing
    structure Io = Parsing.Io
    structure Keyword = Parsing.Keyword
    structure Henv = Henv;
    structure Proof = Proof;
    structure Hthm = Hthm;
    structure Hterm = Hterm;
    structure Htype = Htype;

    open Parsing Htype Hterm Hthm Proof

    fun PASS1_ERR {function, message} =
      Exception.CHK_ERR{origin_structure = "Pass1",
                        origin_function = function, message=message};
    val debug = Debug.get_debug("Pass1");
    fun write_out s = (output(std_err, s); flush_out std_err)
```

4.2.1 Parser The parser is a recursive descent parser. The top expression is a proof file. The exact syntax of the file can be found in [10]. In this pass, we are only interested in theorem references in the justifications, so a lot of input can be skipped.

```
fun parse_proof_file () =
  (parse_item parse_ver ();
   parse_item parse_env() ;
   parse_closure parse_proof ();
   ())
and parse_ver () =
  (get_tag Keyword.VERSION;
   skip_sp ();
   if check_ver_string() then ()
   else raise (PASS1_ERR
               {function = "parse_ver", message="incorrect format"})
     )
and parse_env () =
   (get_tag Keyword.ENV;
    get_name();
    skip_item ();
    skip_item ();
    ())
```

The function `parse_proof` parses a proof. The local variable `thms` contains a list of goals which are the subexpressions after the tag. Then, it calls `new_proof1` to initialise the current proof data structures. Next, `parse_list` is called to parse the proof lines. At the end of the proof, the `pline` table is saved for the second pass.

```
and parse_proof () =
  (get_tag Keyword.PROOF;
   let val name = get_name()
     val thms = parse_list parse_thm ()
   in
     new_proof1 (name,thms);
     parse_list parse_line ();(* handle PARSE_DONE => []
                                   | e => raise e; *)
     Proof.add_pline_table name;
      if (debug > 0) then (write_out("\nProof: "^name^"\n");
                           Proof.print_prooftab()) else ()
   end;
   ())
```

The function `parse_line` processes each proof line.

```
and parse_line () =
  (get_tag Keyword.LINE;
   let val n = get_num ()
     val nl = parse_item parse_just ()
     val th = parse_item parse_thm ()
   in
     if (debug > 1) then write_out("\nline "^makestring n) else ();
     let
       val proved = Proof.add_pline_tab(n,nl,th)
     in
```

```
      if proved then raise PARSE_DONE else ()
    end
  end;
  ())
```

Now, we are looking at the justification field of a proof line. The function **get_name** is used to get the name of the justification which is case insensitive, so the call to **toupper** folds all letters to upper case. As we are only interested in the theorem references, we skip over all other items. The function **get_num** is used to get the line numbers. A list of line numbers is returned by **parse_just**.

```
and parse_just () =
  let
    val toupper = StringUtil.stringTrans
      ("abcdefghijklmnopqrstuvwxyz","ABCDEFGHIJKLMNOPQRSTUVWXYZ")
  in
    case toupper(get_name()) of
      "ABS" => (skip_item(); [get_num ()])
    | "DISCH" => (skip_item(); [get_num ()])
    | "INSTTYPE" => (skip_item(); [get_num ()])
    | "MP" => [get_num(), get_num()]
    | "SUBST" =>
        let
          val l = map (#1) (parse_list_pairs (get_num,skip_item) ())
        in
          (skip_item(); get_num () :: l)
        end
    | "ADDASSUM" => (skip_item(); [get_num()])
    | "APTERM" => (skip_item(); [get_num()])
    | "APTHM" =>
        let val n = [get_num()]
        in
          skip_item(); n
        end
    | "CCONTR" => (skip_item();[get_num()])
    | "CHOOSE" =>
        let val (_,n1) = parse_pair ((parse_item parse_term),get_num) ()
        in
          ([n1,get_num()])
        end
    | "CONJ" => ([get_num(),get_num()])
    | "CONJUNCT1" => ([get_num()])
    | "CONJUNCT2" => ([get_num()])
    | "CONTR" => (skip_item(); [get_num()])
    | "DISJ1" =>
        let val n = [get_num()]
        in
          skip_item(); n
        end
    | "DISJ2" => (skip_item(); [get_num()])
    | "DISJCASES" => ([get_num(),get_num(),get_num()])
    | "EQIMPRULEL" => ([get_num()])
```

```
    | "EQIMPRULER" => ([get_num()])
    | "EQMP" => ([get_num(),get_num()])
    | "EQTINTRO" => ([get_num()])
    | "EXISTS" => (skip_item();(* skip a pair *) [get_num()])
    | "EXT" => ([get_num()])
    | "GEN" => (skip_item(); [get_num()])
    | "IMPANTISYMRULE" => ([get_num(),get_num()])
    | "IMPTRANS" => ([get_num(),get_num()])
    | "INST" => (skip_item(); [get_num()])
    | "MKABS" => ([get_num()])
    | "MKCOMB" => ([get_num(),get_num()])
    | "MKEXISTS" => ([get_num()])
    | "NOTELIM" => ([get_num()])
    | "NOTINTRO" => ([get_num()])
    | "SPEC" => (skip_item(); [get_num()])
    | "SUBS" =>
        let val nl = parse_num_list ()
        in
          (get_num())::nl
        end
    | "SUBSOCCS" =>
        let
          val nlnl = parse_list (parse_subs_list) ()
            val n = get_num()
        in
          n :: (map #2 nlnl)
        end
    | "SUBSTCONV" =>
        let
          val ntl = parse_list_pairs (get_num,(parse_item parse_term)) ()
        in
          skip_item (); skip_item ();
          (map #1 ntl)
        end
    | "SYM" => ([get_num()])
    | "TRANS" => ([get_num(),get_num()])
    | _ => (skip_long_string false [RP]; [])
  end
```

A little auxiliary parsing function for substitution lists.

```
and parse_subs_list () =
  ((parse_num_list ()), get_num())
```

The function `parse_thm` recognises a theorem and returns it.

```
and parse_thm () =
  (get_tag Keyword.THM;
   let val hyp = parse_list parse_term ()
     val concl = parse_item parse_term ()
   in(if (debug > 2) then write_out("<THM>") else ();
     Hthm.mk_thm(hyp,concl))
   end )
```

The function `parse_term` recognises a term. There are four kinds of terms, two of them, namely variables and constants, have type information attached. The appropriate term constructor is called to create a term, and it is returned as the value of this function.

```
and parse_term() =
  let val tag =  get_name() in
    if(tag = Keyword.VAR) then
       let val name = get_name()
         val ty = parse_item parse_type ()
       in (if (debug > 2) then write_out("-"^name) else ();
           mk_var(name,ty))
       end
    else if (tag = Keyword.CONST) then
       let val name = get_const_name()
         val ty = parse_item parse_type ()
       in (if (debug > 2) then write_out("$"^name) else ();
           mk_const(name,ty))
       end
   else if (tag = Keyword.APP) then
       let val rator = parse_item parse_term ()
         val rand = parse_item parse_term ()
       in (if (debug > 2) then write_out("^") else ();
           mk_comb(rator,rand))
       end
   else if (tag = Keyword.ABS) then
       let val v = parse_item parse_term ()
         val body = parse_item parse_term ()
       in (if (debug > 2) then write_out("\\") else ();
           mk_abs(v,body))
       end
   else raise (PASS1_ERR{function="parse_term",
                         message=("Unknown term"^tag)})
  end
```

The function `parse_type` recognises a type. There are three kinds of types: type operators, type constants and type variables. The first two are represented by type operators in the `HType` module. The appropriate type constructor is called to create a type, and it is returned as the value of this function.

```
and parse_type () =
  let val tag = get_name() in
    if (tag = Keyword.TYVAR) then
      let val name = get_tyvar_name()
       in (if (debug > 2) then write_out("="^name) else ();
            mk_vartype name)
       end
    else if (tag = Keyword.TYCONST) then
      let val name = get_name()
      in (if (debug > 2) then write_out("#"^name) else ();
            mk_type {Tyargs=[],Tyop=name})
      end
    else if (tag = Keyword.TYOP) then
       let val name = get_name()
```

```
        val tyl = parse_list parse_type ()
      in (if (debug > 2) then write_out("&"^name) else ();
          mk_type {Tyargs=(rev tyl),Tyop=name})
      end
   else  raise (PASS1_ERR{function="parse_type",
                         message=("Unknown type"^tag)})
 end
```

4.2.2 The user function This is the entry point of the Pass 1 parser. It takes the name of the proof file as its sole argument. This string should include all the suffixes and possibly the path of the file name. It calls the function `mk_command` in the `Io` module to work out any compressing and decompressing command if they are necessary. See Page 141 for details of how the file name is interpreted.

```
   fun parse_file fname =
     (Parsing.init(); Proof.init();
      let val (outname, incmd, outcmd) = Io.mk_command fname
      in
        Io.open_input_socket fname incmd
      end;
      let val plst = parse_proof_file () in
        (Io.close_io_socket(); (plst))
      end) handle e =>
              (Io.close_io_socket(); raise e)

 end; (* functor Pass1FUN *)
```

5 The second pass

This is the main pass of the checker. It parses and checks the proof file. It has the same signature as the module Pass1. It is repeated here for easy reference.

```
signature Pass_sig =
  sig
    val parse_file : string -> unit
  end
```

5.1 The implementation

The Pass 2 parser is implemented as the functor Pass2FUN with the same signature as the Pass 1 parser.

```
functor Pass2FUN (structure Parsing :Parsing_sig and Henv: Henv_sig
                  and Proof: Proof_sig and Hthm: Hthm_sig
                  and Hterm: Hterm_sig and Htype: Htype_sig
                  and Check :Check_sig and Report : Report_sig
                  sharing Hterm = Hthm.Hterm = Proof.Hterm = Check.Hterm
                  and Htype = Hterm.Htype = Hthm.Hterm.Htype =
                           Proof.Htype = Henv.Htype = Check.Htype
                  and Hthm = Proof.Hthm = Check.Hthm
                  and Report = Check.Report = Htype.Report = Hterm.Report
                  and Report.Keyword = Parsing.Keyword
                  and Proof = Check.Proof) : Pass_sig =
  struct
    structure Report = Report
    structure Parsing = Parsing
    structure Io = Parsing.Io
    structure Keyword = Parsing.Keyword
    structure Henv = Henv
    structure Proof = Proof
    structure Hthm = Hthm
    structure Hterm = Hterm
    structure Htype = Htype
    structure Check = Check

    open Parsing Htype Hterm Proof

    fun PASS2_ERR {function, message} =
      Exception.CHK_ERR{origin_structure = "Pass2",
                        origin_function = function, message=message};
    val debug = Debug.get_debug("Pass2");
    fun write_out s = (output(std_err, s); flush_out std_err)
```

5.1.1 Parser The parser is a recursive descent parser. It is organised in the same way as the Pass 1 parser. The top expression is a proof file. The exact syntax of the file can be found in [10]. In this pass, the parser calls the checking function after recognising each proof line.

```
fun parse_proof_file () =
  (parse_item parse_ver ();
   parse_item parse_env ();
   parse_closure parse_proof ();
   ())
and parse_ver () =
  (get_tag Keyword.VERSION;
   skip_sp ();
   if check_ver_string() then ()
   else raise (PASS2_ERR
               {function = "parse_ver", message="incorrect format"})
     )
and parse_env () =
   (get_tag Keyword.ENV;
    let val name = get_name()
      val tyl = map Henv.mk_typeconst(parse_list_pairs (get_name,get_num) ())
      val cl = map Henv.mk_termconst
        (parse_list_pairs (get_name,(parse_item parse_type)) ())
    in
     (if (debug > 0) then write_out("\nCurrent environment: "^name^"\n") else (
);
      Henv.mk_proof_env(name, tyl, cl))
    end
    )
```

At the beginning of a proof, the function `new_proof2` is called to initialise the internal data structures which hold the theorem reference table and the list of goals. Also, the opening tag of a proof is written to the log file. After parsing all lines, the `USED` list in the log file is closed. Then, `chk_proof` is called to verify that all goals have been proved. The proof in the log file is closed. See the `Report` module on Page 151 for details of the log file format.

```
and parse_proof () =
  (get_tag Keyword.PROOF;
   let val name = get_name()
     val thms = parse_list parse_thm ()
   in
     if (debug > 0) then write_out("\nProof: "^name^"\n") else ();
     Report.write_line_opening2(Keyword.PROOF,name);
     Report.write_line_opening(Keyword.USED);
     Report.write_tok Keyword.LB;
     new_proof2 name;
     let
       val ls = parse_list parse_line ()
     in
       Report.write_tok Keyword.RB;
       Report.write_closing_line ();
       Check.chk_proof(name,thms,ls);
       Report.write_closing_line ()
     end
```

```
  end;
  ())
```

The function **parse_line** processes a proof line. After parsing all fields, namely the line number n, the justification $just$ and the derived theorem th, it calls **chk_pline** to check the line. A line number is returned by the checking function. If it is the same as the current line number n, all the goals have been proved. All remaining proof lines can be ignored. The exception PARSE_DONE is raised to pass this signal up. Otherwise, this function returns the line number n.

```
and parse_line () =
  (get_tag Keyword.LINE;
   let val n = get_num ()
     val just = parse_item parse_just ()
     val th = parse_item parse_thm ()
   in
     if (debug > 1) then write_out("\nline "^makestring n) else ();
     if (Check.chk_pline(n, just, th) = n)
     then raise PARSE_DONE
     else n
   end)
```

This function parses the justification field in a proof line, and returns it. The name of the justification is case insensitive.

```
and parse_just () =
  let
    val toupper = StringUtil.stringTrans
      ("abcdefghijklmnopqrstuvwxyz","ABCDEFGHIJKLMNOPQRSTUVWXYZ")
  in
    case toupper(get_name()) of
      "ABS" =>
        let val tm = parse_item parse_term ()
          val n = get_num ()
        in
          (Abs (tm, n))
        end
    | "ASSUME" => (Assume (parse_item parse_term ()))
    | "BETACONV" =>  (BetaConv (parse_item parse_term ()))
    | "DISCH" =>
        let val tm = parse_item parse_term ()
          val n = get_num ()
        in
          (Disch (tm, n))
        end
    | "INSTTYPE" =>
        let val tytyl = parse_list_pairs
          ((parse_item parse_type), (parse_item parse_type)) ()
          val n = get_num ()
        in
          (InstType (tytyl, n))
        end
    | "MP" => (Mp (get_num(), get_num()))
    | "REFL" => (Refl (parse_item parse_term ()))
```

```
| "SUBST" =>
    let
      val l = (parse_list_pairs (get_num,(parse_item parse_term)) ())
      val tm = parse_item parse_term ()
    in
      (Subst (l, tm, get_num ()))
    end
| "AXIOM" =>
    let val s1 = get_name()
      val s2 = get_name()
    in
      (Axiom (s1,s2))
    end
| "DEFINITION" =>
    let val s1 = get_name()
      val s2 = get_name()
    in
      (Definition (s1,s2))
    end
| "DEFEXISTSRULE" =>
    (DefExistsRule (parse_item parse_term ()))
| "HYPOTHESIS" => (Hypothesis)
| "NEWAXIOM" =>
    let val name = get_name()
      val tm = (parse_item parse_term ())
    in
      (NewAxiom (name,tm))
    end
| "NEWCONSTANT" =>
    let val name = get_name()
      val ty = (parse_item parse_type ())
    in
      (NewConstant (name,ty))
    end
| "NEWTYPE" =>
    let val n = get_num()
      val name = get_name()
    in
      (NewType (n,name))
    end
| "NUMCONV" =>
    (Numconv (parse_item parse_term ()))
| "STOREDEFINITION" =>
    let val name = get_name()
      val tm = (parse_item parse_term ())
    in
      (StoreDefinition (name,tm))
    end
| "THEOREM" =>
    let val s1 = get_name()
      val s2 = get_name()
```

```
    in
      (Theorem (s1,s2))
    end

| "ADDASSUM" =>
    let
      val tm = parse_item parse_term ()
      val n = get_num ()
    in
      (AddAssum (tm,n))
    end
| "ALPHA" =>
    let
      val tm1 = parse_item parse_term ()
      val tm2 = parse_item parse_term ()
    in
      (Alpha (tm1,tm2))
    end
| "APTERM" => (ApTerm ((parse_item parse_term ()), get_num()))
| "APTHM" => (ApThm ((get_num()),(parse_item parse_term ())))
| "CCONTR" => (Ccontr ((parse_item parse_term ()), get_num()))
| "CHOOSE" =>
    let val (tm,n1) = parse_pair ((parse_item parse_term),get_num) ()
    in
      (Choose (tm, n1, get_num()))
    end
| "CONJ" => (Conj (get_num(),get_num()))
| "CONJUNCT1" => (Conjunct1 (get_num()))
| "CONJUNCT2" => (Conjunct2 (get_num()))
| "CONTR" => (Contr ((parse_item parse_term ()), get_num()))
| "DISJ1" => (Disj1((get_num()),(parse_item parse_term ())))
| "DISJ2" => (Disj2 ((parse_item parse_term ()), get_num()))
| "DISJCASES" => (DisjCases (get_num(), get_num(), get_num()))
| "EQIMPRULEL" => (EqImpRuleL (get_num()))
| "EQIMPRULER" => (EqImpRuleR (get_num()))
| "EQMP" => (EqMp (get_num(),get_num()))
| "EQTINTRO" =>  (EqTIntro (get_num()))
| "ETACONV" => (EtaConv (parse_item parse_term ()))
| "EXISTS" => (Exists (
              (parse_pair(parse_item parse_term,parse_item parse_term)()),

                      (get_num())))
| "EXT" => (Ext (get_num()))
| "GEN" => (Gen ((parse_item parse_term ()), get_num()))
| "IMPANTISYMRULE" => (ImpAntisymRule (get_num(),get_num()))
| "IMPTRANS" => (ImpTrans (get_num(),get_num()))
| "INST" => (Inst ((parse_list_pairs
                    (parse_item parse_term,parse_item parse_term) ()),
                   (get_num())))
| "MKABS" => (MkAbs (get_num()))
| "MKCOMB" => (MkComb (get_num(),get_num()))
```

```
    | "MKEXISTS" => (MkExists (get_num()))
    | "NOTELIM" => (NotElim (get_num()))
    | "NOTINTRO" => (NotIntro (get_num()))
    | "SPEC" => (Spec ((parse_item parse_term ()), (get_num())))
    | "SUBS" => (Subs ((parse_num_list ()), (get_num())))
    | "SUBSOCCS" =>
        let
          val nlnl = parse_list (parse_subs_list) ()
        in
          (SubsOccs (nlnl, (get_num())))
        end
    | "SUBSTCONV" =>
        let
          val ntl = parse_list_pairs (get_num, (parse_item parse_term)) ()
          val tm1 = parse_item parse_term ()
          val tm2 = parse_item parse_term ()
        in
          (SubstConv (ntl, tm1, tm2))
        end
    | "SYM" => (Sym (get_num()))
    | "TRANS" => (Trans (get_num(), get_num()))
    | _ => raise (PASS2_ERR{function="parse_just",
                             message="Unknown justification"})
  end
```

A little auxiliary parsing function for substitution lists.

```
and parse_subs_list () =
  ((parse_num_list ()), get_num())
```

The functions for parsing theorems, terms and types, namely `parse_thm`, `parse_term` and `parse_type` respectively, are the same as their counterparts in the module `Pass1`.

```
and parse_thm () =
  (get_tag Keyword.THM;
   let val hyp = parse_list parse_term ()
     val concl = parse_item parse_term ()
   in(if (debug > 2) then write_out("<THM>") else ();
     Hthm.mk_thm(hyp,concl))
   end )
and parse_term() =
  let val tag =  get_name() in
    if(tag = Keyword.VAR) then
       let val name = get_name()
         val ty = parse_item parse_type ()
       in (if (debug > 2) then write_out("-"^name) else ();
           mk_var(name,ty))
       end
    else if (tag = Keyword.CONST) then
       let val name = get_const_name()
         val ty = parse_item parse_type ()
       in (if (debug > 2) then write_out("$"^name) else ();
           mk_const(name,ty))
       end
```

```
   else if (tag = Keyword.APP) then
       let val rator = parse_item parse_term ()
         val rand = parse_item parse_term ()
       in (if (debug > 2) then write_out("^") else ();
           mk_comb(rator,rand))
       end
   else if (tag = Keyword.ABS) then
       let val v = parse_item parse_term ()
         val body = parse_item parse_term ()
       in (if (debug > 2) then write_out("\\") else ();
           mk_abs(v,body))
       end
   else raise (PASS2_ERR{function="parse_term",
                         message=("Unknown term"^tag)})
  end
and parse_type () =
  let val tag = get_name() in
    if (tag = Keyword.TYVAR) then
      let val name = get_tyvar_name()
       in (if (debug > 2) then write_out("="^name) else ();
             mk_vartype name)
       end
    else if (tag = Keyword.TYCONST) then
      let val name = get_name()
      in (if (debug > 2) then write_out("#"^name) else ();
            mk_type {Tyargs=[],Tyop=name})
      end
    else if (tag = Keyword.TYOP) then
       let val name = get_name()
         val tyl = parse_list parse_type ()
       in (if (debug > 2) then write_out("&"^name) else ();
             mk_type {Tyargs=(rev tyl),Tyop=name})
       end
    else  raise (PASS2_ERR{function="parse_type",
                           message=("Unknown type"^tag)})
  end
```

5.1.2 The user function This function is the entry point to the Pass 2 parser. It is similar to the entry function in the module `Pass1`. It takes the name of the proof file as its sole argument. This string should include all the suffixes and possibly the path of the file name. It calls the function `mk_command` in the `Io` module to work out the name of the log file and of any compressing and decompressing command if they are necessary. See Page 141 for details of how the file name is interpreted.

```
    fun parse_file fname =
      (Parsing.init();
       Henv.init(); Proof.clear_proof(); Check.init();
       let val (outname, incmd, outcmd) = Io.mk_command fname
       in
         Io.open_input_socket fname incmd;
         Io.open_output_socket outname outcmd;
```

```
        Report.write_log_preamble ()
      end;
      let val plst = parse_proof_file () in
        (Io.close_io_socket(); (plst))
      end) handle e =>
                (Io.close_io_socket(); raise e)

end; (* functor Pass2FUN *)
```

6 The parsing functions

The `Parsing` module includes all the lexical analysis and parsing functions. The parsing of the proof files can be considered in two tiers: the lexical analysis and parsing. Due to the slowness of string functions in SML, we do not work on strings. Instead, the input bytes returned by the input function are converted to abstract characters represented by the type `ctype`. These characters are converted to strings only when it is necessary.

Functions provided by this module are used by both passes. They can be divided into three groups: the character functions which convert input bytes to abstract characters and are local to this module, the *atom functions* which recognise atoms, such as identifiers and numbers, and the *parsing functionals* which recognise larger syntactic structures, such as lists and pairs.

The module is implemented by a functor, `ParsingFUN`. The interface is specified by the signature `Parsing_sig`.

6.1 The specification

The signature `Parsing_sig` specifies the interface of the `Parsing` module. The structure `Io` provides the input functions to the parser. The structure `Keyword` defines the concrete input syntax.

The type `ctype` is an abstract representation of input characters. The parser works with abstract characters except in certain special situations.

```
signature Parsing_sig =
  sig
    structure Io : IO_sig
    structure Keyword : Keyword_sig

    exception PARSE_DONE

    datatype ctype = ALPHA of string | NUM of int | SYM of string
      | LP | RP | LB | RB | LC | RC  | PL | MI | EOF | SP;
```

The first group of functions provided by this module are for parsing atoms, e.g., tags, identifiers and numbers. Their names are all prefixed by `get_`.

```
    val get_const_name : unit -> string
    val get_name : unit -> string
    val get_num : unit -> int
    val get_string : unit -> ctype list
    val get_symbolic_name : unit -> string
    val get_tag : string -> unit
    val get_tyvar_name : unit -> string
```

The function `check_ver_string` verifies the file format string.

```
    val check_ver_string : unit -> bool
```

The function `init` initialises this module. It should be called at the beginning of every pass.

```
    val init : unit -> unit
```

These functions are parsing functionals. They recognise larger syntactic structures, such as lists and pairs.

```
    val parse_closure : (unit -> 'a) -> unit -> 'a list
    val parse_item : (unit -> 'a) -> unit -> 'a
    val parse_list : (unit -> 'a) -> unit -> 'a list
    val parse_num_list : unit -> int list
    val parse_list_pairs : (unit -> 'a) * (unit -> 'b) -> unit -> ('a * 'b)list

    val parse_pair : (unit -> 'a) * (unit -> 'b) -> unit -> 'a * 'b
```

These functions are for skipping certain input at high speed. They are used only in the first pass.

```
    val skip_item : unit -> unit
    val skip_long_string : bool -> ctype list -> unit
    val skip_sp : unit -> unit
  end (* signature *)
```

6.2 The implementation

The functor `ParsingFUN` implements the `Parsing` module. It requires two structures: the first `Io` provides input and file management functions, and the second `Keyword` defines the concrete input syntax.

```
functor ParsingFUN
  (structure Io:IO_sig and Keyword :Keyword_sig) : Parsing_sig =
  struct
    structure Io = Io
    structure Keyword = Keyword

    exception PARSE_DONE

    fun PARSING_ERR {function, message} =
      Exception.CHK_ERR{origin_structure = "Parsing",
                        origin_function = function, message=message};
    val debug = Debug.get_debug("Parsing");
    fun write_out s = (output(std_err, s); flush_out std_err);
```

6.2.1 Abstract characters The type `ctype` is an abstract internal representation of the input characters. The input structure `Io` has a function for inputing a fixed size block of bytes represented as a list of ASCII codes. It is very time-consuming to convert this into SML strings since the string operation is rather slow. So, we work on the abstract characters.

The type of abstract characters consists of four kinds of characters:

- *letters* represented by the constructor `ALPHA`;
- *digits* represented by the constructor `NUM`;
- *symbols* represented by the constructor `SYM`;
- *special characters* each of which has a constructor.

```
    datatype ctype = ALPHA of string | NUM of int | SYM of string
      | LP | RP | LB | RB | LC | RC  | PL | MI | EOF | SP;
```

6.2.2 Mapping functions The function `byte_to_char` maps a byte (ASCII code) to the abstract character of type `ctype`. Symbolic names are defined locally for some ASCII characters. Upper and lower case letters are mapped to `ALPHA` with single character string containing the letter. Digits are mapped to `NUM` with their corresponding numeric value. Other printable characters except `()[]{}+-` are mapped to `SYM` with single character string containing the character. The special characters `()[]{}+-` are mapped to their respective symbolic names, `LP`, `RP`, `LB`, `RB`, `LC`, `RC`, `PL` and `MI`. All other input bytes are mapped to `SP`, the space character which is ignored in most cases. The special character `EOF` is used to indicate the end of file.

This function needs to be modified if the concrete syntax is changed.

```
local
  val ASCII_code_sp = ordof(" ",0)
  and ASCII_code_0 = ordof("0",0)
  and ASCII_code_9 = ordof("9",0)
  and ASCII_code_a = ordof("a",0)
  and ASCII_code_z = ordof("z",0)
  and ASCII_code_A = ordof("A",0)
  and ASCII_code_Z = ordof("Z",0);
in
  fun byte_to_char n =
    if(n <= ASCII_code_sp) then (SP)
    else
      case n of
        40  => (LP) (* "(" *)
      | 41  => (RP) (* ")" *)
      | _ =>
          if(n < ASCII_code_0) then
            case n of
              43 => (PL) (* "+" *)
            | 45 => (MI) (* "-" *)
            | _  => (SYM (chr n))
          else if (n <= ASCII_code_9) then (NUM (n - ASCII_code_0))
          else if (n < ASCII_code_A) then (SYM (chr n))
          else if (n <= ASCII_code_Z) then (ALPHA (chr n))
          else if (n < ASCII_code_a) then
            case n of
              91 => (LB) (* "[" *)
            | 93 => (RB) (* "]" *)
            | _  => (SYM (chr n))
          else if (n <= ASCII_code_z) then (ALPHA (chr n))
               else
                 case n of
                   123 => (LC) (* "{" *)
                 | 125 => (RC) (* "}" *)
                 | _   => (SYM (chr n))
end (* local *)
```

The function `char_to_str` maps an abstract character to its name string. It is mainly used for debugging.

```
  fun char_to_str (LP) = "LP"
    | char_to_str (RP) = "RP"
    | char_to_str (LB) = "LB"
```

```
    | char_to_str (RB) = "RB"
    | char_to_str (LC) = "LC"
    | char_to_str (RC) = "RC"
    | char_to_str (PL) = "PL"
    | char_to_str (MI) = "MI"
    | char_to_str (SP) = " "
    | char_to_str (EOF) = "EOF"
    | char_to_str (ALPHA c) = c
    | char_to_str (SYM c) = c
    | char_to_str (NUM n) = makestring n
```

6.2.3 character functions

This section contains several low level functions which are used by the string functions and parsing functionals to obtain the next input character.

This module keeps an internal input buffer `char_buf` which is a byte array. The current length of the buffer is kept in `char_buf_len`. The identifier `charp` always points to the next available byte. The local function `chk_buf` is called every time a character is accessed. It calls the input function `Io.read_bytes` to get a new block of bytes if the end of the buffer is reached.

```
local
  val char_buf = ref (ByteArray.array(256,0))
  and char_buf_len = ref 0
  and charp = ref 0;

  fun chk_buf () =
    if (!charp >= !char_buf_len) then
      let val (len,cs) = Io.read_bytes () in
        (char_buf_len := len;
         char_buf := cs;
         charp := 0;
         if (debug > 1) then
           write_out(
             ByteArray.extract((!char_buf), (!charp), (!char_buf_len)))
         else ()
           )
      end
    else ()
in
```

The function `init` clears the internal input buffer. It should be called at the beginning of each pass.

```
  fun init () =
    (charp := 0;
     char_buf_len := 0;
     ())
```

The function `peek` reads the next byte without removing it from the buffer. It converts the byte into an abstract character and returns it.

```
  fun peek() =
    (chk_buf ();
     if (!char_buf_len = 0) then (EOF)
     else
           let
             val b = (ByteArray.sub(!char_buf, !charp))
```

```
              in
                (if (debug > 3) then write_out( ("."^(chr b))) else ();
                   byte_to_char b)
              end)
```

The function **next** removes the next byte from the buffer. It converts the byte into an abstract character and returns it.

```
and next() =
  (chk_buf ();
   if (!char_buf_len = 0) then (EOF)
   else let val c = (ByteArray.sub(!char_buf, !charp))
         in
           (charp :=(!charp) + 1;
            if (debug > 2) then write_out( (":"^(chr  c))) else ();
            byte_to_char c)
         end )
```

The function **unget** puts its argument c which is an abstract character back into the buffer. Since the contents of the buffer is not destroyed before the next call to the input function, **unget** makes sure that c is the same as the character in the buffer. It should always be the same since a character can only be put back immediately after it is removed from the buffer. No intervening reading is allowed. It is not possible to put back a character at the beginning of the buffer.

```
and unget c =
  if (!charp = 0) then
    raise (PARSING_ERR{function="unget",
                     message="Can't unget--beginning of buffer"})
  else if (c = byte_to_char(ByteArray.sub(!char_buf, (!charp-1)))) then
    charp :=  (!charp) - 1
       else raise (PARSING_ERR{function="unget",
                             message="Can't unget--not the same char"})
```

The following two functions, **peekb** and **nextb**, are the same as **peek** and **next** except their return values are bytes instead of abstract characters.

```
and peekb() =
  (chk_buf ();
   if (!char_buf_len = 0) then 0
   else
         let val b = (ByteArray.sub(!char_buf, !charp))
         in
           (if (debug > 3) then write_out( (","^(chr  b))) else ();
              b)
         end)
and nextb() =
  (chk_buf ();
   if (!char_buf_len = 0) then 0
   else let val c =  (ByteArray.sub(!char_buf, !charp))
        in
          (charp :=(!charp) + 1;
           if (debug > 2) then write_out( (";"^(chr  c))) else ();
             c)
        end)
```

The function `strip` removes the next byte from the internal buffer. It is usually used after a peek which finds the desired character, thus avoiding using `unget`.

```
    and strip() = (charp :=(!charp) + 1)

  end (* local *);
```

6.2.4 String Functions Functions in this section are for parsing atoms. We first describe the local functions. The function `skip` takes a predicate f as its argument. It skips all characters that satisfy f, i.e., it removes all initial characters from the internal buffer until the first character c such that evaluating $f\,c$ returns `false`.

The function `get_str` is similar to `skip` but it returns a list of characters which satisfy the predicate f. The second argument l is used for cumulating the characters so the function is more efficient. NOTE: the resulting list is in reverse order. The function `get_bytes` is the same as `get_str` except the return value is a list of bytes.

The function `get_long_str` returns a list of character which may contain matched pairs of delimiters. The delimiter pairs it recognises are `LP` and `RP`, `LB` and `RB`, and `LC` and `RC`. It takes two arguments. The first is a list of right (closing) delimiters, and the second is a list for cumulating the characters. It returns its second argument if the delimiter list is empty. Otherwise, it scans through the input characters. If it finds a closing delimiter which is the same as the head of the delimiter list, the head is removed. If it sees an open delimiter, the matching closing delimiter is pushed on to the delimiter list. Then, it calls itself recursively.

```
local
  fun skip f = while (f (peek())) do strip();

  fun get_str f l =
    let val c = peek() in
      if (f c) then (strip(); get_str f (c::l)) else l
    end
  and get_bytes f l =
    let val c = peekb() in
      if (f c) then (strip(); get_bytes f (c::l)) else l
    end
  and get_long_str [] l = l
    | get_long_str (dl as (delim::ds)) l =
      let val c = next() in
        if (delim = c) then get_long_str ds (c::l)
        else
          case c of
            LP => get_long_str (RP::dl) (c::l)
          | LB => get_long_str (RB::dl) (c::l)
          | LC => get_long_str (RC::dl) (c::l)
          | EOF => raise (PARSING_ERR{function="get_long_str",
                                      message="Unexpected end of file"})
          | _ => get_long_str dl (c::l)
      end
```

Here is a list of predicates to be used in conjunction with the above functions to get input characters.

```
  fun is_alpha (ALPHA c) = true
    | is_alpha _ = false
  and is_num (NUM c) = true
```

```
    | is_num _  = false
  and is_sym (SYM c) = true
    | is_sym _ = false
  and is_lp (LP) = true
    | is_lp _ = false
  and is_rp (RP) = true
    | is_rp _ = false
  and is_eof (EOF) = true
    | is_eof _ = false
  and is_sp (SP) = true
    | is_sp _ = false
  and is_notrp RP = true
    | is_notrp _ = false
  and is_delim (LP) = true
    | is_delim (RP) = true
    | is_delim (LC) = true
    | is_delim (RC) = true
    | is_delim (LB) = true
    | is_delim (RB) = true
    | is_delim _ = false
  val not_alpha = (not o is_alpha)
```

Here are local copies of the special characters defined in the `Keyword` structure. They are all ASCII codes (bytes). `idcharl` is a list of non-letters allowed in identifiers. `tyvchar` is the first character of type variables. `specials` is a list of characters allowed to be the first character of a symbolic identifier.

```
  val idcharl = map ord (explode Keyword.idchars)
  and tyvchar = ord Keyword.tyvar
  and specials = setify (map (fn s => ordof(s,0)) Keyword.symbols);
```

The following three predicates are used in conjunction with the string functions above to get alphanumeric identifiers, type variables and symbolic identifiers, respectively.

```
  fun is_id_charOrd c =
    (CType.isAlphaOrd c) orelse (CType.isDigitOrd c) orelse
    (exists (fn x => (c = x))  idcharl)
  and is_tyv_charOrd c =
    (CType.isAlphaOrd c) orelse (CType.isDigitOrd c) orelse
    (c = tyvchar) orelse (exists (fn x => (c = x))  idcharl)
  and is_spec_charOrd c = (exists (fn x => (c = x)) specials)
in
```

Here are the user functions. The function `skip_sp` skips all consecutive space characters. The function `skip_long_string` skips a string until all matching delimiters in the argument list *delim* are found. The string may contain matching delimiters as well. The first argument *flag* is a boolean value if it is `false`, the last delimiter is not removed from the internal buffer. The function `skip_item` skips the next delimited item which may be an item enclosed in parentheses, or a list or a pair.

```
  fun skip_sp () = skip is_sp
  and skip_long_string flag delim =
    (get_long_str delim []; if flag then () else unget(hd delim))
  and skip_item () =
    (skip_sp();
```

```
    case next() of
      (LP) => get_long_str [RP] []
    | (LC) => get_long_str [RC] []
    | (LB) => get_long_str [RB] []
    | _ => [] ;
        () )
```

The function `get_string` returns a string containing only letters. The function `get_num` returns a number containing only digits and optionally preceded by a plus or minus sign.

```
fun get_string () =  (rev(get_str is_alpha []))
and get_num () =
  let
    fun f sum =
      case peek() of
        (NUM n) => (strip(); f (sum * 10 + n))
      | _ => sum
  in
    (skip_sp();
     case (peek()) of
       (PL) => (strip(); f 0)
     | (MI) => (strip(); ~(f 0))
     | (NUM n) => (f 0)
     | _ => raise (PARSING_ERR{function="get_num",
                               message="Expecting a number"}))
  end
```

The function `get_name` returns an alphanumeric identifier. The function `get_tyvar_name` returns a type variable name.

```
fun get_name () =
  (skip_sp();
   let val name = (implode o rev)(map chr(get_bytes (is_id_charOrd) []))

   in
     (if (debug > 1) then write_out("|"^name^"|") else ();
        name)
   end)
and get_tyvar_name () =
  (skip_sp();
   if (tyvchar = peekb()) then
     (implode o rev)(map chr(get_bytes (is_tyv_charOrd) [nextb()]))
   else raise (PARSING_ERR{function="get_tyvar_name",
                           message="Illegal type variable name"})
     )
```

The function `get_const_name` returns a string which is a valid constant name. It may be either alphanumeric or symbolic. The function `get_symbolic_name` returns a symbolic identifier.

```
and get_const_name () =
  (skip_sp();
   if(is_id_charOrd(peekb())) then
     (implode o rev)(map chr(get_bytes (is_id_charOrd) []))
   else if (is_spec_charOrd(peekb())) then
```

```
        get_symbolic_name ()
           else raise (PARSING_ERR{function="get_const_name",
                                   message="Illegal constant name"})
            )
  and get_symbolic_name () =
    let fun sym n l lsym =
      let val c = peekb()
        val lsym' = ListUtil.filter
          (fn s => (if (ordof(s,n) = c) then SOME s else NONE)
           handle Ord => NONE) lsym
      in
        if (debug > 2) then
          (write_out("get_sym_name "^(makestring n));
           (write_out o implode o (map chr)) l; ())
        else ();
        if (null lsym') then ((implode o rev)(map chr l))
        else (strip(); sym (n+1) (c::l) lsym')
      end
    in
      sym 0 [] Keyword.symbols
    end
    handle e => raise
     (PARSING_ERR{function="get_symbolic_name",
                  message="error"})
```

The function check_ver_string verifies the version string which identifies the file format.

```
  fun check_ver_string () =
    let val s = rev (tl(get_long_str [RP] [])) in
      (unget RP;
       (s = (map (byte_to_char o ord) (explode Keyword.versionName))))
    end
```

The function get_tag takes a string *str* which is the tag expected to be read from the input. It scans through the input characters comparing each character with successive character of *str* until the end of it. If the end of *str* is reached and all corresponding characters agree, the required tag is found. Otherwise it raises an exception.

```
  fun get_tag str =
    let
      fun f [] = true
        | f (x::xs) =
          case (peek()) of
            (ALPHA c) => if(c = x) then (strip(); f xs) else false
          | _ => false
    in
      if (f (explode str)) then ()
      else raise (PARSING_ERR {function="get_tag",
                              message="Expecting tag ("^str^")"})
    end
  end (* local *)
```

6.2.5 Parsing functionals Functions in this section are parsing functionals. They process the higher level objects, such as an item, a list or a pair. In general, they take parsing functions as arguments. The functionals parse the delimiters of the object and use the supplied functions to parse the sub-objects.

An item is always enclosed in a pair of parentheses. The function `parse_item` takes a function *kind* which parses the contents between the parentheses. `parse_item` ignores any leading blanks before the open parenthesis `LP`. It applies the parsing function *kind* to process the contents after the `LP`. It then skips any blanks before the closing parenthesis `RP` and gets the `RP`. An exception is raised if neither the `LP` nor the `RP` is found. If the paring function *kind* fails with the special exception `PARSE_DONE`, we skip to the end of the item and propagate the exception. This situation occurs when all goals are proved, so the remaining proof lines are skipped over.

```
fun parse_item kind () =
  (skip_sp ();
   if (LP = peek()) then
     (strip();
      let
        val x = kind()
          handle PARSE_DONE => (skip_long_string true [RP];
                                raise PARSE_DONE)
               | e => raise e
      in
        (skip_sp();
         if (RP = next()) then x
         else raise (PARSING_ERR
                     {function="parse_item", message="expecting RP"}))
      end)
   else raise (PARSING_ERR
               {function="parse_item", message="expecting LP"})
    )
```

A pair is always enclosed in a pair of braces. The function `parse_pair` processes a pair. The argument is a pair of parsing functions which parse the first and second field of the pair, respectively.

```
and parse_pair (f1,f2) () =
  (skip_sp();
   if (LC = peek()) then
     (strip();
      let
        val x1 = f1() and x2 = f2()
      in
        (skip_sp();
         if (RC = next()) then (x1,x2)
         else raise (PARSING_ERR
                     {function = "parse_pair", message = "expecting RC"}))
      end)
   else raise (PARSING_ERR
               {function = "parse_pair", message = "expecting LC"}))
```

A list is always enclosed in a pair of brackets. The function `parse_list` parses lists. Its argument *kind* is a parsing function for a single element. It uses the local function `get_list_items` to process the elements recursively.

A closure is a list without the enclosing brackets. The function `parse_closure` uses the same local function as `parse_list` to process the elements.

NOTE: the list of items returned is in reverse order.

```
local
  fun get_list_items f l =
    get_list_items f ((parse_item f ())::l)
    handle PARSE_DONE => (skip_long_string false [RB]; l)
                           (* raise PARSE_DONE *)
         | CHK_ERR {message = "expecting LP",
                    origin_function = "parse_item",
                    origin_structure = "Parsing"} => l
         | e => raise e
  fun get_list_num l =
    get_list_num ((get_num ())::l)
    handle CHK_ERR {message,origin_function,origin_structure} => l
         | e => raise e
in
  fun parse_list kind () =
    (skip_sp();
     if (LB = peek()) then
       (strip();
        let val items = get_list_items kind [] in
          (skip_sp();
           if (RB = next()) then items
           else raise (PARSING_ERR
                       {function = "parse_list", message = "expecting RB"})
)
        end)
     else raise (PARSING_ERR
                 {function = "parse_list", message = "expecting LB"}))
  and parse_closure kind () =
    (skip_sp(); get_list_items kind [])
  and parse_num_list () =
    (skip_sp();
     if (LB = peek()) then
       (strip();
        let val items = get_list_num [] in
          (skip_sp();
           if (RB = next()) then items
           else raise (PARSING_ERR
                       {function = "parse_num_list", message = "expecting R
B"}))
        end)
     else raise (PARSING_ERR
                 {function = "parse_num_list", message = "expecting LB"}))
end (* local *)
```

The function `parse_list_pairs` is similar to `parse_list_items` except that the elements are pairs. NOTE: the list of pairs returned is in reverse order.

```
local
  fun get_list_pairs f l =
    get_list_pairs f ((parse_pair f ())::l)
    handle  PARSE_DONE => (skip_long_string true [RB];
```

```
                              raise PARSE_DONE)
          | CHK_ERR {message = "expecting LC",
                     origin_function = "parse_pair",
                     origin_structure = "Parsing"} => l
          | e => raise e
  in
    fun parse_list_pairs kind () =
      (skip_sp();
     if (LB = peek()) then
       (strip();
        let val items = get_list_pairs kind [] in
          (skip_sp();
           if (RB = next()) then items
           else raise (PARSING_ERR
                        {function = "parse_list", message = "expecting RB"}))
        end)
     else raise (PARSING_ERR
                  {function = "parse_list", message = "expecting LB"}))
  end;(* local *)

end;(* functor Parsing *)
```

7 Keywords

The module `keyword` specifies the concrete strings of the proof file and the log file. Throughout the program, symbolic names defined in this module are used instead of concrete strings. If the formats are changed, only definitions in this module need to be modified.

7.1 The specification

The signature `Keyword_sig` specifies a structure containing all the keywords the lexical analyser and the parser understand. These keywords constitute the concrete syntax of the proof format.

```
signature Keyword_sig =
  sig
    val alphas: string list     (* Alphanumeric keywords *)
    and symbols: string list    (* Symbolic identifiers  *)
    and idchars: string         (* Special characters in alphanumeric id *)
    and tyvar: string           (* The leading charcter of type variables *)
    and LP: string              (* Begin of an item *)
    and RP: string              (* End of an item   *)
    and LC: string              (* Begin of a pair *)
    and RC: string              (* End of a pair  *)
    and LB: string              (* Begin of a list *)
    and RB: string              (* End of a list   *)
    and PLUS: string
    and MINUS: string
    and PM: string list         (* The optional signs before a number *)
    and versionName: string
    and log_versionName: string
    and TYOP: string
    and TYVAR: string
    and TYCONST: string
    and VAR: string
    and CONST: string
    and APP: string
    and ABS: string
    and THM: string
    and LINE: string
    and PROOF: string
    and VERSION: string
    and ENV: string
    and TIMESTAMP: string
    and USED: string
    and PROVED: string
    and UNSOLVED: string
  end;
```

7.2 The implementation

Here is the keyword structure. `alphas` is a list of tags, i.e., the first atom in an expression that identifies what kind of expression this is. `symbols` is a list of allowable symbolic identifiers. All initial substrings of these strings can be the names of constants. `idchars` are characters, in addition to letters and digits, allowed in alphanumeric identifiers. `tyvar` is the initial character of a type variable. `LP` and `RP` are the left and right parentheses. They enclose an expression. `LC` and `RC` are the left and right braces which enclose a pair. `LB` and `RB` are the left and right brackets which enclose a list. `PM` is a list of characters which may appear in front of a number, i.e., the optional signs. `versionName` is a string identifying the version of the input file format. `log_versionName` is a string identifying the version of the output file format.

`VERSION` is a special tag. All characters after the blanks following this tag until the closing `RP` are taken as a single string. A list of symbolic names are defined for the tags of all kinds of expressions. They are exported and used by the parser.

```
structure HOLProofKey :Keyword_sig =
  struct
    val alphas = ["o", "v", "c", "V", "C", "A", "L",
                  "THM", "LINE", "PROOF", "VERSION", "ENV",
                  "TIMESTAMP", "USED", "PROVED", "UNSOLVED"]
    and symbols = ["~", "**", "++", "<--", "<->", "-->", "---", "><", ">>",
                   ">=", "<==", "<=>", "===", "==>", "\\/", "//", "/\\",
                   "!?", "!!", "!\\", "?!", "??", "?\\", ":=", "<>", ",","@",
                   "<-", "<<", "<=", "->", "=>", "=="]
    and idchars = "_'%"
    and tyvar = "*"
    and LP = "(" and RP = ")"
    and LC = "{" and RC = "}"
    and LB = "[" and RB = "]"
    and PLUS = "+" and MINUS = "-"
    and PM = ["-", "+"] (* There should be no space between these signs
                            and the following digits. The first should be
                            the minus sign, the second the plus. *)
    and versionName = "PRF FORMAT 1.0 EXTENDED"
    and log_versionName = "HOL CHECKER 0.1";

    val idchar_codes = map ord (explode idchars)
    and tyvar_code =  ord tyvar;

    val [TYOP, TYVAR, TYCONST, VAR, CONST, APP, ABS,
         THM, LINE, PROOF, VERSION, ENV,
         TIMESTAMP, USED, PROVED, UNSOLVED] = alphas;
  end;
```

8 The checking rules

The module `Check` is the critical part of the checker. It implements the checking rules. For each basic inference rule *basicrule*, a checking function `chk_`*basicrule* is defined. The functions return `true` if the inference is correct, otherwise, they return `false`. If the theorem is not in the expected form, an exception is raised due to the failure in pattern matching.

8.1 The specification

```
signature Check_sig =
  sig
    structure Report: Report_sig
    structure Htype: Htype_sig
    structure Hterm: Hterm_sig
    structure Hthm: Hthm_sig
    structure Henv: Henv_sig
    structure Proof: Proof_sig
    structure HtermSet: ORD_SET
```

The main functions provided by this module are `init` which should be called to initialise the module at the beginning of a proof; `chk_pline` which is called for each proof line; and `chk_proof` which is called at the end of each proof.

```
    val init : unit -> unit

    val chk_pline: (int * Proof.justification * Hthm.hthm) -> int
    val chk_proof: (string * Hthm.hthm list * int list) -> unit
```

Below are checking functions for each kind of justification. They do not need to be exported. They should be removed from the signature when the program is stable.

```
    val chk_Abs: (int * Hterm.hterm * int * Hthm.hthm) -> bool
    val chk_Assume: (Hterm.hterm * Hthm.hthm) -> bool
    val chk_BetaConv: (Hterm.hterm * Hthm.hthm) -> bool
    val chk_Disch: (int * Hterm.hterm * int * Hthm.hthm) -> bool
    val chk_InstType:
      (int * (Htype.htype * Htype.htype)list * int * Hthm.hthm) -> bool
    val chk_Mp: (int * int * int * Hthm.hthm) -> bool
    val chk_Refl: (Hterm.hterm * Hthm.hthm) -> bool
    val chk_Subst:
      (int * (int * Hterm.hterm)list * Hterm.hterm * int * Hthm.hthm) -> bool

    val chk_Axiom: (string * string * Hthm.hthm) -> bool
    val chk_Definition: (string * string * Hthm.hthm) -> bool
    val chk_DefExistsRule: (Hterm.hterm * Hthm.hthm) -> bool
    val chk_Hypothesis: Hthm.hthm -> bool
    val chk_NewAxiom: (string * Hterm.hterm * Hthm.hthm) -> bool
```

```
  val chk_NewConstant: (string * Htype.htype) -> bool
  val chk_NewType: (int * string) -> bool
  val chk_Numconv: (Hterm.hterm * Hthm.hthm) -> bool
  val chk_StoreDefinition: (string * Hterm.hterm * Hthm.hthm) -> bool
  val chk_Theorem: (string * string * Hthm.hthm) -> bool

  val chk_AddAssum: (int * Hterm.hterm * int * Hthm.hthm) -> bool
  val chk_Alpha: (Hterm.hterm * Hterm.hterm * Hthm.hthm) -> bool
  val chk_ApTerm: (int * Hterm.hterm * int * Hthm.hthm) -> bool
  val chk_ApThm: (int * int * Hterm.hterm * Hthm.hthm) -> bool
  val chk_Ccontr: (int * Hterm.hterm * int * Hthm.hthm) -> bool
  val chk_Choose: (int * Hterm.hterm * int * int * Hthm.hthm) -> bool
  val chk_Conj: (int * int * int * Hthm.hthm) -> bool
  val chk_Conjunct1: (int * int * Hthm.hthm) -> bool
  val chk_Conjunct2: (int * int * Hthm.hthm) -> bool
  val chk_Disj1: (int * int * Hterm.hterm * Hthm.hthm) -> bool
  val chk_Disj2: (int * Hterm.hterm * int * Hthm.hthm) -> bool
  val chk_DisjCases: (int * int * int * int * Hthm.hthm) -> bool
  val chk_EqImpRuleL: (int * int * Hthm.hthm) -> bool
  val chk_EqImpRuleR: (int * int * Hthm.hthm) -> bool
  val chk_EqMp: (int * int * int * Hthm.hthm) -> bool
  val chk_EqTIntro: (int * int * Hthm.hthm) -> bool
  val chk_EtaConv: (Hterm.hterm * Hthm.hthm) -> bool
  val chk_Exists: (int * Hterm.hterm * Hterm.hterm * int * Hthm.hthm) -> bool

  val chk_Ext: (int * int * Hthm.hthm) -> bool
  val chk_Gen: (int * Hterm.hterm * int * Hthm.hthm) -> bool
  val chk_ImpAntisymRule: (int * int * int * Hthm.hthm) -> bool
  val chk_ImpTrans: (int * int * int * Hthm.hthm) -> bool
  val chk_Inst: (int * (Hterm.hterm * Hterm.hterm) list * int * Hthm.hthm)
    -> bool
  val chk_MkAbs: (int * int * Hthm.hthm) -> bool
  val chk_MkComb: (int * int * int * Hthm.hthm) -> bool
  val chk_MkExists: (int * int * Hthm.hthm) -> bool
  val chk_NotElim: (int * int * Hthm.hthm) -> bool
  val chk_NotIntro: (int * int * Hthm.hthm) -> bool
  val chk_Spec: (int * Hterm.hterm * int * Hthm.hthm) -> bool
  val chk_Subs: (int * int list * int * Hthm.hthm) -> bool
  val chk_SubsOccs: (int * (int list * int) list * int * Hthm.hthm) -> bool
  val chk_SubstConv: (int * (int * Hterm.hterm) list *
                      Hterm.hterm * Hterm.hterm * Hthm.hthm) -> bool
  val chk_Sym: (int * int * Hthm.hthm) -> bool
  val chk_Trans: (int * int * int * Hthm.hthm) -> bool
end
```

8.2 The implementation

```
functor CheckFUN (structure Report: Report_sig and Htype:Htype_sig
                  and Hterm:Hterm_sig and Hthm: Hthm_sig
                  and Henv: Henv_sig and Proof: Proof_sig
                  and HtermSet: ORD_SET
```

```
                  sharing Report = Htype.Report = Hterm.Report = Hthm.Report
                  and Htype = Hterm.Htype = Hthm.Hterm.Htype =
                    Henv.Htype = Proof.Htype
                  and Hterm = Hthm.Hterm = Henv.Hterm = Proof.Hterm
                  and Hthm = Proof.Hthm
                  and HtermSet = Hthm.HtermSet
                  sharing type Hterm.hterm = HtermSet.item) : Check_sig =
struct
  structure Report = Report;
  structure Htype = Htype;
  structure Hterm = Hterm;
  structure Hthm = Hthm;
  structure Henv = Henv;
  structure Proof = Proof;
  structure HtermSet = HtermSet;

  open Htype Hterm Hthm Henv Proof;

  fun Check_ERR{function,message} =
    Exception.CHK_ERR{message = message,
                      origin_function = function,
                      origin_structure = "Check"};

  val debug = ref 0;
  fun write_out s = (output(std_err, s); flush_out std_err);
  fun init () = (
                debug := Debug.get_debug "Check";
                ());
```

8.2.1 Functions for outputting to log file These two functions are convenient for writing to the log file. The *tag* is the name of the justification. The function `write_used` is used for the justifications `DEFINITION` and `THEOREM`. The first string s_1 is the name of the theory and the second string s_2 is the name of the theorem. The function `write_used2` is for the justifications `NEWAXIOM` and `STOREDEFINITION`.

```
  local
    open Report
    open Keyword Io
  in
   fun write_used (tag,s1,s2,thm) =
     (write_output_string (LP ^ tag ^ " " ^ s1 ^ " " ^ s2);
      Hthm.pr_hthm thm;
      write_closing_line ())
   and write_used2 (tag,s1,tm,thm) =
     (write_output_string (LP ^ tag ^ " " ^ s1);
      Hterm.pr_hterm tm;
      Hthm.pr_hthm thm;
      write_closing_line ())
  end
```

8.3 Checking functions

All checking functions are described below in detail. They are grouped into three subsections: primitive rules, miscellaneous functions and derived ules. For each rule, the syntax in the proof file is given in a box first. It is followed by a specification, the name of the rule and the type of the ML function which implements the rule in HOL and a more detail description. The checking function is shown at the end. In the case of a derived rule, a proof in terms of primitive rules and other derived rules is given. Care has been taken not to introduce circular dependency in these proofs. Some of the proofs were adapted from [5] with some modifications.

8.3.1 Primitive rules

- **Abstraction**

  ```
  ABS term NUMBER
  ```

  $$\frac{\Gamma \vdash t_1 = t_2}{\Gamma \vdash (\lambda x.t_1) = (\lambda x.t_2)}$$

  `ABS : term -> thm -> thm`

  The `term` argument must be a variable, and must not be free in the assumptions of the hypothesis Γ. The `NUMBER` refers to the theorem in a line having the given line number. This theorem must be an equation.

```
fun chk_Abs (line,term,n1,thm) =
  let val thm1 = (get_thm (line,n1))
    val hyp1 = hyp thm1 and con1 = concl thm1
    val (left,right) = dest_eq (concl thm)
    val (lv,lbody) = dest_abs left and (rv,rbody) = dest_abs right
    val hyps = hyp thm
  in
    well_typed term andalso is_var term andalso
    is_eq con1 andalso (HtermSet.equal(hyp1, hyps)) andalso
    (lv = rv) andalso (term = rv) andalso
    not(mem lv (freesl (HtermSet.listItems hyps))) andalso
    (lbody = (lhs con1)) andalso (rbody = (rhs con1))
  end;
```

- **Assumption introduction**

  ```
  ASSUME term
  ```

  $$\frac{}{t \vdash t}$$

  `ASSUME : term -> thm`

  The term t must be of type $: bool$. There should be only a single assumption, and it must be the same as the conclusion.

```
fun chk_Assume (term, thm) =
  let val [ass] = HtermSet.listItems (hyp thm)
    val conc = concl thm
  in
    is_bool_ty term andalso (conc = term) andalso (conc = ass)
  end;
```

- **β-conversion**

```
BETACONV term
```

$$\overline{\vdash (\lambda x.t_1)t_2 = t_1[t_2/x]}$$

`BETA_CONV : term -> thm`

The `term` argument must be a β-redex in the form $(\lambda x.t_1)t_2$. The right-hand side of the resulting theorem is obtained by substituting t_2 for x in t_1 with suitable renaming of variables in t_1 to avoid variable capture.

```
fun chk_BetaConv (term, thm) =
  let val (rat,rnd) = dest_comb term
    val (v,body) = dest_abs rat
    val conc = concl thm
  in
    well_typed term andalso
    is_eq conc andalso ((lhs conc) = term) andalso
    (HtermSet.isEmpty (hyp thm)) andalso
    term_subst_chk [((v,rnd),v)] body (rhs conc) body
  end;
```

- **Discharging an assumption**

```
DISCH term NUMBER
```

$$\frac{\Gamma \vdash t_2}{\Gamma - \{t_1\} \vdash t_1 \supset t_2}$$

`DISCH : term -> thm -> thm`

The input term t_1 must be of type $: bool$. The `NUMBER` refers to the theorem in the line having the given number. The expression $\Gamma - \{t_1\}$ denotes the set subtraction of $\{t_1\}$ from Γ. If t_1 is not in Γ, the result of the subtraction is Γ itself.

```
fun chk_Disch (line, term, n1, thm) =
  let val thm1 = get_thm (line,n1)
    val (ante,conc) = dest_imp (concl thm)
  in
    is_bool_ty term andalso
    (ante = term) andalso (conc = (concl thm1)) andalso
    (HtermSet.equal((hyp thm),
     (HtermSet.difference((hyp thm1),(HtermSet.singleton term)))))
  end;
```

- **Type instantiation**

```
INSTTYPE type_type_list NUMBER
```

$$\frac{\Gamma \vdash t}{\Gamma \vdash t[\sigma_1, \ldots, \sigma_n/\alpha_1, \ldots, \alpha_n]}$$

`INST_TYPE : (type # type)list -> thm -> thm`

The first argument is a list of type pairs $[(\sigma_1, \alpha_1); \ldots; (\sigma_n, \alpha_n)]$ which specifies the simultaneous type substitutions to be made in the theorem referred to by `NUMBER`. The second fields α_i of the pairs must be type variables, and α_i must occur in any assumption in Γ.

All occurrences of α_i in t are replaced by the corresponding σ_i. Furthermore, if distinct variables in t become identical after the instantiation, they will be renamed.

```
fun chk_InstType (line, type_type_list,n1,thm) =
  let
    val thm1 = get_thm (line,n1)
    val tyvar_list = map snd type_type_list
    val hypp = hyp thm
    val hypl = HtermSet.listItems hypp
  in
    every is_vartype tyvar_list andalso
    every (type_OK o fst) type_type_list andalso
    HtermSet.equal(hypp, (hyp thm1)) andalso
    null(intersect tyvar_list (tyvarsl hypl)) andalso
    term_inst_chk type_type_list (freesl hypl) (concl thm) (concl thm1)
  end;
```

- **Modus Ponens**

`MP NUMBER NUMBER`

$$\frac{\Gamma_1 \vdash t_1 \supset t_2 \quad \Gamma_2 \vdash t_1}{\Gamma_1 \cup \Gamma_2 \vdash t_2}$$

`MP : thm -> thm -> thm`

The theorem referred to by the first `NUMBER` must be an implication. The theorem referred to by the second `NUMBER` must match the antecedent of the first theorem exactly.

```
fun chk_Mp (line, n1,n2,thm) =
  let val thm1 = get_thm (line,n1) and thm2 = get_thm (line,n2)
    val (ante,conc) = dest_imp (concl thm1)
  in
    (conc = concl thm) andalso (ante = concl thm2) andalso
    HtermSet.equal((hyp thm), (HtermSet.union((hyp thm1),(hyp thm2))))
  end;
```

- **Reflexivity**

`REFL term`

$$\overline{\vdash t = t}$$

`REFL : term -> thm`

The term t must be of type : *bool*. The resulting theorem must be an equation with identical terms on both sides.

```
fun chk_Refl (term,thm) =
  let val conc = concl thm
  in
    well_typed term andalso HtermSet.isEmpty(hyp thm) andalso
    is_eq conc andalso (lhs conc = term) andalso (rhs conc = term)
  end;
```

- **Substitution**

```
SUBST NUMBER_term_list term NUMBER
```

$$\frac{\Gamma_1 \vdash t_1 = t_1' \quad \cdots \quad \Gamma_n \vdash t_n = t_n' \quad \Gamma \vdash t[t_1, \ldots, t_n]}{\Gamma_1 \cup \cdots \cup \Gamma_n \cup \Gamma \vdash t[t_1', \ldots, t_n'/t_1, \ldots, t_n]}$$

`SUBST : (thm # term)list -> term -> thm -> thm`

The first argument is a list of pairs whose first fields are equational theorems $t_i = t_i'$, and whose second fields are simple variables x_i. The second argument is a term of the form $t[x_1, \ldots, x_n]$. It should match the conclusion of the theorem referred to by **NUMBER**. The variables x_i in the term mark the places where substitutions using the theorems $\vdash t_i = t_i'$ are to be done. The type of x_i must be the same as of t_i. Variables in t may be renamed to avoid capture.

```
fun chk_Subst (line, num_term_list,term,n1,thm) =
  let val thm1 = get_thm (line,n1)
    val thms = map (fn n => get_thm(line,(fst n))) num_term_list
    val hyps = fold HtermSet.union (map hyp thms) HtermSet.empty
    val lrl = map (dest_eq o concl) thms
    val (_,vl) = ListUtil.unzip num_term_list
  in
    every (is_eq o concl) thms andalso
    HtermSet.equal((hyp thm), (HtermSet.union(hyps,(hyp thm1)))) andalso
    term_subst_chk (ListUtil.zip(lrl,vl)) term (concl thm) (concl thm1)
  end;
```

8.3.2 Miscellaneous Functions

- **Retrieving an axiom**

```
AXIOM STRING STRING
```

`axiom : string -> string -> thm`

This justification indicates that the theorem in this inference step is an axiom whose name is the second string and which is stored in the theory specified as the first string. The well-typedness of the axiom should be checked. An entry is written to the log file.

```
fun chk_Axiom (s1,s2,thm) =
  (write_used ("AXIOM", s1, s2, thm);
   is_bool_ty(concl thm) andalso
   every is_bool_ty (HtermSet.listItems(hyp thm)))
```

- **Retrieving a definition**

```
DEFINITION STRING STRING
```

`definition : string -> string -> thm`

This justification indicates that the theorem in this inference step is a previously defined definition whose name is the second string and which is stored in the theory specified as the first string. The well-typedness of the theorem should be checked. An entry is written to the log file.

```
fun chk_Definition (s1,s2,thm) =
  (write_used ("DEFINITION", s1, s2, thm);
   is_bool_ty(concl thm) andalso
   every is_bool_ty (HtermSet.listItems(hyp thm)))
```

- **Create a definitional theorem**

```
DEFEXISTSRULE term
```

`DEF_EXISTS_RULE : term -> thm`

This justification introduces a new definitional theorem. The input term must be an equation and both sides must be of the same type.

```
fun chk_DefExistsRule (tm,thm) =
  let val (left,right) = dest_eq (snd(strip_forall tm))
    val (l,r) = dest_eq(snd(strip_forall(snd(dest_exists(concl thm)))))
  in
    (well_typed left) andalso (well_typed right) andalso
    ((type_of left) = (type_of right)) andalso
    (left = l) andalso (r = right)
  end
```

- **Hypothesis**

```
HYPOTHESIS
```

This justification indicates that the theorem is one of the initial theorems of the proof. It should have been proven in a previous proof. An entry is written to the log file.

```
fun chk_Hypothesis thm =
  (write_used ("HYPOTHESIS", "", "", thm);
  is_bool_ty (concl thm))
```

- **Introducing a new axiom**

```
NEWAXIOM STRING term
```

$$\frac{}{\vdash \forall x_1 \ldots x_n . t[x_1, \ldots, x_n]}$$

`new_axiom : (string # term) -> thm`

This justification introduces a new axiom. It is in the form given as the `term`. The `STRING` is the name of the axiom. All free variables in the input term are automatically generalised. An entry is written to the log file.

```
fun chk_NewAxiom (s,term,thm) =
  let val (vs,body) = (strip_forall (concl thm)) in
    write_used2 ("NEWAXIOM", s, term, thm);
    (term = body) andalso (every (fn x => mem x vs) (frees term))
  end
```

- **Introducing a new constant**

```
NEWCONSTANT STRING type
```

`new_constant : string -> type -> void`

This justification declares a new constant whose name is the given `STRING` and whose type is the given `type`. The name should be unique, i.e., different from any existing constant, and the type should be well-formed. The current signature should be updated. There should be no conclusion (derived theorem) in this inference step. To satisfy the type checker, the theorem `TRUTH` is used as a dummy.

```
fun chk_NewConstant (string,ty) =
  if (type_OK ty) then
```

```
    let val _  =  add_const(string,ty)
      handle (CHK_ERR{origin_function=f,
                      origin_structure=s,message=m}) =>
        raise (Check_ERR{function="chk_NewConstant",
                         message=(s^"."^f^":"^m)})
           | e => raise e
    in true end
  else raise (Check_ERR{function="chk_NewConstant",
                        message="type of new constant not well-formed"})
```

- **Introducing a new type**

  `NEWTYPE NUMBER STRING`

  `new_type : int -> string -> void`

  This justification declares a new type constructor whose name is the given `STRING` and whose arity is the given `NUMBER`. The name should be unique, i.e., different from any existing type constructor. The current type structure should be updated. The theorem in this inference step is not used. To satisfy the type checker, the theorem `TRUTH` is used as a dummy.

```
fun chk_NewType (num,string) =
  let val _  = add_type(string,num)
    handle (CHK_ERR{origin_function=f,origin_structure=s,message=m}) =>
      raise (Check_ERR{function="chk_NewType",
                       message=(s^"."^f^":"^m)})
         | e => raise e
  in true end
```

- **Definition of non-zero numbers**

  `NUMCONV term`

$$\overline{\vdash n = \mathsf{SUC}\, m}$$

  `num_CONV : term -> thm`

  The input term must be a constant denoting a non-zero natural number. m is a numeric constant denoting the predecessor of n.

```
fun chk_Numconv (term,thm) =
  let val (left,right) = dest_eq (concl thm)
    val n = StringCvt.atoi(fst(dest_const left))
    and (s,m) =
       ((fst o dest_const) ## (StringCvt.atoi o fst o dest_const))
        (dest_comb right)
  in
    (term = left) andalso (n = (m + 1)) andalso (s = "SUC")
  end
```

- **Storing a definition**

  `STOREDEFINITION STRING term`

  `store_definition : (string # term) -> thm`

  This justification introduces a new definition. In fact, making a new definition is a three-step process:

1. a theorem asserting the existence of the definition is derived with the justification `DEFEXISTSRULE`;
2. a new constant is declared with the justification `NEWCONSTANT`;
3. the definition is saved in the current theory with the justification `STOREDEFINITION`.

Since this file format allows only the four kinds of primitive terms, special syntactic status of constants, i.e., infix or binder, are not recognised. The input term must be a series of conjunctions. Each conjunct is an equation and both sides are of the same type. An entry is written to the log file.

```
fun chk_StoreDefinition (s,term,thm) =
  let
   fun chk_tm ((l,r),t) =
    t andalso (well_typed l) andalso (well_typed r) andalso
    ((type_of l) = (type_of r))
   val conjs = strip_conj (snd(strip_forall term))
   val lrlst = map (dest_eq o snd o strip_forall) conjs
  in
    write_used2 ("STOREDEFINITION", s, term, thm);
    (fold chk_tm lrlst true) andalso (term = (concl thm))
  end
```

- **Retrieving a theorem**

THEOREM STRING STRING

`theorem : string -> string -> thm`

This justification indicates that the theorem in this inference step has been derived previously. It has been stored in the theory, whose name is the first string, under the name specified as the second string. The well-typedness of the theorem should be checked. An entry is written to the log file.

```
fun chk_Theorem (s1,s2,thm) =
  (write_used ("THEOREM", s1, s2, thm);
   is_bool_ty(concl thm) andalso
   every is_bool_ty (HtermSet.listItems(hyp thm)))
```

8.3.3 Derived Rules

- **Adding an assumption**

ADDASSUM term NUMBER

$$\frac{\Gamma \vdash t}{\Gamma,\ t' \vdash t}$$

`ADD_ASSUM : term -> thm -> thm`

The input term is the new assumption t' to be added to the theorem. It must be of type `bool`.

1.	$t' \vdash t'$	[`ASSUME`]
2.	$\Gamma \vdash t$	[Hypothesis]
3.	$\Gamma \vdash t' \supset t$	[`DISCH` 2]
4.	$\Gamma,\ t' \vdash t$	[`MP` 3,1]

```
fun chk_AddAssum (line, term, n, thm) =
  let val thm1 = get_thm(line, n)
  in
    (concl thm = concl thm1) andalso (ty_is_bool(type_of term)) andalso
    (HtermSet.equal((HtermSet.delete((hyp thm), term)), (hyp thm1)))
  end
```

- **α-conversion**

`ALPHA term term`

$$\overline{\vdash t_1 = t_2}$$

`ALPHA : term -> term -> thm`

The input terms t_1 and t_2 must be α-equivalent, otherwise, it fails.

```
fun chk_Alpha (term1, term2, thm) =
  let val (left,right) = dest_eq (concl thm)
  in
    well_typed term1 andalso well_typed term2 andalso
    aconv left right andalso (term1 = left) andalso (term2 = right)
  end
```

- **Application of a term to a theorem**

`APTERM term NUMBER`

$$\frac{\Gamma \vdash t_1 = t_2}{\Gamma \vdash t\, t_1 = t\, t_2}$$

`AP_TERM : term -> thm -> thm`

The input term must have a function type whose domain is the type of the left-hand side (or right-hand side as the type of both sides must be the same) of the input theorem.

1.	$\Gamma \vdash t_1 = t_2$	[Hypothesis]
2.	$\vdash t\, t_1 = t\, t_1$	[`REFL`]
3.	$\Gamma \vdash t\, t_1 = t\, t_2$	[`SUBST` 1,2]

```
fun chk_ApTerm (line, term, n, thm) =
  let val ((lt,left), (rt,right)) =
         (dest_comb ## dest_comb)(dest_eq(concl thm))
      val thm1 = get_thm(line, n)
      val (lft,rgt) = dest_eq (concl thm1)
  in
    (lt = rt) andalso (term = lt) andalso
    (lft = left) andalso (rgt = right) andalso
    (HtermSet.equal((hyp thm), (hyp thm1))) andalso
    (domain_of(type_of term) = (type_of left))
  end
```

- **Application of a theorem to a term**

```
APTHM NUMBER term
```

$$\frac{\Gamma \vdash t_1 = t_2}{\Gamma \vdash t_1\, t = t_2\, t}$$

`AP_THM : thm -> term -> thm`

The input term must have the same type as the domain of the left-hand side (or the right-hand side) of the input theorem.

1.	$\Gamma \vdash t_1 = t_2$	[Hypothesis]
2.	$\vdash t_1\, t = t_1\, t$	[REFL]
3.	$\Gamma \vdash t_1\, t = t_2\, t$	[SUBST 1,2]

```
fun chk_ApThm (line, n, term, thm) =
  let val ((left,lt), (right,rt)) =
          (dest_comb ## dest_comb)(dest_eq(concl thm))
      val thm1 = get_thm(line, n)
      val (lft,rgt) = dest_eq (concl thm1)
  in
    (lt = rt) andalso (lt = term) andalso
    (left = lft) andalso (right = rgt) andalso
    (HtermSet.equal((hyp thm), (hyp thm1))) andalso
    (domain_of(type_of left) = (type_of term))
  end
```

- **Classical contradiction rule**

```
CCONTR term NUMBER
```

$$\frac{\Gamma \vdash \mathsf{F}}{\Gamma - \{\neg t\} \vdash t}$$

`CCONTR : term -> thm -> thm`

The theorem referred to by the `NUMBER` must have F as its conclusion. The input term t should be of type `:bool`, and the negation of it should occur in the assumption of the input theorem.

1.	$\vdash \neg = \lambda b.\, b \supset \mathsf{F}$	[Definition of $\neg$]
2.	$\vdash \neg t = (\lambda b.\, b \supset \mathsf{F})t$	[`AP_THM` 1]
3.	$\vdash (\lambda b.\, b \supset \mathsf{F})t = t \supset \mathsf{F}$	[`BETA_CONV`]
4.	$\vdash \neg t = t \supset \mathsf{F}$	[`TRANS` 2,3]
5.	$\Gamma \vdash \mathsf{F}$	[Hypothesis]
6.	$\Gamma \vdash \neg t \supset \mathsf{F}$	[`DISCH` 5]
7.	$\Gamma \vdash (t \supset \mathsf{F}) \supset \mathsf{F}$	[`SUBST` 4,6]
8.	$t = \mathsf{F} \vdash t = \mathsf{F}$	[`ASSUME`]
9.	$\Gamma,\ t = \mathsf{F} \vdash (\mathsf{F} \supset \mathsf{F}) \supset \mathsf{F}$	[`SUBST` 8,7]
10.	$\mathsf{F} \vdash \mathsf{F}$	[`ASSUME`]
11.	$\vdash \mathsf{F} \supset \mathsf{F}$	[`DISCH` 10]
12.	$\Gamma,\ t = \mathsf{F} \vdash \mathsf{F}$	[`MP` 9,11]
13.	$\vdash \mathsf{F} = \forall b.\, b$	[Definition of F]
14.	$\Gamma,\ t = \mathsf{F} \vdash \forall b.\, b$	[`SUBST` 13,12]
15.	$\Gamma,\ t = \mathsf{F} \vdash t$	[`SPEC` 14]
16.	$\vdash \forall b.\, (b = \mathsf{T}) \vee (b = \mathsf{F})$	[Axiom `EXCLUDED_MIDDLE`]
17.	$\vdash (t = \mathsf{T}) \vee (t = \mathsf{F})$	[`SPEC` 16]
18.	$t = \mathsf{T} \vdash t = \mathsf{T}$	[`ASSUME`]
19.	$t = \mathsf{T} \vdash \mathsf{T} = t$	[`SYM` 18]
20.	$\vdash \mathsf{T}$	[Theorem `TRUTH`]
21.	$t = \mathsf{T} \vdash t$	[`EQ_MP` 19,20]
22.	$\Gamma \vdash t$	[`DISJ_CASES` 17,21,15]

```
fun chk_Ccontr (line, term, n, thm) =
  let val thm1 = get_thm(line, n)
      and not_term =
        mk_comb(mk_const("~",mk_funtype(bool_ty,bool_ty)), term)
      and F = mk_const("F", bool_ty)
      val hyp1 = (HtermSet.delete((hyp thm1), not_term))
        handle NotFound => (hyp thm1)
  in
    (concl thm1 = F) andalso (concl thm = term) andalso
    (HtermSet.equal((hyp thm), hyp1))
  end
```

- **∃-elimination**

```
CHOOSE term NUMBER NUMBER
```

$$\frac{\Gamma_1 \vdash \exists x.\, t[x] \quad \Gamma_2,\ t[v] \vdash t'}{\Gamma_1 \cup \Gamma_2 \vdash t'}$$

`CHOOSE : (term # thm) -> thm -> thm`

The input `term` must be a variable v and its type must be the same as the existentially quantified variable x in the first theorem. $t[v]$ is a term occurring in the assumptions of the second theorem. It is the same as $t[x]$, the body of the first theorem, up to α-conversion. The variable v must not occur free in the conclusion of the first theorem, i.e., $\exists x.\, t[x]$, and neither can it occur free in Γ_2 or t'.

1.	$\vdash \exists = \lambda P.\, P(\varepsilon\ P)$	[Definition of $\exists$[1]]
2.	$\vdash \exists(\lambda x.\, t[x]) = (\lambda P.\, P(\varepsilon\ P))(\lambda x.\, t[x])$	[`AP_THM` 1]
3.	$\Gamma_1 \vdash \exists(\lambda x.\, t[x])$	[Hypothesis]
4.	$\Gamma_1 \vdash (\lambda P.\, P(\varepsilon\ P))(\lambda x.\, t[x])$	[`EQ_MP` 2,3]
5.	$\vdash (\lambda P.\, P(\varepsilon\ P))(\lambda x.\, t[x]) = (\lambda x.\, t[x])(\varepsilon(\lambda x.\, t[x]))$	[`BETA_CONV`]
6.	$\Gamma_1 \vdash (\lambda x.\, t[x])(\varepsilon(\lambda x.\, t[x])$	[`EQ_MP` 5,4]
7.	$\vdash (\lambda x.\, t[x])v = t[v]$	[`BETA_CONV`]
8.	$\vdash t[v] = (\lambda x.\, t[x])v$	[`SYM` 7]
9.	$\Gamma_2,\ t[v] \vdash t'$	[Hypothesis]
10.	$\Gamma_2 \vdash t[v] \supset t'$	[`DISCH` 9]
11.	$\Gamma_2 \vdash (\lambda x.\, t[x])v \supset t'$	[`SUBST` 8,10]
12.	$(\lambda x.\, t[x])v \vdash (\lambda x.\, t[x])v$	[`ASSUME`]
13.	$\Gamma_2,\ (\lambda x.\, t[x])v \vdash t'$	[`MP` 11,12]
14.	$\Gamma_1 \cup \Gamma_2 \vdash t'$	[`SELECT_ELIM` 6,13]

```
fun chk_Choose(line, term, n1, n2, thm) =
  let val thm1 = get_thm(line, n1) and thm2 = get_thm(line, n2)
    val (evar,ebody) = dest_exists(concl thm1)
    val hyp1 = hyp thm1 and hyp2 = hyp thm2
    val hyp2l = HtermSet.listItems hyp2
    val wit = ListUtil.findOne (aconv ebody) hyp2l
  in
    case wit of
      NONE =>  false
    | SOME tm  =>
        let
          val hyp2' = HtermSet.delete(hyp2, tm)
        in
          (is_var term) andalso not(mem term(frees(concl thm1))) andalso
          (not (mem term (frees (concl thm2)))) andalso
          (not (mem term (freesl (HtermSet.listItems hyp2')))) andalso
          (concl thm = concl thm2) andalso
          (HtermSet.equal((hyp thm), (HtermSet.union(hyp1, hyp2'))))
        end
  end
```

- **$\wedge$-introduction**

```
CONJ NUMBER NUMBER
```

$$\frac{\Gamma_1 \vdash t_1 \quad \Gamma_2 \vdash t_2}{\Gamma_1 \cup \Gamma_2 \vdash t_1 \wedge t_2}$$

`CONJ : thm -> thm -> thm`

The two `NUMBER`s refer to two theorems which are to be combined by the $\wedge$ operator.

[1] with suitable type instantiation.

1.	$\vdash \wedge = \lambda b_1\, b_2. \forall b. (b_1 \supset (b_2 \supset b)) \supset b$	[Definition of $\wedge$]
2.	$\vdash \$\wedge t_1 = \$\wedge t_1$	[`REFL`]
3.	$\vdash (\lambda b_1\, b_2. \forall b. (b_1 \supset (b_2 \supset b)) \supset b) t_1 =$ $\lambda b_2. \forall b. (t_1 \supset (b_2 \supset b)) \supset b$	[`BETA_CONV`]
4.	$\vdash \$\wedge t_1 = (\lambda b_1\, b_2. \forall b. (b_1 \supset (b_2 \supset b)) \supset b) t_1$	[`SUBST` 1,2]
5.	$\vdash \$\wedge t_1 = \lambda b_2. \forall b. (t_1 \supset (b_2 \supset b)) \supset b$	[`SUBST` 3,4]
6.	$\vdash t_1 \wedge t_2 = t_1 \wedge t_2$	[`REFL`]
7.	$\vdash (\lambda b_2. \forall b. (t_1 \supset (b_2 \supset b)) \supset b) t_2$ $= \forall b. (t_1 \supset (t_2 \supset b)) \supset b)$	[`BETA_CONV`]
8.	$\vdash t_1 \wedge t_2 = (\lambda b_2. \forall b. (t_1 \supset (b_2 \supset b)) \supset b) t_2$	[`SUBST` 5,6]
9.	$\vdash t_1 \wedge t_2 = \forall b. (t_1 \supset (t_2 \supset b)) \supset b$	[`SUBST` 7,8]
10.	$t_1 \supset (t_2 \supset b) \vdash t_1 \supset (t_2 \supset b)$	[`ASSUME`]
11.	$\Gamma_1 \vdash t_1$	[Hypothesis]
12.	$\Gamma_1,\ t_1 \supset (t_2 \supset b) \vdash t_2 \supset b$	[`MP` 10,11]
13.	$\Gamma_2 \vdash t_2$	[Hypothesis]
14.	$\Gamma_1 \cup \Gamma_2,\ t_1 \supset (t_2 \supset b) \vdash b$	[`MP` 12,13]
15.	$\Gamma_1 \cup \Gamma_2 \vdash (t_1 \supset (t_2 \supset b)) \supset b$	[`DISCH` 14]
16.	$\Gamma_1 \cup \Gamma_2 \vdash \forall b. (t_1 \supset (t_2 \supset b)) \supset b$	[`GEN` 15]
17.	$\vdash \forall b. (t_1 \supset (t_2 \supset b)) \supset b = t_1 \wedge t_2$	[`SUBST` 9,6]
18.	$\Gamma_1 \cup \Gamma_2 \vdash t_1 \wedge t_2$	[`SUBST` 17,16]

```
fun chk_Conj(line, n1, n2, thm) =
  let val thm1 = get_thm(line, n1) and thm2 = get_thm(line, n2)
    val hyp1 = hyp thm1 and hyp2 = hyp thm2
    and concls = map concl [thm1, thm2]
    val (AND,conjs) = strip_comb (concl thm)
    val hyps = hyp thm
  in
    (well_typed AND) andalso (fst(dest_const AND) = "/\\") andalso
    (every is_bool_ty concls) andalso (conjs = concls) andalso
    (every is_bool_ty (HtermSet.listItems hyps)) andalso
    (HtermSet.equal(hyps, (HtermSet.union(hyp1, hyp2))))
  end
```

- **$\wedge$-elimination(left)**

```
CONJUNCT1 NUMBER
```

$$\frac{\Gamma \vdash t_1 \wedge t_2}{\Gamma \vdash t_1}$$

`CONJUNCT1 : thm -> thm`

This inference step extracts the left conjunct from the theorem referred to by `NUMBER`. The conclusion of the input theorem must be a conjunction.

1.	$\vdash \wedge = \lambda b_1\, b_2.\forall b.\,(b_1 \supset (b_2 \supset b)) \supset b$	[Definition of $\wedge$]
2.	$\vdash \$\wedge t_1 = \$\wedge t_1$	[`REFL`]
3.	$\vdash (\lambda b_1\, b_2.\forall b.\,(b_1 \supset (b_2 \supset b)) \supset b)t_1$ $= \lambda b_2.\forall b.\,(t_1 \supset (b_2 \supset b)) \supset b$	[`BETA_CONV`]
4.	$\vdash \$\wedge t_1 = (\lambda b_1\, b_2.\forall b.\,(b_1 \supset (b_2 \supset b)) \supset b)t_1$	[`SUBST` 1,2]
5.	$\vdash \$\wedge t_1 = \lambda b_2.\forall b.\,(t_1 \supset (b_2 \supset b)) \supset b$	[`SUBST` 3,4]
6.	$\vdash t_1 \wedge t_2 = t_1 \wedge t_2$	[`REFL`]
7.	$\vdash (\lambda b_2.\forall b.\,(t_1 \supset (b_2 \supset b)) \supset b)t_2$ $= \forall b.\,(t_1 \supset (t_2 \supset b)) \supset b)$	[`BETA_CONV`]
8.	$\vdash t_1 \wedge t_2 = (\lambda b_2.\forall b.\,(t_1 \supset (b_2 \supset b)) \supset b)t_2$	[`SUBST` 5,6]
9.	$\vdash t_1 \wedge t_2 = \forall b.\,(t_1 \supset (t_2 \supset b)) \supset b$	[`SUBST` 7,8]
10.	$\Gamma \vdash t_1 \wedge t_2$	[Hypothesis]
11.	$\Gamma \vdash \forall b.\,(t_1 \supset (t_2 \supset b)) \supset b$	[`SUBST` 9,10]
12.	$\Gamma \vdash (t_1 \supset (t_2 \supset t_1)) \supset t_1$	[`SPEC` 11]
13.	$t_1 \vdash t_1$	[`ASSUME`]
14.	$t_1 \vdash t_2 \supset t_1$	[`DISCH` 13]
15.	$\vdash t_1 \supset (t_2 \supset t_1)$	[`DISCH` 14]
16.	$\Gamma \vdash t_1$	[`MP` 12,15]

```
fun chk_Conjunct1(line, n, thm) =
  let val thm1 = get_thm(line, n)
    val (AND,[conj1,conj2]) = strip_comb (concl thm1)
    and hyps = (hyp thm1)
  in
    (every is_bool_ty (HtermSet.listItems hyps)) andalso
    (HtermSet.equal(hyps, (hyp thm1))) andalso
    (well_typed AND) andalso (fst(dest_const AND) = "/\\") andalso
    (conj1 = (concl thm))
  end
```

- **$\wedge$-elimination(right)**

  `CONJUNCT2 NUMBER`

$$\frac{\Gamma \vdash t_1 \wedge t_2}{\Gamma \vdash t_2}$$

`CONJUNCT2 : thm -> thm`

This inference step extracts the right conjunct from the theorem referred to by `NUMBER`. The conclusion of the input theorem must be a conjunction.

1.	$\vdash \wedge = \lambda b_1\, b_2. \forall b. (b_1 \supset (b_2 \supset b)) \supset b$	[Definition of $\wedge$]
2.	$\vdash \$\wedge t_1 = \$\wedge t_1$	[REFL]
3.	$\vdash (\lambda b_1\, b_2. \forall b. (b_1 \supset (b_2 \supset b)) \supset b) t_1$ $= \lambda b_2. \forall b. (t_1 \supset (b_2 \supset b)) \supset b$	[BETA_CONV]
4.	$\vdash \$\wedge t_1 = (\lambda b_1\, b_2. \forall b. (b_1 \supset (b_2 \supset b)) \supset b) t_1$	[SUBST 1,2]
5.	$\vdash \$\wedge t_1 = \lambda b_2. \forall b. (t_1 \supset (b_2 \supset b)) \supset b$	[SUBST 3,4]
6.	$\vdash t_1 \wedge t_2 = t_1 \wedge t_2$	[REFL]
7.	$\vdash (\lambda b_2. \forall b. (t_1 \supset (b_2 \supset b)) \supset b) t_2$ $= \forall b. (t_1 \supset (t_2 \supset b)) \supset b)$	[BETA_CONV]
8.	$\vdash t_1 \wedge t_2 = (\lambda b_2. \forall b. (t_1 \supset (b_2 \supset b)) \supset b) t_2$	[SUBST 5,6]
9.	$\vdash t_1 \wedge t_2 = \forall b. (t_1 \supset (t_2 \supset b)) \supset b$	[SUBST 7,8]
10.	$\Gamma \vdash t_1 \wedge t_2$	[Hypothesis]
11.	$\Gamma \vdash \forall b. (t_1 \supset (t_2 \supset b)) \supset b$	[SUBST 9,10]
12.	$\Gamma \vdash (t1_1 \supset (t_2 \supset t_2)) \supset t_2$	[SPEC 11]
13.	$t_2 \vdash t_2$	[ASSUME]
14.	$\vdash t_2 \supset t_2$	[DISCH 13]
15.	$\vdash t_1 \supset (t_2 \supset t_2)$	[DISCH 14]
16.	$\Gamma \vdash t_2$	[MP 12,15]

```
fun chk_Conjunct2 (line, n, thm) =
  let val thm1 = get_thm(line, n)
    val (AND,[conj1,conj2]) = strip_comb (concl thm1)
    and hyps = hyp thm1
  in
    (every is_bool_ty (HtermSet.listItems hyps)) andalso
    (HtermSet.equal(hyps, (hyp thm))) andalso
    (well_typed AND) andalso (fst(dest_const AND) = "/\\") andalso
    (conj2 = (concl thm))
  end
```

- **Intuitionistic contradiction rule**

```
CONTR term NUMBER
```

$$\frac{\Gamma \vdash \mathsf{F}}{\Gamma \vdash t}$$

`CONTR : term -> thm -> thm`

The theorem referred to by `NUMBER` should have falsity (F) as its conclusion.

1.	$\vdash \forall t. \mathsf{F} \supset t$	[Theorem FALSITY]
2.	$\Gamma \vdash \mathsf{F}$	[Hypothesis]
3.	$\vdash \mathsf{F} \supset t$	[SPEC 1]
4.	$\Gamma \vdash t$	[MP 3,2]

```
fun chk_Contr(line, term, n, thm) =
  let val thm1 = get_thm(line, n)
      and hyps = hyp thm
  in
    (well_typed(concl thm1)) andalso
    (fst (dest_const (concl thm1)) = "F") andalso
    (every is_bool_ty (HtermSet.listItems hyps)) andalso
```

```
    (HtermSet.equal(hyps, (hyp thm1))) andalso
    (is_bool_ty term) andalso (term = concl thm)
  end
```

- **Right $\vee$-introduction**

```
DISJ1 NUMBER term
```

$$\frac{\Gamma \vdash t_1}{\Gamma \vdash t_1 \vee t_2}$$

`DISJ1 : thm -> term -> thm`

The result of this inference rule is a disjunctive theorem whose second disjunct is the input term. This term must be of type `:bool`.

1.	$\vdash \vee = \lambda b_1\, b_2.\, \forall b.\, (b_1 \supset b) \supset (b_2 \supset b) \supset b$	[Definition of $\vee$]
2.	$\vdash \$\vee t_1 = \$\vee t_1$	[REFL]
3.	$(\lambda b_1\, b_2.\, \forall b.\, (b_1 \supset b) \supset (b_2 \supset b) \supset b)t_1$ $= \lambda b_2.\, \forall b.\, (t_1 \supset b) \supset (b_2 \supset b) \supset b$	[BETA_CONV]
4.	$\vdash \$\vee t_1 = (\lambda b_1\, b_2.\, \forall b.\, (b_1 \supset b) \supset (b_2 \supset b) \supset b)t_1$	[SUBST 1,2]
5.	$\vdash \$\vee t_1 = \lambda b_2.\, \forall b.\, (t_1 \supset b) \supset (b_2 \supset b) \supset b$	[SUBST 3,4]
6.	$\vdash t_1 \vee t_2 = t_1 \vee t_2$	[REFL]
7.	$(\lambda b_2.\, \forall b.\, (t_1 \supset b) \supset (b_2 \supset b) \supset b)t_2$ $= \forall b.\, (t_1 \supset b) \supset (t_2 \supset b) \supset b$	[BETA_CONV]
8.	$\vdash t_1 \vee t_2 = (\lambda b_2.\, \forall b.\, (t_1 \supset b) \supset (b_2 \supset b) \supset b)t_2$	[SUBST 5,6]
9.	$\vdash t_1 \vee t_2 = \forall b.\, (t_1 \supset b) \supset (t_2 \supset b) \supset b$	[SUBST 7,8]
10.	$\Gamma \vdash t_1$	[Hypothesis]
11.	$t_1 \supset b \vdash t_1 \supset b$	[ASSUME]
12.	$\Gamma,\ t_1 \supset b \vdash b$	[MP 11,10]
13.	$\Gamma,\ t_1 \supset b \vdash (t_2 \supset b) \supset b$	[DISCH 12]
14.	$\Gamma \vdash (t_1 \supset b) \supset (t_2 \supset b) \supset b$	[DISCH 13]
15.	$\Gamma \vdash \forall b.\, (t_1 \supset b) \supset (t_2 \supset b) \supset b$	[GEN 14]
16.	$\vdash \forall b.\, (t_1 \supset b) \supset (t_2 \supset b) \supset b = t_1 \vee t_2$	[SUBST 9,6]
17.	$\Gamma \vdash t_1 \vee t_2$	[SUBST 16,15]

```
fun chk_Disj1(line, n, term, thm) =
  let val thm1 = get_thm(line, n)
    val (OR, [disj1,disj2]) = strip_comb(concl thm)
  in
    (is_bool_ty disj1) andalso (is_bool_ty disj2) andalso
    (well_typed OR) andalso ((fst(dest_const OR)) = "\\/") andalso
    (concl thm1 = disj1) andalso (term = disj2) andalso
    (HtermSet.equal((hyp thm), (hyp thm1)))
  end
```

- **Left $\vee$-introduction**

```
DISJ2 term NUMBER
```

$$\frac{\Gamma \vdash t_2}{\Gamma \vdash t_1 \vee t_2}$$

`DISJ2 : term -> thm -> thm`

The result of this inference rule is a disjunctive theorem whose first disjunct is the input term. This term must be of type `:bool`.

1.	$\vdash \vee = \lambda b_1\, b_2. \forall b. (b_1 \supset b) \supset (b_2 \supset b) \supset b$	[Definition of $\vee$]
2.	$\vdash \$\vee t_1 = \$\vee t_1$	[`REFL`]
3.	$(\lambda b_1\, b_2. \forall b. (b_1 \supset b) \supset (b_2 \supset b) \supset b)t_1$ $= \lambda b_2. \forall b. (t_1 \supset b) \supset (b_2 \supset b) \supset b$	[`BETA_CONV`]
4.	$\vdash \$\vee t_1 = (\lambda b_1\, b_2. \forall b. (b_1 \supset b) \supset (b_2 \supset b) \supset b)t_1$	[`SUBST` 1,2]
5.	$\vdash \$\vee t_1 = \lambda b_2. \forall b. (t_1 \supset b) \supset (b_2 \supset b) \supset b$	[`SUBST` 3,4]
6.	$\vdash t_1 \vee t_2 = t_1 \vee t_2$	[`REFL`]
7.	$(\lambda b_2. \forall b. (t_1 \supset b) \supset (b_2 \supset b) \supset b)t_2$ $= \forall b. (t_1 \supset b) \supset (t_2 \supset b) \supset b$	[`BETA_CONV`]
8.	$\vdash t_1 \vee t_2 = (\lambda b_2. \forall b. (t_1 \supset b) \supset (b_2 \supset b) \supset b)t_2$	[`SUBST` 5,6]
9.	$\vdash t_1 \vee t_2 = \forall b. (t_1 \supset b) \supset (t_2 \supset b) \supset b$	[`SUBST` 7,8]
10.	$\Gamma \vdash t_2$	[Hypothesis]
11.	$t_2 \supset b \vdash t_2 \supset b$	[`ASSUME`]
12.	$\Gamma,\ t_2 \supset b \vdash b$	[`MP` 11,10]
13.	$\Gamma,\ t_2 \supset b \vdash (t_2 \supset b) \supset b$	[`DISCH` 12]
14.	$\Gamma \vdash (t_1 \supset b) \supset (t_2 \supset b) \supset b$	[`DISCH` 13]
15.	$\Gamma \vdash \forall b. (t_1 \supset b) \supset (t_2 \supset b) \supset b$	[`GEN` 14]
16.	$\vdash \forall b. (t_1 \supset b) \supset (t_2 \supset b) \supset b = t_1 \vee t_2$	[`SUBST` 9,6]
17.	$\Gamma \vdash t_1 \vee t_2$	[`SUBST` 16,15]

```
fun chk_Disj2(line, term, n, thm) =
  let val thm1 = get_thm(line, n)
    val (OR, [disj1,disj2]) = strip_comb(concl thm)
  in
    (is_bool_ty disj1) andalso (is_bool_ty disj2) andalso
    (well_typed OR) andalso ((fst(dest_const OR)) = "\\/") andalso
    ((concl thm1) = disj2) andalso (term = disj1) andalso
    (HtermSet.equal((hyp thm), (hyp thm1)))
  end
```

- **$\vee$-elimination**

```
DISJCASES NUMBER NUMBER NUMBER
```

$$\frac{\Gamma \vdash t_1 \vee t_2 \quad \Gamma_1,\ t_1 \vdash t \quad \Gamma_2,\ t_2 \vdash t}{\Gamma \cup \Gamma_1 \cup \Gamma_2 \vdash t}$$

`DISJ_CASES : thm -> thm -> thm -> thm`

The theorem referred to by the first `NUMBER` must be a disjunction. The assumptions of the second and the third theorems must include the first and second disjunct of the first theorem, respectively.

1.	$\vdash \vee = \lambda b_1\, b_2.\, \forall b.\, (b_1 \supset b) \supset (b_2 \supset b) \supset b$	[Definition of $\vee$]
2.	$\vdash \$\vee t_1 = \$\vee t_1$	[REFL]
3.	$(\lambda b_1\, b_2.\, \forall b.\, (b_1 \supset b) \supset (b_2 \supset b) \supset b)t_1 =$ $\lambda b_2.\, \forall b.\, (t_1 \supset b) \supset (b_2 \supset b) \supset b$	[BETA_CONV]
4.	$\vdash \$\vee t_1 = (\lambda b_1\, b_2.\, \forall b.\, (b_1 \supset b) \supset (b_2 \supset b) \supset b)t_1$	[SUBST 1,2]
5.	$\vdash \$\vee t_1 = \lambda b_2.\, \forall b.\, (t_1 \supset b) \supset (b_2 \supset b) \supset b$	[SUBST 3,4]
6.	$\vdash t_1 \vee t_2 = t_1 \vee t_2$	[REFL]
7.	$(\lambda b_2.\, \forall b.\, (t_1 \supset b) \supset (b_2 \supset b) \supset b)t_2$ $= \forall b.\, (t_1 \supset b) \supset (t_2 \supset b) \supset b$	[BETA_CONV]
8.	$\vdash t_1 \vee t_2 = (\lambda b_2.\, \forall b.\, (t_1 \supset b) \supset (b_2 \supset b) \supset b)t_2$	[SUBST 5,6]
9.	$\vdash t_1 \vee t_2 = \forall b.\, (t_1 \supset b) \supset (t_2 \supset b) \supset b$	[SUBST 7,8]
10.	$\Gamma \vdash t_1 \vee t_2$	[Hypothesis]
11.	$\Gamma \vdash \forall b.\, (t_1 \supset b) \supset (t_2 \supset b) \supset b$	[SUBST 9,10]
12.	$\Gamma \vdash (t_1 \supset t) \supset (t_2 \supset t) \supset t$	[SPEC 11]
13.	$\Gamma_1,\ t_1 \vdash t$	[Hypothesis]
14.	$\Gamma_1 \vdash t_1 \supset t$	[DISCH 13]
15.	$\Gamma \cup \Gamma_1 \vdash (t_2 \supset t) \supset t$	[MP 12,14]
16.	$\Gamma_2,\ t_2 \vdash t$	[Hypothesis]
17.	$\Gamma_2 \vdash t_2 \supset t$	[DISCH 16]
18.	$\Gamma \cup \Gamma_1 \cup \Gamma_2 \vdash t$	[MP 15,17]

```
  fun chk_DisjCases(line, n1, n2, n3, thm) =
    let val thm1 = get_thm(line, n1)
        and thm2 = get_thm(line, n2) and thm3 = get_thm(line, n3)
        val (OR,[disj1,disj2]) = strip_comb (concl thm1)
    in
      (well_typed OR) andalso ((fst(dest_const OR)) = "\\/") andalso
      (HtermSet.member((hyp thm2), disj1)) andalso
      (HtermSet.member((hyp thm3), disj2)) andalso
      ((concl thm2) = (concl thm3)) andalso
      ((concl thm) = (concl thm2)) andalso
      (HtermSet.equal((hyp thm), (HtermSet.union((hyp thm1),
        (HtermSet.union((HtermSet.delete((hyp thm2),disj1)),
         (HtermSet.delete((hyp thm3),disj2))))))))
end
```

- **Implication from equality (left)**

  `EQIMPRULEL NUMBER`

$$\frac{\Gamma \vdash t_1 = t_2}{\Gamma \vdash t_2 \supset t_1}$$

`EQ_IMP_RULE : thm -> (thm # thm)`

The theorem referred to by `NUMBER` must be an equation, and both sides of it must be of type `:bool`. The resulting theorem of this justification is the second theorem returned by the function `EQ_IMP_RULE`. It is an implication whose antecedent is the right-hand side of the hypothesis and whose conclusion is the left-hand side of the hypothesis.

1.	$\Gamma \vdash t_1 = t_2$	[Hypothesis]
2.	$t_2 \vdash t_2$	[ASSUME]
3.	$\Gamma \vdash t_2 = t_1$	[SYM 1]
4.	$\Gamma,\ t_2 \vdash t_1$	[SUBST 3,2]
5.	$\Gamma \vdash t_2 \supset t_1$	[DISCH 4]

```
fun chk_EqImpRuleL(line, n, thm) =
  let val thm1 = get_thm(line, n)
    val (left,right) = dest_eq (concl thm1)
  in
    (is_bool_ty left) andalso (is_bool_ty right) andalso
    ((right,left) = dest_imp(concl thm)) andalso
    (HtermSet.equal((hyp thm), (hyp thm1)))
  end
```

- **Implication from equality (right)**

EQIMPRULER NUMBER

$$\frac{\Gamma \vdash t_1 = t_2}{\Gamma \vdash t_1 \supset t_2}$$

`EQ_IMP_RULE : thm -> (thm # thm)`

The theorem referred to by `NUMBER` must be an equation, and both sides of it must be of type `:bool`. The resulting theorem of this justification is the first theorem returned by the function `EQ_IMP_RULE`. It is an implication whose antecedent is the left-hand side of the hypothesis and whose conclusion is the right-hand side of the hypothesis .

1.	$\Gamma \vdash t_1 = t_2$	[Hypothesis]
2.	$t_1 \vdash t_1$	[`ASSUME`]
3.	$\Gamma,\ t_1 \vdash t_2$	[`SUBST` 1,2]
4.	$\Gamma \vdash t_1 \supset t_2$	[`DISCH` 3]

```
fun chk_EqImpRuleR(line, n, thm) =
  let val thm1 = get_thm(line, n)
    val (left,right) = dest_eq (concl thm1)
  in
    (is_bool_ty left) andalso (is_bool_ty right) andalso
    ((left,right) = dest_imp(concl thm)) andalso
    (HtermSet.equal((hyp thm), (hyp thm1)))
  end
```

- **Modus Ponens for equality**

EQMP NUMBER NUMBER

$$\frac{\Gamma_1 \vdash t_1 = t_2 \quad \Gamma_2 \vdash t_1}{\Gamma_1 \cup \Gamma_2 \vdash t_2}$$

`EQ_MP : thm -> thm -> thm`

The first theorem should be an equation. The second theorem should be identical to the left-hand side of the first. The resulting theorem is the right-hand side of the first theorem.

1.	$\Gamma_1 \vdash t_1 = t_2$	[Hypothesis]
2.	$\Gamma_2 \vdash t_1$	[Hypothesis]
3.	$\Gamma_1 \cup \Gamma_2 \vdash t_2$	[`SUBST` 1,2]

```
fun chk_EqMp(line, n1, n2, thm) =
  let val thm1 = get_thm(line, n1) and thm2 = get_thm(line, n2)
    val (left,right) = dest_eq (concl thm1)
  in
    (left = (concl thm2)) andalso (right = (concl thm)) andalso
    (HtermSet.equal((hyp thm), (HtermSet.union((hyp thm1),(hyp thm2)))))
  end
```

- **Equality-with-T introduction**

```
EQTINTRO NUMBER
```

$$\frac{\Gamma \vdash t}{\Gamma \vdash t = \mathsf{T}}$$

`EQT_INTRO : thm -> thm` This inference introduces an equality. The left-hand side of the conclusion must be the same as the hypothesis. The right-hand side must be a single constant `T`.

1.	$\vdash \forall b_1\, b_2.\, (b_1 \supset b_2) \supset (b_2 \supset b_1) \supset (b_1 = b_2)$	[`IMP_ANTISYM_AX`]
2.	$\vdash \forall b_2.\, (t \supset b_2) \supset (b_2 \supset t) \supset (t = b_2)$	[`SPEC` 1]
3.	$\vdash (t \supset \mathsf{T}) \supset (\mathsf{T} \supset t) \supset (t = \mathsf{T})$	[`SPEC` 2]
4.	$\vdash \mathsf{T}$	[`TRUTH`]
5.	$\vdash t \supset \mathsf{T}$	[`DISCH` 4]
6.	$\vdash (\mathsf{T} \supset t) \supset (t = \mathsf{T})$	[`MP` 3,5]
7.	$\Gamma \vdash t$	[Hypothesis]
8.	$\Gamma \vdash \mathsf{T} \supset t$	[`DISCH` 7]
9.	$\Gamma \vdash t = \mathsf{T}$	[`MP` 6,8]

```
fun chk_EqTIntro(line, n, thm) =
  let val thm1 = get_thm(line, n)
    val (lhs,rhs) = dest_eq (concl thm)
    and T = mk_const("T", bool_ty)
  in
    (rhs = T) andalso (lhs = (concl thm1)) andalso
    (HtermSet.equal((hyp thm1), (hyp thm)))
  end
```

- **η-conversion**

```
ETACONV term
```

$$\frac{}{\vdash (\lambda x'.\, t\ x') = t}$$

`ETA_CONV : term -> thm`

The variable x' does not occur free in t. The input `term` is the same as the left-hand side of the resulting theorem.

1.	$\vdash \forall f.\, (\lambda x.\, f\ x) = f$	[`ETA_AX`]
2.	$\vdash (\lambda x.\, t\ x) = t$	[`SPEC` 1]
3.	$\vdash (\lambda x'.\, t\ x') = (\lambda x.\, t\ x)$	[α-conversion]
4.	$\vdash (\lambda x'.\, t\ x') = t$	[`TRANS` 3,2]

```
fun chk_EtaConv(term, thm) =
  let val (left,right) = dest_eq (concl thm)
  in
    (left = term) andalso (null (HtermSet.listItems(hyp thm))) andalso
    (right = (fst o dest_comb o snd o dest_abs)left)
  end
```

- **∃-introduction**

EXISTS term term NUMBER

$$\frac{\Gamma \vdash t[t_2/x]}{\Gamma \vdash \exists x.\, t[x]}$$

`EXISTS :(term # term) -> thm -> thm`

The first term is an existentially quantified term which matches exactly the conclusion of the resulting theorem, i.e., $\exists x.\, t[x]$. The second term t_2 must be of the same type as the bound variable x. When t_2 is substituted into t for the bound variable x, it results in a theorem which is the same as the input theorem referred to by `NUMBER`.

1.	$\vdash (\lambda x.\, t_1[x])t_2 = t_1[t_2]$	[`BETA_CONV`]
2.	$\vdash t_1[t_2] = (\lambda x.\, t_1[x])t_2$	[`SYM` 1]
3.	$\Gamma \vdash t_1[t_2]$	[Hypothesis]
4.	$\Gamma \vdash (\lambda x.\, t_1[x])t_2$	[`EQ_MP` 2,3]
5.	$\Gamma \vdash (\lambda x.\, t_1[x])(\varepsilon(\lambda x.\, t_1[x]))$	[`SELECT_INTRO` 4]
6.	$\vdash \exists = \forall P.\, P(\varepsilon\ P)$	[Definition of $\exists$]
7.	$\vdash \exists(\lambda x.\, t_1[x]) = (\lambda P.\, P(\varepsilon\ P))(\lambda x.\, t_1[x])$	[`AP_THM` 6]
8.	$\vdash (\lambda P.\, P(\varepsilon\ P))(\lambda x.\, t_1[x]) = (\lambda x.\, t_1[x])(\varepsilon(\lambda x.\, t_1[x]))$	[`BETA_CONV`]
9.	$\vdash \exists(\lambda x.\, t_1[x]) = (\lambda x.\, t_1[x])(\varepsilon(\lambda x.\, t_1[x]))$	[`TRANS` 7,8]
10.	$\vdash (\lambda x.\, t_1[x])(\varepsilon(\lambda x.\, t_1[x])) = \exists(\lambda x.\, t_1[x])$	[`SYM` 9]
11.	$\Gamma \vdash \exists(\lambda x.\, t_1[x])$	[`EQ_MP` 10,5]

```
fun chk_Exists(line, term1, term2, n, thm) =
  let val thm1 = get_thm(line, n)
    val (x,body) = dest_exists (concl thm)
  in
    (term1 = (concl thm)) andalso
    (term_subst_chk [((x,term2),x)] body (concl thm1) body) andalso
    (HtermSet.equal((hyp thm), (hyp thm1)))
  end
```

- **Extensionality**

EXT NUMBER

$$\frac{\Gamma \vdash \forall x.\, t_1\ x = t_2\ x}{\Gamma \vdash t_1 = t_2}$$

`EXT : thm -> thm`

The variable x' in the proof below is a new variable which does not occur free anywhere in the input theorem. Both the hypothesis and the conclusion must be equality. Both

[1] with appropriate type instantiation.

sides of the hypothesis must be function applications, and their operands must be the same variable. Both sides of the conclusion must be the same as the function on the corresponding side of the hypothesis.

1.	$\Gamma \vdash \forall x.\, t_1\ x = t_2\ x$	[Hypothesis]
2.	$\Gamma \vdash t_1\ x' = t_2\ x'$	[SPEC 1]
3.	$\Gamma \vdash (\lambda x'.\, t_1\ x') = (\lambda x'.\, t_2\ x')$	[ABS 2]
4.	$\vdash (\lambda x'.\, t_1\ x') = t_1$	[ETA_CONV]
5.	$\vdash t_1 = (\lambda x'.\, t_1\ x')$	[SYM 4]
6.	$\Gamma \vdash t_1 = (\lambda x'.\, t_2\ x')$	[TRANS 5,3]
7.	$\vdash (\lambda x'.\, t_2\ x') = t_2$	[ETA_CONV]
8.	$\Gamma \vdash t_1 = t_2$	[TRANS 6,7]

```
fun chk_Ext(line, n, thm) =
  let val (left,right) = dest_eq(concl thm)
    val thm1 = get_thm(line, n)
    val (x, con) = dest_forall (concl thm1)
    val (left',right') = dest_eq con
  in
    (mk_comb(left,x) = left') andalso
    (mk_comb(right,x) = right') andalso
    (HtermSet.equal((hyp thm), (hyp thm1))) andalso
    (not(mem x (freesl (HtermSet.listItems(hyp thm))))) andalso
    (not (free_in x left)) andalso (not(free_in x right))
  end
```

- **Generalisation($\forall$-introduction)**

```
GEN term NUMBER
```

$$\frac{\Gamma \vdash t}{\Gamma \vdash \forall x.\, t}$$

`GEN : term -> thm -> thm`

The input term x is a variable which does not occur free in the assumption Γ.

1.	$\Gamma \vdash t$	[Hypothesis]
2.	$\Gamma \vdash t = \mathsf{T}$	[EQT_INTRO 1]
3.	$\Gamma \vdash (\lambda x.\, t) = (\lambda x.\, \mathsf{T})$	[ABS 2]
4.	$\vdash \forall(\lambda x.\, t) = \forall(\lambda x.\, t)$	[REFL]
5.	$\vdash \forall = (\lambda P.\, P = (\lambda x.\, \mathsf{T}))$	[Definition of $\forall$[2]]
6.	$\vdash \forall(\lambda x.\, t) = (\lambda P.\, P = (\lambda x.\, \mathsf{T}))(\lambda x.\, t)$	[SUBST 5,4]
7.	$\vdash (\lambda P.\, P = (\lambda x.\, \mathsf{T}))(\lambda x.\, t) = ((\lambda x.\, t) = (\lambda x.\, \mathsf{T}))$	[BETA_CONV]
8.	$\vdash \forall(\lambda x.\, t) = ((\lambda x.\, t) = (\lambda x.\, \mathsf{T}))$	[TRANS 6,7]
9.	$\vdash ((\lambda x.\, t) = (\lambda x.\, \mathsf{T})) = \forall(\lambda x.\, \mathsf{T})$	[SYM 8]
10.	$\Gamma \vdash \forall(\lambda x.\, t)$	[EQ_MP 9,3]

```
fun chk_Gen(line, term, n, thm) =
  let val thm1 = get_thm(line, n)
    val (x,body) = dest_forall(concl thm)
```

[2] with appropriate type instantiation.

```
    val fs = freesl (HtermSet.listItems(hyp thm))
  in
    (body = concl thm1) andalso (not (mem x fs)) andalso
    (HtermSet.equal((hyp thm), (hyp thm1)))
  end
```

- **Deducing equality from implications**

IMPANTISYMRULE NUMBER NUMBER

$$\frac{\Gamma_1 \vdash t_1 \supset t_2 \quad \Gamma_2 \vdash t_2 \supset t_1}{\Gamma_1 \cup \Gamma_2 \vdash t_1 = t_2}$$

`IMP_ANTISYM_RULE : thm -> thm -> thm`

The two hypotheses should be implications. The antecedent of first must be the same as the conclusion of the second, and vice versa. The conclusion should be an equality whose left-hand side must be the same as the antecedent of the first hypothesis and whose right-hand side must be the same as the conclusion of the first hypothesis.

1.	$\vdash \forall b_1\, b_2.\, (b_1 \supset b_2) \supset (b_2 \supset b_1) \supset (b_1 = b_2)$	[IMP_ANTISYM_AX]
2.	$\Gamma_1 \vdash t_1 \supset t_2$	[Hypothesis]
3.	$\Gamma_2 \vdash t_2 \supset t_1$	[Hypothesis]
4.	$\vdash \forall b_2.\, (t_1 \supset b_2) \supset (b_2 \supset t_1) \supset (t_1 = b_2)$	[SPEC]
5.	$\vdash (t_1 \supset t_2) \supset (t_2 \supset t_1) \supset (t_1 = t_2)$	[SPEC]
6.	$\Gamma_1 \vdash (t_2 \supset t_1) \supset (t_1 = t_2)$	[MP 5,2]
7.	$\Gamma_1 \cup \Gamma_2 \vdash t_1 = t_2$	[MP 6,3]

```
fun chk_ImpAntisymRule(line, n1, n2, thm) =
  let val thm1 = get_thm(line, n1) and thm2 = get_thm(line, n2)
    val (left,right) = dest_eq (concl thm)
  in
    ((left,right) = dest_imp(concl thm1)) andalso
    ((right,left) = dest_imp(concl thm2)) andalso
    (HtermSet.equal((hyp thm), (HtermSet.union((hyp thm1),(hyp thm2)))))
  end
```

- **Transitivity of implications**

IMPTRANS NUMBER NUMBER

$$\frac{\Gamma_1 \vdash t_1 \supset t_2 \quad \Gamma_2 \vdash t_2 \supset t_3}{\Gamma_1 \cup \Gamma_2 \vdash t_1 \supset t_3}$$

`IMP_TRANS : thm -> thm -> thm`

Both theorems referred to by the numbers must be implications. The conclusion of the first theorem must be the same as the antecedent of the second theorem.

1.	$\Gamma_1 \vdash t_1 \supset t_2$	[Hypothesis]
2.	$\Gamma_2 \vdash t_2 \supset t_3$	[Hypothesis]
3.	$t_1 \vdash t_1$	[ASSUME]
4.	$\Gamma_1 \cup \{t_1\} \vdash t_2$	[MP 1,3]
5.	$\Gamma_1 \cup \Gamma_2 \cup \{t_1\} \vdash t_3$	[MP 2,4]
6.	$\Gamma_1 \cup \Gamma_2 \vdash t_1 \supset t_3$	[DISCH 5]

```
fun chk_ImpTrans(line, n1, n2, thm) =
  let val thm1 = get_thm(line, n1) and thm2 = get_thm(line, n2)
    val (left',right') = dest_imp(concl thm1)
    and (left'',right'') = dest_imp(concl thm2)
    val (left,right) = dest_imp(concl thm)
  in
    (left = left') andalso (right' = left'') andalso
    (right = right'') andalso
    (HtermSet.equal((hyp thm), (HtermSet.union((hyp thm1),(hyp thm2)))))
  end
```

- **Instantiation of free variables**

INST term_term_list NUMBER

  $$\frac{\Gamma \vdash t}{\Gamma \vdash t[t_1, \ldots, t_n/x_1, \ldots, x_n]}$$

  `INST :(term # term) list -> thm -> thm`

  The `term_term_list` is a list of pairs. Their second fields are variables which do not occur free in the assumption Γ. Their first fields are terms having the same type as the corresponding variables. They are substituted for the variables in the theorem referred to by the `NUMBER`.

  1. $\Gamma \vdash t$ [Hypothesis]
  2. $\Gamma \vdash \forall x_1 \ldots x_n . t[x_1, \ldots, x_n]$ [GENL[3] 1]
  3. $\Gamma \vdash t[t_1, \ldots, t_n/x_1, \ldots, x_n]$ [SPECL[4] 2]

```
fun chk_Inst(line, ttl, n, thm) =
  let val thm1 = get_thm(line, n)
    val vars = map snd ttl
    val fs = freesl (HtermSet.listItems(hyp thm1))
    val con1 = concl thm1
    val (tl1,tl2) = ListUtil.unzip ttl
    val ttl' = ListUtil.zip(tl2,tl1)
  in
    (every (fn x => not (mem x fs)) vars) andalso
    (term_subst_chk
     (ListUtil.zip(ttl',vars)) con1 (concl thm) con1) andalso
    (HtermSet.equal((hyp thm), (hyp thm1)))
  end
```

- **Abstraction introduction on equality**

MKABS NUMBER

  $$\frac{\Gamma \vdash \forall x. t_1 = t_2}{\Gamma \vdash (\lambda x. t_1) = (\lambda x. t_2)}$$

  `MK_ABS : thm -> thm`

  The theorem referred to by the `NUMBER` must be an equality inside a single universal quantification.

[3] `GENL` is an iterative version of `GEN` that applies `GEN` repeatedly to a list of variables.
[4] `SPECL` is an iterative version of `SPEC` that applies `SPEC` repeatedly to a list of terms.

1.	$\Gamma \vdash \forall x.\, t_1 = t_2$	[Hypothesis]
2.	$\vdash (\lambda x.\, t_1)x' = t_1[x'/x]$	[`BETA_CONV`]
3.	$\Gamma \vdash t_1[x'/x] = t_2[x'/x]$	[`SPEC` 1]
4.	$\vdash (\lambda x.\, t_2)x' = t_2[x'/x]$	[`BETA_CONV`]
5.	$\vdash t_2[x'/x] = (\lambda x.\, t_2)x'$	[`SYM` 4]
6.	$\Gamma \vdash (\lambda x.\, t_1)x' = t_2[x'/x]$	[`TRANS` 2,3]
7.	$\Gamma \vdash (\lambda x.\, t_1)x' = (\lambda x.\, t_2)x'$	[`TRANS` 5,6]
8.	$\Gamma \vdash \forall x'.\, (\lambda x.\, t_1)x' = (\lambda x.\, t_2)x'$	[`GEN` 7]
9.	$\Gamma \vdash (\lambda x.\, t_1) = (\lambda x.\, t_2)$	[`EXT` 8]

```
fun chk_MkAbs(line, n, thm) =
  let val thm1 = get_thm(line, n)
    val (x,body) = dest_forall(concl thm1)
    val (left,right) = dest_eq (concl thm)
  in
      (left = mk_abs(x,lhs body)) andalso
      (right = mk_abs(x, rhs body)) andalso
      (HtermSet.equal((hyp thm), (hyp thm1)))
  end
```

- **Equality of combinations**

  `MKCOMB NUMBER NUMBER`

$$\frac{\Gamma_1 \vdash f = g \quad \Gamma_2 \vdash x = y}{\Gamma_1 \cup \Gamma_2 \vdash f\,x = g\,y}$$

`MK_COMB : (thm # thm) -> thm`

Both theorems referred to by the `NUMBER`s must be equations. Both sides of the first theorem are functions whose domains are the type of x in the second theorem.

1.	$\Gamma_1 \vdash f = g$	[Hypothesis]
2.	$\Gamma_2 \vdash x = y$	[Hypothesis]
3.	$\vdash f\,x = f\,x$	[`REFL`]
4.	$\Gamma_1 \vdash f\,x = g\,x$	[`SUBST` 1,3]
5.	$\Gamma_1 \vdash f\,x = g\,y$	[`SUBST` 2,4]

```
fun chk_MkComb(line, n1, n2, thm) =
  let val thm1 = get_thm(line, n1) and thm2 = get_thm(line, n2)
    val (f,g) = dest_eq(concl thm1) and (x,y) = dest_eq(concl thm2)
    val (left,right) = dest_eq(concl thm)
  in
    (left = mk_comb(f,x)) andalso (right = mk_comb(g,y)) andalso
    (HtermSet.equal((hyp thm), (HtermSet.union((hyp thm1),(hyp thm2)))))
  end
```

- **Converting $\forall$ to $\exists$**

  `MKEXISTS NUMBER`

$$\frac{\Gamma \vdash \forall x.\, t_1 = t_2}{\Gamma \vdash (\exists x.\, t_1) = (\exists x.\, t_2)}$$

`MK_EXISTS : thm -> thm`

The theorem referred to by the NUMBER must be an equality inside a single universal quantification. The terms t_1 and t_2 must be of type :bool.

1.	$\Gamma \vdash \forall x.\, t_1[x] = t_2[x]$	[Hypothesis]
2.	$\Gamma \vdash t_1[x'/x] = t_2[x'/x]$	[SPEC 1]
3a.	$\Gamma \vdash t_1[x'/x] \supset t_2[x'/x]$	[EQ_IMP_RULE(right) 2]
3b.	$\Gamma \vdash t_2[x'/x] \supset t_1[x'/x]$	[EQ_IMP_RULE(left) 2]
4.	$t_1[x'/x] \vdash t_1[x'/x]$	[ASSUME]
5.	$\Gamma \cup \{t_1[x'/x]\} \vdash t_2[x'/x]$	[MP 3a,4]
6.	$\Gamma \cup \{t_1[x'/x]\} \vdash \exists x.\, t_2[x]$	[EXISTS 5]
7.	$\exists x.\, t_1[x] \vdash \exists x.\, t_1[x]$	[ASSUME]
8.	$\Gamma \cup \{\exists x.\, t_1[x]\} \vdash \exists x.\, t_2[x]$	[CHOOSE 7,6]
9.	$\Gamma \vdash \exists x.\, t_1[x] \supset \exists x.\, t_2[x]$	[DISCH 8]
9.	$t_2[x'/x] \vdash t_2[x'/x]$	[ASSUME]
10.	$\Gamma \cup \{t_2[x'/x]\} \vdash t_1[x'/x]$	[MP 3b,9]
11.	$\Gamma \cup \{t_2[x'/x]\} \vdash \exists x.\, t_1[x]$	[EXISTS 10]
12.	$\exists x.\, t_2[x] \vdash \exists x.\, t_2[x]$	[ASSUME]
13.	$\Gamma \cup \{\exists x.\, t_2[x]\} \vdash \exists x.\, t_1[x]$	[CHOOSE 12,11]
14.	$\Gamma \vdash \exists x.\, t_2[x] \supset \exists x.\, t_1[x]$	[DISCH 13]
15.	$\Gamma \vdash \exists x.\, t_1[x] = \exists x.\, t_2[x]$	[IMP_ANTISYM_RULE 9,14]

```
fun chk_MkExists(line, n, thm) =
  let val thm1 = get_thm(line, n)
    val (x,body) = dest_forall (concl thm1)
    val (left',right') = dest_eq body
    val (left,right) = dest_eq (concl thm)
  in
    ((x,left') = dest_exists left) andalso
    ((x,right') = dest_exists right) andalso
    (HtermSet.equal((hyp thm), (hyp thm1)))
  end
```

- **¬-elimination**

NOTELIM NUMBER

$$\frac{\Gamma \vdash \neg t}{\Gamma \vdash t \supset \mathsf{F}}$$

NOT_ELIM : thm -> thm

The theorem referred to by NUMBER must be a negation.

1.	$\vdash \neg = \lambda b.\, b \supset \mathsf{F}$	[Definition of $\neg$]
2.	$\Gamma \vdash \neg t$	[Hypothesis]
3.	$\Gamma \vdash (\lambda b.\, b \supset \mathsf{F})t$	[SUBST 1,2]
4.	$\vdash (\lambda b.\, b \supset \mathsf{F})t = t \supset \mathsf{F}$	[BETA_CONV]
5.	$\Gamma \vdash t \supset \mathsf{F}$	[SUBST 4,3]

```
fun chk_NotElim(line, n, thm) =
  let val thm1 = get_thm(line, n)
    val t = dest_not (concl thm1)
    val (t',f) = dest_imp (concl thm)
```

```
  in
    (t = t') andalso (well_typed f) andalso
    (fst(dest_const f) = "F") andalso
    (HtermSet.equal((hyp thm), (hyp thm1)))
  end
```

- **¬-introduction**

  `NOTINTRO NUMBER`

$$\frac{\Gamma \vdash t \supset \mathsf{F}}{\Gamma \vdash \neg t}$$

`NOT_INTRO : thm -> thm`

The theorem referred to by `NUMBER` must be an implication whose conclusion is the constant F.

1.	$\vdash \neg = \lambda b.\, b \supset \mathsf{F}$	[Definition of ¬]
2.	$\Gamma \vdash t \supset \mathsf{F}$	[Hypothesis]
3.	$\vdash \neg t = \neg t$	[REFL]
4.	$\vdash \neg t = (\lambda b.\, b \supset \mathsf{F})t$	[SUBST 1,3]
5.	$\vdash (\lambda b.\, b \supset \mathsf{F})t = t \supset \mathsf{F}$	[BETA_CONV]
6.	$\vdash \neg t = t \supset \mathsf{F}$	[SUBST 5,4]
7.	$\vdash t \supset \mathsf{F} = \neg t$	[SUBST 6,3]
8.	$\Gamma \vdash \neg t$	[SUBST 7,2]

```
fun chk_NotIntro(line, n, thm) =
  let val thm1 = get_thm(line, n)
    val t = dest_not (concl thm)
    val (t',f) = dest_imp (concl thm1)
  in
    (t = t') andalso (well_typed f) andalso
    (fst(dest_const f) = "F") andalso
    (HtermSet.equal((hyp thm), (hyp thm1)))
  end
```

- **Specialisation (∀-elimination)**

  `SPEC term NUMBER`

$$\frac{\Gamma \vdash \forall x.\, t}{\Gamma \vdash t[t'/x]}$$

`SPEC : term -> thm -> thm`

The theorem referred to by the `NUMBER` must be universally quantified. The input term must have the same type as the bound variable x of the theorem.

1.	$\vdash \forall = (\lambda P.\, P = (\lambda x.\, \mathsf{T}))$	[Definition of $\forall$[5]]
2.	$\Gamma \vdash \forall(\lambda x.\, t)$	[Hypothesis]
3.	$\Gamma \vdash (\lambda P.\, P = (\lambda x.\, \mathsf{T}))(\lambda x.\, t)$	[SUBST 1,2]
4.	$\vdash (\lambda P.\, P = (\lambda x.\, \mathsf{T}))(\lambda x.\, t) = ((\lambda x.\, t) = (\lambda x.\, \mathsf{T}))$	[BETA_CONV]
5.	$\Gamma \vdash (\lambda x.\, t) = (\lambda x.\, \mathsf{T})$	[SUBST 4,3]
6.	$\vdash (\lambda x.\, t)\ t' = (\lambda x.\, t)\ t'$	[REFL]
7.	$\Gamma \vdash (\lambda x.\, t)\ t' = (\lambda x.\, \mathsf{T})\ t'$	[SUBST 5,6]
8.	$\vdash (\lambda x.\, t)\ t' = t[t'/x]$	[BETA_CONV]
9.	$\vdash (\lambda x.\, t)\ t' = (\lambda x.\, t)\ t'$	[REFL]
10.	$\vdash t[t'/x] = (\lambda x.\, t)\ t'$	[SUBST 8,9]
11.	$\Gamma \vdash t[t'/x] = (\lambda x.\, \mathsf{T})\ t'$	[SUBST 10,7]
12.	$\vdash (\lambda x.\, \mathsf{T})\ t' = \mathsf{T}$	[BETA_CONV]
13.	$\Gamma \vdash t[t'/x] = \mathsf{T}$	[SUBST 12,11]
14.	$\vdash t[t'/x] = t[t'/x]$	[REFL]
15.	$\Gamma \vdash \mathsf{T} = t[t'/x]$	[SUBST 13,14]
16.	$\vdash \mathsf{T}$	[Theorem TRUTH]
17.	$\Gamma \vdash t[t'/x]$	[SUBST 15,16]

```
fun chk_Spec(line, term, n, thm) =
  let val thm1 = get_thm(line, n)
    val (x,body) = dest_forall (concl thm1)
  in
   if (term_subst_chk [((x,term),x)] body (concl thm) body)
   then if (HtermSet.equal((hyp thm), (hyp thm1)))
        then true
        else raise (Check_ERR{function="chk_Spec",
                              message="Assumption set not equal"})
   else  raise (Check_ERR{function="chk_Spec",
                              message="Substitution check failed"})
  end
```

- **Substitution (for all instances)**

SUBS NUMBER_list NUMBER

$$\frac{\Gamma_1 \vdash x_1 = t_1 \ldots \Gamma_n \vdash x_n = t_n \quad \Gamma \vdash t}{\Gamma_1 \cup \ldots \cup \Gamma_n \cup \Gamma \vdash t[t_1, \ldots, t_n/x_1, \ldots x_n]}$$

`SUBS : thm list -> thm -> thm`

This is a generalised version of the primitive rule SUBST. It replaces all occurrences of the variables x_i in t by the corresponding term t_i.

Since the term substitution checking function **term_subst_chk** requires a template to indicate where the substitutions occur, we build one using the function **mk_subs_templ**.

```
fun chk_Subs(line, nl, n, thm) =
  let val thms = map (fn n' => get_thm(line, n')) nl
      and thm1 = get_thm(line, n)
      val con1 = concl thm1
      val vtl = map (dest_eq o concl) thms
      val sub_list =
        ListUtil.zip (vtl,(map (mk_gen_var o type_of o (#1)) vtl))
```

[5]with appropriate type instantiation.

```
    val templ = mk_subs_templ sub_list con1
fun out_ppstream outstrm n =
  System.PrettyPrint.mk_ppstream{linewidth = n,
              flush = fn () => flush_out outstrm,
              consumer = outputc outstrm}
  val ppstrm = out_ppstream std_out 78
in
  if ((!debug) > 1) then
    (Hterm.pp_qterm ppstrm templ;
     System.PrettyPrint.flush_ppstream ppstrm)
  else ();
  (term_subst_chk sub_list templ (concl thm) con1) andalso
  (HtermSet.equal((hyp thm),
   (HtermSet.union ((hyp thm1),
                    (fold HtermSet.union
                     (map hyp thms) HtermSet.empty)))))
end
```

- **Substitution (for some instances)**

SUBSOCCS NUMBER_list_NUMBER_list NUMBER

$$\frac{\Gamma_1 \vdash x_1 = t_1 \ldots \Gamma_n \vdash x_n = t_n \quad \Gamma \vdash t}{\Gamma_1 \cup \ldots \cup \Gamma_n \cup \Gamma \vdash t[t_1, \ldots, t_n/x_1, \ldots x_n]}$$

`SUBS_OCCS : ((num)list # thm)list -> thm -> thm`

This is another version of the primitive rule `SUBST`. It is more selective than `SUBS`. The first argument is a list of pairs whose general form is

$$([k_{i1}; \ldots; k_{im}], \Gamma_i \vdash x_i = t_i)$$

This inference replaces only the occurrences of x_i in t specified by the numbers in the list $[k_{i1}; \ldots; k_{1m}]$. The occurrences are numbered from left to right starting from 1. In addition to the similar checks to `SUBS`, checking of whether only the specific occurrences are replaced needs to be carried out. This is done using a template which is created by the function `mk_suboccs_template`. This function replaces the occurrences of x_i in t which are to be substituted by a new variable, and it replaces the x_i in the substitution list with the same variable. Then it uses the standard term substitution checking function `term_subst_chk` to check the results.

The local function `mk_vlist` creates a list of new variables for the x_is. The first argument is a triple $(nlnl, thms, fs)$ where $nlnl$ is the `NUMBER_list_NUMBER_list` argument to this inference rule acting as a counter, $thms$ is a list of equational terms obtained from the substitution theorems $\Gamma_i \vdash x_i = t_i$, and fs is the list of free variables occurring in all theorems. Its second argument is a list of triples whose first fields are the two side of the substitution theorem, whose second fields are the new variables and whose third fields are the occurrence lists. When reaching the end of the list $nlnl$, it returns its second argument.

Because the function `mk_subs_occu_templ` which creates the template requires the occurrence list in ascending order and the function `get_thm` also requires the line number in ascending order, the list $nlnl$ is sorted first. The substitution theorems are fetched and bound to $thms$. $allhyps$ is bound to the assumptions of all input theorems. fvs are the free variables of all the assumptions and the conclusion of $thm1$. The template is bound to $templ$.

```
fun chk_SubsOccs(line, nlnl, n, thm) =
  let
    fun mk_vlist ([],[], fs) l = l
        | mk_vlist (((nl,n)::nlnl), thm::thms, fs) l =
          let val(x,tm) = dest_eq thm
            val v = variant fs x
          in
            mk_vlist (nlnl,thms,(v :: fs)) (((tm,x),v,nl)::l)
          end
    fun mk_subsoccs_template (((tm,x),v,nl),tm') =
      #1(mk_subs_occu_templ tm' x v 0 nl)
    val nlnl' =
         map (fn (nl,n) => ((ListMergeSort.sort (op>) nl),n))
          (ListMergeSort.sort(fn ((nl1,n1),(nl2,n2)) => n1 > n2)nlnl)
    val thms = map (fn ((l:int list),n') => get_thm(line, n')) nlnl'
    and thm1 = get_thm(line, n)
    val con1 = concl thm1
    val allhyps = HtermSet.union ((hyp thm1),
                  (fold HtermSet.union (map hyp thms) HtermSet.empty))
    val fvs = (frees con1) @ (freesl (HtermSet.listItems allhyps))
    val ol = mk_vlist(nlnl',(map concl thms), fvs) []
    val sublst = map (fn ((tm,x),v,nl) => ((x,tm),v)) ol
    val templ = fold mk_subsoccs_template ol con1
    fun out_ppstream outstrm n =
      System.PrettyPrint.mk_ppstream{linewidth = n,
                                     flush = fn () => flush_out outstrm,
                                     consumer = outputc outstrm}
    val ppstrm = out_ppstream std_out 78
  in
    if ((!debug) > 1) then
      (Hterm.pp_qterm ppstrm templ;
       System.PrettyPrint.flush_ppstream ppstrm)
    else ();
    (term_subst_chk sublst templ (concl thm) con1) andalso
    (HtermSet.equal((hyp thm), allhyps))

  end
```

- **Substitution (conversion)**

SUBSTCONV NUMBER_term_list term term

$$\frac{\Gamma_1 \vdash t_1 = t_1' \quad \cdots \quad \Gamma_n \vdash t_n = t_n'}{\Gamma_1 \cup \cdots \cup \Gamma_n \vdash t[t_1, \ldots, t_n/x_1, \ldots, x_n] = t[t_1', \ldots, t_n'/x_1, \ldots, x_n]}$$

`SUBST_CONV : (thm # term)list -> term -> term -> thm`

This is a conversion performing substitution in a term similar to the primitive rule `SUBST`. The elements of the first argument to this conversion have the following form:

$$(\Gamma_i \vdash t_i = t_i', "x_i")$$

The first term argument $term_1 = t[x_1, \ldots, x_n]$, contains the variables x_i marking the places where substitution is required. The second term argument $term_2$ should match $term_1$ with occurrences of x_i replaced by the corresponding t_i which is the left-hand side of the corresponding theorem in the list. The term $term_2$ is the left-hand side of the resulting theorem. The right-hand side is obtained by replacing the occurrences of x_i in t by the corresponding t'_i. The term $term_1$ is used as the template.

```
fun chk_SubstConv(line, ntl, term1, term2, thm) =
  let
    val thms = map (fn (n,tm) => get_thm(line, n)) ntl
    val allhyps = fold HtermSet.union (map hyp thms) HtermSet.empty
    val sublst = ListUtil.zip((map (dest_eq o concl) thms),
                              (map (#2) ntl))
    val conc = concl thm
    val (left,right) = dest_eq conc
  in
    (term_subst_chk sublst term1 right left) andalso
    (left = term2) andalso (HtermSet.equal((hyp thm), allhyps))
  end
```

- **Symmetry of equality**

```
SYM NUMBER
```

$$\frac{\Gamma \vdash t_1 = t_2}{\Gamma \vdash t_2 = t_1}$$

`SYM : thm -> thm`

The theorem referred to by `NUMBER` must be an equation.

1.	$\Gamma \vdash t_1 = t_2$	[Hypothesis]
2.	$\vdash t_1 = t_1$	[`REFL`]
3.	$\Gamma \vdash t_2 = t_1$	[`SUBST` 1,2]

```
fun chk_Sym(line, n, thm) =
  let val thm1 = get_thm(line, n)
    val (left,right) = dest_eq (concl thm1)
  in
    ((right,left) = dest_eq (concl thm)) andalso
    (HtermSet.equal((hyp thm), (hyp thm1)))
  end
```

- **Transitivity of equality**

```
TRANS NUMBER NUMBER
```

$$\frac{\Gamma_1 \vdash t_1 = t_2 \quad \Gamma_2 \vdash t_2 = t_3}{\Gamma_1 \cup \Gamma_2 \vdash t_1 = t_3}$$

`TRANS : thm -> thm -> thm`

Both theorems referred to by the `NUMBER`s must be equations. The right-hand side of the first theorem must be the same as the left-hand side of the second.

1.	$\Gamma_1 \vdash t_1 = t_2$	[Hypothesis]
2.	$\Gamma_2 \vdash t_2 = t_3$	[Hypothesis]
3.	$\Gamma_1 \cup \Gamma_2 \vdash t_1 = t_3$	[`SUBST` 2,1]

```
fun chk_Trans(line, n1, n2, thm) =
  let val thm1 = get_thm(line, n1) and thm2 = get_thm(line, n2)
    val (left',right') = dest_eq (concl thm1)
    and (left'',right'') = dest_eq (concl thm2)
  in
    (right' = left'') andalso
    ((left', right'') = dest_eq (concl thm)) andalso
    (HtermSet.equal((hyp thm),(HtermSet.union ((hyp thm1),(hyp thm2)))))
  end
```

8.3.4 Checking proof lines The function `chk_pline` is the checking function dispatcher. It is called by the parser after recognising a line.

```
local
  open System.PrettyPrint Report
  fun out_ppstream outstrm n =
    mk_ppstream{linewidth = n,
                flush = fn () => flush_out outstrm,
                consumer = outputc outstrm};
  val check_message = ref "";
in
  fun chk_pline (thisline, just, thm) =
    let
      val n' = get_last_lineNo ()
    in
      if not((thisline >= n') orelse (n' = 0)) then
        raise (Check_ERR {function = "chk_pline",
                          message = "incorrect line number"})
      else (* line number OK *)
        let
          val line_OK =
            (case just of
               (Hypothesis) => chk_Hypothesis thm
             | (Abs (term, n)) => chk_Abs (thisline,term,n,thm)
             | (Assume term) => chk_Assume (term, thm)
             | (BetaConv term) => chk_BetaConv (term,thm)
             | (Disch (term,n)) => chk_Disch (thisline,term,n,thm)
             | (InstType (ttl,n)) => chk_InstType (thisline,ttl,n,thm)
             | (Mp (n1,n2)) => chk_Mp (thisline,n1,n2,thm)
             | (Refl term) => chk_Refl (term, thm)
             | (Subst (itmlist,term,n)) =>
                 chk_Subst(thisline,itmlist,term,n,thm)
             | (Axiom (s1,s2)) => chk_Axiom(s1,s2,thm)
             | (Definition (s1,s2)) => chk_Definition(s1,s2,thm)
             | (DefExistsRule term) => chk_DefExistsRule(term,thm)
             | (NewAxiom (s,term)) => chk_NewAxiom(s,term,thm)
             | (NewConstant (s,ty)) => chk_NewConstant(s,ty)
             | (NewType (n,s)) => chk_NewType(n,s)
             | (Numconv term) => chk_Numconv(term,thm)
             | (StoreDefinition (s,term)) => chk_StoreDefinition(s,term,thm)
             | (Theorem (s1, s2)) => chk_Theorem(s1,s2,thm)
```

```
      | (AddAssum (term,n)) => chk_AddAssum(thisline,term,n,thm)
      | (Alpha (term1,term2)) => chk_Alpha(term1,term2,thm)
      | (ApTerm(term,n)) => chk_ApTerm(thisline,term,n,thm)
      | (ApThm (n,term)) => chk_ApThm(thisline,n,term,thm)
      | (Ccontr (term,n)) => chk_Ccontr(thisline,term,n,thm)
      | (Choose (term,n1,n2)) => chk_Choose(thisline,term,n1,n2,thm)
      | (Conj (n1,n2)) => chk_Conj(thisline,n1,n2,thm)
      | (Conjunct1 n) => chk_Conjunct1(thisline,n,thm)
      | (Conjunct2 n) => chk_Conjunct2(thisline,n,thm)
      | (Contr (term,n)) => chk_Contr(thisline,term,n,thm)
      | (Disj1 (n,term)) => chk_Disj1(thisline,n,term,thm)
      | (Disj2 (term,n)) => chk_Disj2(thisline,term,n,thm)
      | (DisjCases(n1,n2,n3)) => chk_DisjCases(thisline,n1,n2,n3,thm)
      | (EqImpRuleL n) => chk_EqImpRuleL(thisline,n,thm)
      | (EqImpRuleR n) => chk_EqImpRuleR(thisline,n,thm)
      | (EqMp (n1,n2)) => chk_EqMp(thisline,n1,n2,thm)
      | (EqTIntro n) => chk_EqTIntro(thisline,n,thm)
      | (EtaConv term) => chk_EtaConv(term,thm)
      | (Exists ((tm1,tm2),n)) => chk_Exists(thisline,tm1,tm2,n,thm)
      | (Ext n) => chk_Ext(thisline,n,thm)
      | (Gen (term,n)) => chk_Gen(thisline,term,n,thm)
      | (ImpAntisymRule (n1,n2)) =>
          chk_ImpAntisymRule(thisline,n1,n2,thm)
      | (ImpTrans (n1,n2)) => chk_ImpTrans(thisline,n1,n2,thm)
      | (Inst (ttl,n)) => chk_Inst(thisline,ttl,n,thm)
      | (MkAbs n) => chk_MkAbs(thisline,n,thm)
      | (MkComb (n1,n2)) => chk_MkComb(thisline,n1,n2,thm)
      | (MkExists n) => chk_MkExists(thisline,n,thm)
      | (NotElim n) => chk_NotElim(thisline,n,thm)
      | (NotIntro n) => chk_NotIntro(thisline,n,thm)
      | (Spec (term,n)) => chk_Spec(thisline,term,n,thm)
      | (Subs (nl,n)) => chk_Subs(thisline,nl,n,thm)
      | (SubsOccs (nlnl,n)) => chk_SubsOccs(thisline,nlnl,n,thm)
      | (SubstConv(ntl,tm1,tm2)) =>
          chk_SubstConv(thisline,ntl,tm1,tm2,thm)
      | (Sym n) => chk_Sym(thisline,n,thm)
      | (Trans (n1,n2)) => chk_Trans(thisline,n1,n2,thm)
          ) handle  (CHK_ERR{message=m,origin_function=f,...}) =>
                   (check_message := (f ^":"^ m);
                    write_out(!check_message);
                    false)
                           | e =>
                   (check_message := "Unexpected error"; false)
in
  (if ((!debug) > 0)
     then (pp_pline (out_ppstream std_err 78)
            (mk_pline (thisline, just, thm)))
   else ();
   if line_OK then add_pline(thisline,just,thm)
   else(raise (Check_ERR{function = "chk_pline",
                         message =
```

```
                                ("Proof line "^(makestring thisline)^
                                 " fails:"^(!check_message))}))
          )
       end (* let *)
    end (* let *)
```

The function **chk_proof** is called when the end of a proof is found in Pass 2. The first argument *name* is the name of the proof, which is a string. The second is a list of goals. It writes the result of checking the proof to the log file, i.e., whether all goals have been proved, and so on.

```
  fun chk_proof (name, thmlst, nlst) =
    let
      fun splitpp [] (lp,lu) = (lp,lu)
        | splitpp ((Goal{Thm,Proved})::gs) (lp,lu) =
          if Proved then splitpp gs ((Thm::lp),lu)
            else splitpp gs (lp, (Thm::lu))
      in
        (output(std_out, ("Proof "^name^" has been checked\n"));
         if (null thmlst) then
           (write_line_opening Keyword.PROVED;
            write_list Hthm.pr_hthm [Proof.get_final_thm()];
            write_closing_line())
         else
           let
             val (thms1,thms2) = splitpp (goal_table()) ([],[])
             val out_ppstr = out_ppstream std_out 78
           in
             (if (null thms1) then ()
              else(if ((length thms1) > 1)
                   then output(std_out,"\nFollowing theorems are proved:\n")
                   else output(std_out,"\nFollowing theorem is proved:\n");
                   map (fn thm => pp_hthm out_ppstr thm) thms1;
                   System.PrettyPrint.flush_ppstream out_ppstr;
                   write_line_opening Keyword.PROVED;
                   write_list Hthm.pr_hthm thms1;
                   write_closing_line()
                   );
              if (null thms2) then ()
              else(if ((length thms2) > 1)
                   then output(std_out,"\nFollowing goals are NOT proved:\n")

                   else output(std_out,"\nFollowing goal is not proved:\n");
                   map (fn thm => pp_hthm out_ppstr thm) thms2;
                   System.PrettyPrint.flush_ppstream out_ppstr;
                   write_line_opening Keyword.UNSOLVED;
                   write_list Hthm.pr_hthm thms1;
                   write_closing_line());
              output(std_out, "\nUsing the following hypotheses:");
              Proof.pp_proof_thm out_ppstr)
           end;
         clear_proof ())
```

```
        end

  end(* local *)

   end; (* functor *)
```

9 Proof objects

The `proof` module models HOL proofs. A proof is a sequence of inferences. Each element of this sequence is called a *proof line* which represents a single inference step. It has a line number, a justification indicating which inference rule is applied, and a theorem derived by this inference.

The interface of the `proof` module is specified by the signature `Proof_sig`. The module is implemented by the functor `ProofFUN`. It requires the modules of type, term and theorem.

Its main task is to maintain several internal data structures which contain information about the proof. They are used to communicate between the two passes. The functions available to other modules can be roughly divided into two groups: those used in Pass 1 and those used in Pass 2.

9.1 The specification

The signature `Proof_sig` specifies the interface of the proof module. A proof is represented by the type `proof`. Each proof is a triple `(name, goals, lines)`. The `name` of a proof is a string. The `goals` are conjectures to be proved in the proof, and they are represented by the type `Hthm list`. The field `lines` is a sequence of proof lines represented by a list whose elements are of type `pline`.

Each `pline` is again a triple `(int * justification * hthm)`. The first field is the line number, the second is the justification, and the third is the derived theorem. The justification is represented by the type `justification`. The representations of these two types, `proof` and `pline` are private to this module.

```
signature Proof_sig =
  sig
    structure Hthm: Hthm_sig
    structure Hterm: Hterm_sig
    structure Htype: Htype_sig

    type pline
    type proof
```

The type `justification` represents the justifications allowed in the proof lines. There are 53 different justifications which can be divided into three groups: the primitive rules, derived rules and miscellaneous function such as using a theorem stored in a theory. They are described in detail in the checking module in Chapter 8.

```
    datatype justification =
      Hypothesis
      | Assume of Hterm.hterm
      | Refl of Hterm.hterm
      | Subst of (int*Hterm.hterm)list * Hterm.hterm * int
      | BetaConv of Hterm.hterm
      | Abs of Hterm.hterm * int
      | InstType of (Htype.htype * Htype.htype)list * int
      | Disch of Hterm.hterm * int
```

```
    | Mp of int * int
    | Axiom of (string * string)
    | Definition of (string * string)
    | DefExistsRule of (Hterm.hterm)
    | NewAxiom of (string *Hterm.hterm)
    | NewConstant of (string * Htype.htype)
    | NewType of (int * string)
    | Numconv of Hterm.hterm
    | StoreDefinition of (string * Hterm.hterm)
    | Theorem of (string * string)
    | AddAssum of  Hterm.hterm * int
    | Alpha of Hterm.hterm * Hterm.hterm
    | ApTerm of Hterm.hterm * int
    | ApThm of int * Hterm.hterm
    | Ccontr of Hterm.hterm * int
    | Choose of Hterm.hterm * int * int
    | Conj of int * int
    | Conjunct1 of int
    | Conjunct2 of int
    | Contr of Hterm.hterm * int
    | Disj1 of int * Hterm.hterm
    | Disj2 of Hterm.hterm * int
    | DisjCases of int * int * int
    | EqImpRuleL of int
    | EqImpRuleR of int
    | EqMp of int * int
    | EqTIntro of int
    | EtaConv of Hterm.hterm
    | Exists of (Hterm.hterm * Hterm.hterm) * int
    | Ext of int
    | Gen of Hterm.hterm * int
    | ImpAntisymRule of int * int
    | ImpTrans of int * int
    | Inst of (Hterm.hterm * Hterm.hterm)list * int
    | MkAbs of int
    | MkComb of int * int
    | MkExists of int
    | NotElim of int
    | NotIntro of int
    | Spec of Hterm.hterm * int
    | Subs of int list * int
    | SubsOccs of (int list * int) list * int
    | SubstConv of (int * Hterm.hterm)list * Hterm.hterm * Hterm.hterm
    | Sym of int
    | Trans of int * int
```

The functions below are for resetting and initialising the internal data structures at various points. The function `init` is for cold start, i.e., it should be called before processing a file. The function `clear_proof` should be called when the second pass starts. The functions `new_proof1` and `new_proof2` should be called at the beginning of every proof in Pass 1 and Pass 2, respectively.

```
val init: unit -> unit
val clear_proof: unit -> unit
val new_proof1: (string * Hthm.hthm list) -> unit
val new_proof2: string -> unit
```

The type `goal` represents goals in proof tables. If the field `Proved` is `true`, the goals have been proved.

```
datatype goal = Goal of {Thm: Hthm.hthm, Proved: bool}
```

The function `lazy_mode` is used to set or clear the lazy mode. When the lazy mode is set, the checker compresses the goals with the theorem derived in each step. When a match is found, the goal is flagged as proved. When all goals in a proof are proved, the checker skips the remaining proof lines.

```
val lazy_mode : bool -> unit
```

The function `goal_table` returns the current goal list. The function `last_pline` returns the line number of the last line in the current proof. The function `get_final_thm` returns the theorem in the last proof line.

```
val goal_table: unit -> goal list
val last_pline: unit -> int
val get_final_thm: unit -> Hthm.hthm
```

The following three functions are used in Pass 1. The function `add_pline_tab` should be called for each proof line to update the `pline` table. The function `add_pline_table` should be called at the end of a proof to save the current `pline` table. The function `print_prooftab` outputs the current `pline` table. It is mainly for debugging. The structure and the use of `pline` tables will be explained below.

```
val add_pline_tab : (int * int list * Hthm.hthm) -> bool
val add_pline_table : string -> unit
val print_prooftab : unit -> unit
```

The functions below are for use in Pass 2. The function `add_pline` should be called for every proof line. It saves the proof line in the current proof. The function `get_last_lineNo` returns the line number of the previous line. The function `get_thm` is called to fetch a theorem. Given the line number of the required theorem, it retrieves it from the current proof. The function `find_thm` takes a theorem, and searches the current proof to find the theorem. If it finds it, `SOME` is returned, otherwise, `NONE` is returned.

```
val add_pline: (int * justification * Hthm.hthm) -> int
val get_last_lineNo: unit -> int
val get_thm: (int * int) -> Hthm.hthm
val find_thm : Hthm.hthm -> Hthm.hthm option
```

The functions `mk_pline` and `mk_proof` are constructors for the types `pline` and `proof`, respectively.

```
val mk_pline: (int * justification * Hthm.hthm) -> pline
val mk_proof: (string * Hthm.hthm list * pline list) -> proof
```

The functions `pp_just` and `pp_pline` are the pretty printers for the justifications and proof lines, respectively. The function `pp_proof_thm` is used only in debugging.

```
  val pp_just : System.PrettyPrint.ppstream -> justification -> unit
  val pp_pline : System.PrettyPrint.ppstream -> pline -> unit
  val pp_proof_thm : System.PrettyPrint.ppstream -> unit
end;
```

9.2 The implementation

The functor `ProofFUN` implements the proof module. It requires three modules: type, term and theorem.

```
functor ProofFUN(structure Hthm: Hthm_sig
                 and Hterm: Hterm_sig and Htype: Htype_sig
                 sharing Htype = Hterm.Htype = Hthm.Hterm.Htype
                 and Hterm = Hthm.Hterm): Proof_sig =
  struct
    structure Hthm = Hthm;
    structure Hterm = Hterm;
    structure Htype = Htype;
```

The internal data structures in this module need the dynamic array structure and binary dictionary structure. The functors creating these structures are from the SML/NJ library.

A dynamic array is an array which can grow in size. When an attempt to access an element with an index greater than the current size, the dynamic array is enlarged to accommodate the element, thus, it provides an unbounded storage space. It is used to store the theorem references. Using dynamic arrays, there is no hard limit on the number of proof lines in a proof.

The integer-keyed dictionary is used to stored theorems. The keys are the line numbers.

```
    structure Sarray = StaticArrayFUN(type elemType = int);
    structure Darray = DynamicArray (Sarray);
    structure Int_key : ORD_KEY =
      struct
        type ord_key = int
        fun cmpKey (i,j:ord_key) =
          if (i = j) then LibBase.Equal
          else if (i < j) then LibBase.Less
               else LibBase.Greater
      end;
    structure Dict = BinaryDict(Int_key);

    open Darray;
```

9.2.1 Defining types The type `justification` represents the justifications. The meanings of various fields are explained in the `Check` module in Chapter 8.

```
    datatype justification =
      Hypothesis
      | Assume of Hterm.hterm
      | Refl of Hterm.hterm
      | Subst of (int*Hterm.hterm)list * Hterm.hterm * int
      | BetaConv of Hterm.hterm
      | Abs of Hterm.hterm * int
      | InstType of (Htype.htype * Htype.htype)list * int
      | Disch of Hterm.hterm * int
      | Mp of int * int
      | Axiom of (string * string)
      | Definition of (string * string)
      | DefExistsRule of (Hterm.hterm)
      | NewAxiom of (string * Hterm.hterm)
```

```
    | NewConstant of (string * Htype.htype)
    | NewType of (int * string)
    | Numconv of Hterm.hterm
    | StoreDefinition of (string * Hterm.hterm)
    | Theorem of (string * string)
    | AddAssum of  Hterm.hterm * int
    | Alpha of Hterm.hterm * Hterm.hterm
    | ApTerm of Hterm.hterm * int
    | ApThm of int * Hterm.hterm
    | Ccontr of Hterm.hterm * int
    | Choose of Hterm.hterm * int * int
    | Conj of int * int
    | Conjunct1 of int
    | Conjunct2 of int
    | Contr of Hterm.hterm * int
    | Disj1 of int * Hterm.hterm
    | Disj2 of Hterm.hterm * int
    | DisjCases of int * int * int
    | EqImpRuleL of int
    | EqImpRuleR of int
    | EqMp of int * int
    | EqTIntro of int
    | EtaConv of Hterm.hterm
    | Exists of (Hterm.hterm * Hterm.hterm) * int
    | Ext of int
    | Gen of Hterm.hterm * int
    | ImpAntisymRule of int * int
    | ImpTrans of int * int
    | Inst of (Hterm.hterm * Hterm.hterm)list * int
    | MkAbs of int
    | MkComb of int * int
    | MkExists of int
    | NotElim of int
    | NotIntro of int
    | Spec of Hterm.hterm * int
    | Subs of int list * int
    | SubsOccs of (int list * int) list * int
    | SubstConv of (int * Hterm.hterm) list * Hterm.hterm * Hterm.hterm
    | Sym of int
    | Trans of int * int ;;
```

The record type `pline` represents proof lines with the following three fields:

`Line` — the line number;

`Just` — the justification;

`Thm` — the derived theorem.

```
datatype pline =
  Pline of {Line: int, Just: justification, Thm: Hthm.hthm};
```

The record type `proof` represents proofs with the following three fields:

`Name` — the name of the proof;

`Goals` — a list of theorems to be derived in this proof;

`Proof` — a list of proof lines.

```
datatype proof =
  Proof of {Name: string, Goals: Hthm.hthm list, Proof: pline list};
```

The type `pline_tab` represents `pline` tables. They are used to store theorem reference information found in the first pass. They also store the goals of proofs. Every proof will have a `pline` table. Each table has the following fields:

`Name` — the name of the proof;

`Last` — the line number of the last proof line;

`Goals` — the goals of the proof;

`TabHyp` — hypothesis table;

`TabLine` — inference line table.

The `Goals` field is a list of goals. Each element of the list represents a single goal. It has two fields: `Thm` is the goal, and `Proved` is a boolean value indicating whether the theorem is found in one of the proof lines. The fields `TabHyp` and `TabLine` are dynamic arrays whose elements are line numbers. These arrays are use to record the highest line which refers to a theorem. For example, if `TabLine[2]` equals 5, then the last reference to the theorem in Line 2 is in Line 5. This means that the derived theorem of Line 2 is no longer needed after Line 5 is checked. Since the indexes must be non-negative number, two arrays are used. One for the hypotheses whose line numbers are negative, and the other for the remaining lines. The `TabHyp` index is the absolute value of the hypothesis line number, e.g., the element `TabHyp[3]` is the hypothesis line -3. The default value of these array elements is 0.

```
datatype goal = Goal of {Thm: Hthm.hthm, Proved: bool};
datatype pline_tab =
  PlineTab of {Name:string, Last:int, Goals: goal list,
               TabHyp:Darray.array, TabLine:Darray.array};
```

9.2.2 Exception This function defines the exception in this module.

```
fun PROOF_ERR{function,message} =
  Exception.CHK_ERR{message = message,
                    origin_function = function,
                    origin_structure = "Proof"};

val debug = Debug.get_debug("Proof");
```

9.2.3 Internal data structures The main purpose of the internal data structures is to store information about the proofs to be found in the first pass so that it can be used in the later pass. There are four dynamic objects for working with the current proof. Their names are prefixed by `current_`, and their usages are described below. They are initialised at the beginning of a proof, i.e., when the tag `PROOF` is found. In the first pass, information is cumulated in three of the dynamic structures. At the end of a proof, a structure of type `pline_tab` is created to group all information in these three structures. It is then inserted into the list containing all tables of the proofs in the current file to be used by the next pass.

The identifier current_goal refers to a list of goals of the current proof. From the observation of some proof generated by the HOL system, it has been found that sometimes the proof continues after the theorem matching the goal has been derived. In such a case, it is not necessary to check the entire proof if all goals are proved before the end of the proof is reached. The derived theorem of each line is compared with the goals. If a goal is found to be equal to the theorem, the goal is deleted from the list of current goals. When the last goal is deleted, we can ignore any remaining lines in the proof. By including this mechanism, it is hoped that the checker may be able to save some time so it is more efficient. However, the usefulness of this mechanism in practical, large proofs is still not clear, because it may not save time as comparing the goals with the derived theorem in every line takes time.

The line number of the last line is recorded in the Last field of the pline table. However, if there is no goal, we are obliged to check the entire proof. This laziness feature is enabled if the value of lazy is true. The default value is false when the checker starts. The value of lazy should not be changed in the middle of a proof.

At the beginning of a proof, proof_goal is bound to the same list of goals as current_goal. In Pass 1, as the proof lines are processed, when a goal is found to be the same as a theorem, it is deleted from current_goal. At the end of the proof, what remains in the list current_goal will be unsolved. A pline table is built for the proof, in which Goals contains the list bound to proof_goal with all goals proved (i.e., not in current_goal) flagged as such.

The identifier current_ptab refers to a pair of dynamic arrays containing theorem reference information. This information is used in the same way as in the description of type pline_tab above.

The identifier current_pline refers to the line number of the latest line being processed.

The identifier current_proof refers to a dictionary of theorems keyed by their line numbers. This dictionary is used only in Pass 2. Initially, it is empty. When a proof line is processed, the derived theorem is added to the dictionary if it is referenced by later line(s), i.e., the value of the corresponding element in the pline table array is greater than the current line number. When the theorem is retrieved from the dictionary for the last line referencing it, it is deleted. In this way, the size of the dictionary is kept to a minimum.

The name final_thm is bound to the theorem of the last line in a proof. This is assumed to be the goal of the proof if there is no goal specified explicitly at the beginning of the proof.

```
val default_ptab = ((Darray.array(10,0)), (Darray.array(10,0)))
val current_ptab = ref default_ptab
and proof_goal = ref([]:Hthm.hthm list)
and current_goal = ref ([]:Hthm.hthm list)
and goal_tab = ref([]:goal list)
and pline_tables = ref ([]:pline_tab list)
and current_pline = ref 0
and last_line = ref 0
and dummy_thm = (Hthm.mk_thm([],Hterm.mk_const("T",Htype.bool_ty)))
and dummy_proof = (Dict.mkDict()):Hthm.hthm Dict.dict

val final_thm = ref dummy_thm
and current_proof = ref dummy_proof

val lazy = ref false
fun lazy_mode b = (lazy := b)
```

9.2.4 Functions for Pass 1 The function add_pline_tab processes a proof line in Pass 1. It takes a triple as its argument. The first field n is the line number, the second ks is a list of line numbers to which this line references, the last thm is the derived theorem. It returns a boolean value **true** if all the goals of the current proof have been found, otherwise, **false**.

The current pline table, i.e., the arrays pointed to by **current_ptab**, is updated. The values of the array elements whose indices occur in the line number list ks are updated to the current line number. This is performed by the local function **update_tab**. Likewise, the current line number **current_pline** is updated. The local function **chk** checks the current goal list. If one of the goals is the same as the theorem thm, it is deleted. If this is the last goal, **true** is returned to indicate that all the goals have been proved. This checking is done only if lazy mode is enabled.

```
fun add_pline_tab (n, ks, thm) =
  let
    fun update_tab 0 = ()
      | update_tab k =
      if k > 0 then update(#2(!current_ptab), k, n)
      else  update(#1(!current_ptab), ~k, n)
    and check_goal thm =
      let
        fun chk th [] = false
          | chk th (th' :: thms) =
            if (Hthm.thm_eq th th') then
              (current_goal := thms;
              if (null thms) then true else false)
            else chk th thms
      in
         chk thm (!current_goal)
      end
  in
    (map update_tab ks; current_pline := n;
     if (!lazy) then check_goal(thm) else false)
  end
```

The function `add_pline_table` is called at the end of a proof. It builds a `pline` table (of type `pline_tab`) and adds it to the `pline` table list `pline_tables`.

```
fun add_pline_table name =
  let
    val gls = (!current_goal)
    val gl0 = null gls
    fun chk_goal th =
      if gl0 then true
      else (not(exists (fn th' => Hthm.thm_eq th th') gls))
    val (tabh,tabl) = !current_ptab
    val gs =
      if not(!lazy) then []
      else map (fn th => Goal{Thm=th,Proved=chk_goal th}) (!proof_goal)
  in
    pline_tables := (PlineTab{Name=name,Last=(!current_pline),
                              Goals = gs,
                              TabHyp=tabh,TabLine=tabl})::(!pline_tables);
    ()
  end
```

The internal function `ptab_value` retrieves the value of an element from the current pline arrays.

The function `print_prooftab` outputs the current `pline` table arrays to the `std_out` stream. It is mainly for debugging.

```
fun ptab_value n =
  if (n > 0) then sub(#2(!current_ptab), n)
      else  sub(#1(!current_ptab), ~n)

fun print_prooftab () =
  let
    val (hyptab,linetab) = (!current_ptab)
    val hb = bound hyptab
    and lb = bound linetab
    val i = ref hb
  in
    (while((!i) > 0) do
    (output(std_out,
            ("-"^makestring (!i) ^" = "^ makestring (sub(hyptab,(!i)))^"\n"
));
     i := (!i) - 1);

    while((!i) < lb) do
    (output(std_out,
            (makestring (!i) ^" = "^ makestring (sub(linetab,(!i)))^"\n"));

     i := (!i) + 1);
    ())
  end

fun reset_line_num () = (current_pline := 0);
```

9.2.5 Functions for Pass 2 Functions in this section are used in Pass 2. The function `print_cur_proof` prints out the current proof dictionary. It is mainly for debugging.

```
fun print_cur_proof () =
  let
    val thms =  Dict.listItems(!current_proof)
    fun prt (ln,thm) = output(std_out,((makestring ln)^" "))
  in
    output(std_out,("current_proof:[ "));
    map prt thms;
    output(std_out, "]\n")
  end
```

The function `add_pline` should be called for every proof line after it is checked. It inserts the derived theorem into the current proof dictionary if it is referenced by later line(s). It returns the line number of the expected last proof line in the proof.

```
fun add_pline (line,just,thm) =
  (if (ptab_value line > line) then
     (if (debug > 1) then
        output(std_out,("add_pline: "^(makestring line)^"\n"))
      else ();
     current_proof := Dict.insert( (!current_proof),line,thm);
      if (debug > 1) then print_cur_proof() else ())
```

```
      else();
      current_pline := line;
      if (line = (!last_line)) then final_thm := thm else ();
      (!last_line));

  fun get_last_lineNo () = !current_pline;
```

The function `get_thm` retrieves a theorem from the current proof dictionary. It takes two line numbers as its arguments. The first one ln is the line number of the required theorem. The second n is the line number of the line which uses the theorem. If n is the last line that references the theorem in line ln, the theorem is removed from the dictionary. However, no hypothesis will be removed as they are reported at the end of the proof.

```
local
  fun get_th n [] = NONE
    | get_th n ((l as Pline{Line=line,...}) :: thmlst) =
      if (n = line) then (SOME l) else get_th n thmlst
  fun find_th th [] = NONE
    | find_th th ((Pline{Thm=thm,...}) :: thmlst) =
      if (Hthm.thm_eq th thm) then (SOME thm) else find_th th thmlst
in
  fun get_thm (ln, n) =
    if (ln < (ptab_value n)) orelse (n < 0) then
        Dict.find((!current_proof),n)
    else
        let
          val (dict,th) = Dict.remove(!current_proof, n)
        in
          current_proof := dict;
          th
        end
    handle Dict.NotFound =>
      raise (PROOF_ERR {function="get_thm",
                        message="Line "^(makestring n)^" not found"})
       | e => raise e
  and find_thm thm = NONE (* find_th thm (!current_proof)    *)
end;
```

9.2.6 Constructors and field selector These two functions, `mk_pline` and `mk_proof`, are the constructors for the types `pline` and `proof`, respectively.

The functions `pline_No`, `pline_Just` and `pline_Thm` return the respective fields of a `pline`.

```
fun mk_pline (line,just,thm) =
  Pline{Line=line, Just=just, Thm=thm}
and mk_proof (name, goals, lines) =
  Proof{Name=name, Goals=goals, Proof=lines};

fun pline_No (Pline{Line,...}) = Line
and pline_Just (Pline{Just,...}) = Just
and pline_Thm (Pline{Thm,...}) = Thm;
```

9.2.7 Initialisation The function `init` initialises the internal data structures. It should be called at the beginning of the first pass. The function `new_proof1` initialises the current goal list and `pline` table. It should be called at the beginning of every proof in Pass 1.

```
fun init () =
  (current_proof := dummy_proof;
   current_ptab := ((Darray.array(10,0)), (Darray.array(10,0)));
   pline_tables := [];
   ())
and new_proof1 (name,thms) =
  (current_goal := thms;
   proof_goal := thms;
   current_ptab := ((Darray.array(10,0)), (Darray.array(10,0)));
   ())
```

The next two functions, `clear_proof` and `new_proof2`, are used in the second pass to initialise the internal data structures in a way similar to the above first pass functions.

```
fun clear_proof () =
  (current_proof := dummy_proof; ())
and new_proof2 name =
  let
    fun find_ptab name [] =
      raise (PROOF_ERR {function="new_proof2",
                        message=("Proof "^name^" not found")})
     | find_ptab name
         ((tab as PlineTab{Name,Goals,Last,TabHyp,TabLine})::xs) =
       if (name = Name) then tab
       else find_ptab name xs
    val (PlineTab ptab) = find_ptab name (!pline_tables)
  in
    current_ptab := ((#TabHyp ptab),(#TabLine ptab));
    goal_tab := (#Goals ptab);
    last_line := #Last ptab;
    current_proof := dummy_proof;
    current_pline := ~(Darray.bound(#1(!current_ptab)));
    ()
  end
```

The following three functions are provided to allow other modules to access some internal data. The function `goal_table` returns the current goal list. The function `last_pline` returns the line number of the last line in the current proof. The function `get_final_thm` returns the theorem in the last proof line.

```
and goal_table () = (!goal_tab)
and last_pline () = (!last_line)
and get_final_thm () = (!final_thm)
```

9.2.8 Pretty Printer The function `pp_proof_thm` prints the theorems in the `current_proof` theorem table. Each theorem is preceded by its line number. This is used in debugging. The functions `pp_just` and `pp_pline` print a justification and a proof line, respectively, to a `ppstream`. They use the system pretty printer structure.

```
local
  open System.PrettyPrint;
  fun with_ppstream ppstrm =
    {add_string = add_string ppstrm,
```

```
         add_break = add_break ppstrm,
         begin_block = begin_block ppstrm,
         end_block = fn () => end_block ppstrm,
         flush_ppstream = fn () => flush_ppstream ppstrm};
    in

      fun  pp_proof_thm ppstrm =
        let
          val {add_string, add_break,
               begin_block, end_block, flush_ppstream} =
            with_ppstream ppstrm
          and pp_hthm = Hthm.pp_hthm ppstrm
          and thms = Dict.listItems(!current_proof)
          fun pp_thmno n =
            (add_string "\n<";
             add_string (Format.format "%d" [(Format.INT n)]);
             add_string ">")
          and prt (ln,thm) =
            (pp_thmno ln; add_string "  "; pp_hthm thm; add_break (1,0))
        in
          if (debug > 1) then
            output(std_out, ("pp_proof_thm:"^(makestring (length thms))^"\n"))
          else ();
          if null thms then ()
          else (begin_block CONSISTENT 0;
                map prt thms;
                end_block ();
                add_string "\n";
                flush_ppstream())
        end
```

9.2.9 Pretty printer for justification Each justification is enclosed in a pair of parentheses. The name of the justification is printed first followed by the other fields. A theorem is printed as a line number surrounded by angle brackets. Terms are printed using the term pretty printer.

```
      fun pp_just ppstrm just =
        let
          val {add_string, add_break,
               begin_block, end_block, flush_ppstream} =
            with_ppstream ppstrm;
          val pp_hthm = Hthm.pp_hthm ppstrm
          and pp_qterm = Hterm.pp_qterm ppstrm
          and pp_htype = Htype.pp_htype ppstrm
          fun pp_thmno n =
            (add_string "<";
             add_string (Format.format "%d" [(Format.INT n)]);
             add_string ">")
          and pp_arity n =
             if (n = 0) then ()
             else (add_string "("; add_string (makestring n);
                   add_string ")")
```

```
      fun pp_list pf l =
        let
          fun pplist pf [] = ()
            | pplist pf [x] = pf x
            | pplist pf (x::(l as (y::ys))) =
              (pf x; add_string";"; add_break(1,0); pplist pf l)
        in
          (begin_block CONSISTENT 0;
           add_string "[";
           pplist pf l;
           add_string "]";
           end_block ())
        end
      and pp_pair (f1,f2) (x1,x2) =
        (begin_block CONSISTENT 0;
         add_string"("; f1 x1; add_string",";
         f2 x2; add_string")";
         end_block ())
    in
      case just of
        Hypothesis => add_string "Hypothsis"
      | (Assume term) => (add_string "ASSUME "; pp_qterm term)
      | (Refl term) => (add_string "REFL "; pp_qterm term)
      | (Subst (itml,tm,n)) => (add_string "SUBST ";
                                pp_list (pp_pair (pp_thmno,pp_qterm)) itml;
                                add_break (1,3);
                                pp_qterm tm; add_string",";
                                pp_thmno n)
      | (BetaConv term) => (add_string "BETA_CONV "; pp_qterm term)
      | (Abs (term,n)) => (add_string "ABS ";
                           pp_qterm term; add_string","; pp_thmno n)
      | (InstType (ttl,n)) => (add_string "INST_TYPE ";
                                pp_list (pp_pair (pp_htype,pp_htype)) ttl;
                                pp_thmno n)
      | (Disch (term,n)) => (add_string "DISCH ";
                              pp_qterm term; add_string","; pp_thmno n)
      | (Mp (n1,n2)) => (add_string "MP ";
                         pp_thmno n1; add_string","; pp_thmno n2)

      | (Axiom (s1,s2)) => (add_string "AXIOM ";
                            add_string s1; add_string"."; add_string s2)
      | (Definition (s1,s2)) => (add_string "DEFINITION ";
                            add_string s1; add_string"."; add_string s2)
      | (DefExistsRule term) => (add_string "DEF_EXISTS_RULE";
                                 pp_qterm term)
      | (NewAxiom (s,term)) => (add_string "NEW_AXIOM ";
                            pp_qterm term)
      | (NewConstant (s, ty)) => (add_string "NEW_CONSTANT ";
                                  pp_qterm (Hterm.mk_const(s,ty)))
      | (NewType (n,s)) => (add_string "NEW_TYPE ";
                            pp_arity n; add_string s)
```

```
| (Numconv term) => (add_string "num_CONV ";
                     pp_qterm term)
| (StoreDefinition (s,term)) => (add_string "STORE_DEFINITION ";
                                 add_string s; pp_qterm term)
| (Theorem (s1,s2)) => (add_string "THEOREM ";
                       add_string s1; add_string"."; add_string s2)
| (AddAssum (term, n)) => (add_string "ADDASSUM ";
                          pp_qterm term; pp_thmno n)
| (Alpha (term1,term2)) => (add_string "ALPHA ";
                           pp_qterm term1; pp_qterm term2)
| (ApTerm (term,n)) => (add_string "APTERM ";
                       pp_qterm term; pp_thmno n)
| (ApThm (n,term)) => (add_string "APTHM ";
                      pp_thmno n; pp_qterm term)
| (Ccontr (term,n)) => (add_string "CCONTR ";
                       pp_qterm term; pp_thmno n)
| (Choose (term,n1,n2)) => (add_string "CHOOSE ";
                           pp_qterm term; pp_thmno n1; pp_thmno n2)
| (Conj (n1,n2)) => (add_string "CONJ ";
                    pp_thmno n1; pp_thmno n2)
| (Conjunct1 n) => (add_string "CONJUNCT1 "; pp_thmno n)
| (Conjunct2 n) => (add_string "CONJUNCT2 "; pp_thmno n)
| (Contr (term,n)) => (add_string "CONTR ";
                      pp_qterm term; pp_thmno n)
| (Disj1 (n,term)) => (add_string "DISJ1 ";
                      pp_thmno n; pp_qterm term)
| (Disj2 (term,n)) => (add_string "DISJ2 ";
                      pp_qterm term; pp_thmno n)
| (DisjCases (n1,n2,n3)) => (add_string "DISJCASES ";
                            pp_thmno n1; pp_thmno n2; pp_thmno n3)
| (EqImpRuleL n) => (add_string "EQIMPRULEL "; pp_thmno n)
| (EqImpRuleR n) => (add_string "EQIMPRULER "; pp_thmno n)
| (EqMp (n1,n2)) => (add_string "EQMP "; pp_thmno n1; pp_thmno n2)
| (EqTIntro n) => (add_string "EQTINTRO "; pp_thmno n)
| (EtaConv term) => (add_string "ETACONV "; pp_qterm term)
| (Exists ((term1,term2),n)) => (add_string "EXISTS ";
                                pp_qterm term1; pp_qterm term2;
                                pp_thmno n)
| (Ext n) => (add_string "EXT "; pp_thmno n)
| (Gen (term,n)) => (add_string "GEN ";
                    pp_qterm term; pp_thmno n)
| (ImpAntisymRule (n1,n2)) => (add_string "IMPANTISYMRULE ";
                              pp_thmno n1; pp_thmno n2)
| (ImpTrans (n1,n2)) => (add_string "IMPTRANS ";
                        pp_thmno n1; pp_thmno n2)
| (Inst (ttl,n)) => (add_string "INST ";
                    pp_list (pp_pair (pp_qterm, pp_qterm)) ttl;
                    add_break(1,3);
                    pp_thmno n)
| (MkAbs n) => (add_string "MKABS "; pp_thmno n)
| (MkComb (n1,n2)) => (add_string "MKCOMB ";
```

```
                          pp_thmno n1; pp_thmno n2)
     | (MkExists n) => (add_string " "; pp_thmno n)
     | (NotElim n) => (add_string "NOTELIM "; pp_thmno n)
     | (NotIntro n) => (add_string "NOTINTRO "; pp_thmno n)
     | (Spec (term,n)) => (add_string "SPEC ";
                          pp_qterm term; pp_thmno n)
     | (Subs (nl,n)) => (add_string "SUBS ";
                        pp_list pp_thmno nl; pp_thmno n)
     | (SubsOccs (nlnl,n)) =>
         (add_string "SUBSOCCS ";
          pp_list(pp_pair(pp_list pp_thmno,pp_thmno)) nlnl;
          add_break (1,3); pp_thmno n)
     | (SubstConv (ntml,term1,term2)) =>
         (add_string "SUBSTCONV ";
          pp_list (pp_pair(pp_thmno,pp_qterm))ntml;
          add_break(1,3);
          pp_qterm term1; pp_qterm term2)
     | (Sym n) => (add_string "SYM "; pp_thmno n)
     | (Trans (n1,n2)) => (add_string "TRANS ";
                          pp_thmno n1; pp_thmno n2)
   end
```

9.2.10 Pretty printer for proof line The proof lines are printed in the following form:

$\langle linenumber\rangle$: $\langle justification\rangle$
$\langle theorem\rangle$

```
  fun pp_pline ppstrm (Pline{Line,Just,Thm}) =
    let
      val {add_string, add_break,
           begin_block, end_block, flush_ppstream} =
        with_ppstream ppstrm;
      val pp_hthm = Hthm.pp_hthm ppstrm
      and pp_just = pp_just ppstrm;
      fun mk_str n = Format.format "%6d: " [(Format.INT n)]
    in
      (begin_block CONSISTENT 0;
       add_string (mk_str Line);
       pp_just Just;
       add_break (1,6);
       case Just of
         (NewType _) => ()
       | (NewConstant _) => ()
       | _ => pp_hthm Thm;
       add_string "\n";
       end_block ();
       flush_ppstream())
    end
end(* local *)

end;   (* end of functor ProofFUN *)
```

10 Proof environment

The proof checker works within an environment which consists of a *type structure* and a *constant structure*. The type structure contains all currently known type operators and their arities. The constant structure contains all known constants and their types. The proof file format `prf` specifies that every proof file has an environment expression at the beginning. It gives the type and constant structures to be used in checking the proofs in the file. There are three default environments. The checker starts with the default environment `HOL` which contains the same types and constants as when the HOL system starts. This default environment will be augmented first by the environment specified at the beginning of a proof file. Then, as checking progresses, new types and constants may be added by the justifications `new_type` and `new_constant`, respectively.

10.1 The specification

The environment module has the following signature:

```
signature Henv_sig =
 sig
   structure Htype : Htype_sig
   structure Hterm : Hterm_sig
```

10.1.1 New types The type `htypeConst` represents an element in the type structure. The type structure is part of the current environment of the checker. A type constant has a name and an arity.

The type `htermConst` represents a constant in the signatures (or constant structure) of the current environment. It contains a set of constants known to the current theory.

The type `henv` represents the current environment. It consists of the type structure and the constant structure.

```
type htypeConst
type htermConst
type henv
```

10.1.2 Functions and identifiers The function `mk_proof_env` creates a new proof environment. It is called when an `ENV` expression is found by the parser. It takes a triple `(name, tylst, clst)` as its argument. The name `envList` is bound to the list of default environments.

```
val mk_proof_env : (string * htypeConst list * htermConst list) -> unit
val envList :  (string * htypeConst list * htermConst list) list
val mk_typeconst : (string * int) -> htypeConst
val mk_termconst : (string * Htype.htype) -> htermConst
```

The function `init` should be called to initialise the environment before starting reading a proof file.

```
val init: unit -> unit
```

The function `envName` returns the name of the current environment. A list of all type operators in the current environment is returned by the function `knownTypes`. Similarly, the function `knownConstants` returns a list of constants in the current environment.

```
val envName : unit -> string
val knownTypes : unit -> htypeConst list
val knownConstants : unit -> htermConst list
```

The functions add_type and add_const add a new type operator and a new constant to the current environment, respectively.

```
val add_type : (string * int) -> unit
val add_const : (string * Htype.htype) -> unit
```

10.1.3 Checking type well-formedness The function type_OK takes a type *ty*. It checks whether *ty* is well-formed under the current environment. A type is well-formed if and only if (i) it is a type variable; or (ii) its name appears in the type structure and the number of arguments equals to the arity given in the structure. The function well_typed returns true if its argument term is well-typed in the current environment.

```
    val type_OK : Htype.htype -> bool
    val well_typed : Hterm.hterm -> bool
end; (* signature *)
```

10.2 The implementation

```
functor HenvFUN (structure Htype : Htype_sig
                 and Hterm : Hterm_sig
                 sharing Htype = Hterm.Htype)  : Henv_sig =
  struct
    structure Htype = Htype;
    structure Hterm = Hterm;
```

10.2.1 Types The type htypeConst represents type constructors. It is a pair consisting of a string and a number. The string is the name of the type constructor. The number is its arity. A type structure is a list of type constructors.

```
datatype htypeConst = TYCON of {Name:string, Arity:int};
```

The type htermConst represents Constant terms. The type henv represents a proof environment which consists of a type structure and a constant structure.

```
datatype htermConst = CCON of {Name:string, Ty:Htype.htype};
datatype henv = HENV of {Name: string,
                         Tys: htypeConst list,
                         Tms: htermConst list};

open ListUtil Htype Hterm;
```

10.2.2 Error handling The function ENV_ERR defines exceptions of this module.

```
fun ENV_ERR{function,message} =
  Exception.CHK_ERR{origin_structure="Henv",
                    origin_function = function, message = message};
  val debug = ref 0;
```

10.2.3 Constructors and destructors These are the constructors and destructors of the type operators and constant terms.

```
fun mk_typeconst (name,arity) = TYCON {Name=name, Arity=arity}
and mk_termconst (name,ty) = CCON {Name=name, Ty=ty}
and dest_typeconst (TYCON{Name,Arity}) = (Name, Arity)
```

```
and dest_termconst (CCON{Name,Ty}) = (Name, Ty)
```

10.2.4 Current environment An empty environment is bound to the name `dummy_env`. The internal variable `current_env` is always pointing to an environment to be used as the current environment. Initially, the current environment is the dummy environment. This is set in the function `init`.

```
val dummy_env =  (HENV {Name="", Tys=[], Tms=[]});
val current_env = ref dummy_env;

fun init () =
  (debug := Debug.get_debug "Henv";
   current_env := dummy_env; ());
```

The functions `envName`, `knownTypes` and `knownConstants` return respectively the name, type structures and constant structures of the current environment. The function `init` should be called to clear the current environment when the checker begins.

```
fun envName () =
  let val HENV {Name,...} = !current_env in Name end
and knownTypes () =
  let val HENV {Tys,...} = !current_env in  Tys end
and knownConstants () =
  let val HENV {Tms,...} = !current_env in Tms end
```

10.2.5 Default environments We first initialise the default environments. To improve the readability of these environments, some identifiers are defined locally. Amongst these local identifiers, some are symbolic, and they stand for type variables, such as `*`, and type operators, such as `-->`. Others are normal identifiers. They stand for type constants and type operators, such as `bool` and `list` and so on.

```
local
  nonfix *;

  val * = mk_vartype "*"
  and ** = mk_vartype "**"
  and *** = mk_vartype "***";

  val bool = bool_ty
  and num = mk_type {Tyargs=[],Tyop="num"}
  and ind = mk_type {Tyargs=[],Tyop="ind"}
  and one = mk_type {Tyargs=[],Tyop="one"}
  and tree = mk_type {Tyargs=[],Tyop="tree"};

  infixr 3 -->;
  infix 4 ++;
  infix 5 ##;

  fun ty1 --> ty2 = mk_funtype (ty1,ty2)
  and ty1 ++ ty2 = mk_type {Tyargs=[ty1,ty2],Tyop="sum"}
  and ty1 ## ty2 = mk_type {Tyargs=[ty1,ty2],Tyop="prod"}
  and list ty = mk_type {Tyargs=[ty],Tyop="list"}
  and ltree ty = mk_type {Tyargs=[ty],Tyop="ltree"};
```

```
in
```

There are three default environments, `MIN`, `LOG` and `HOL`. See Section 16.4 of [5] for the definition of the first two environments. Each environment in the list includes all the types and constants of its preceding environments, e.g., `HOL` includes all the types and constants of `MIN` and `LOG`. `HOL` is the initial environment when the HOL system starts.

```
val envList = [
   ("HOL",
    map mk_typeconst
    [("fun", 2), ("prod", 2), ("num", 0), ("list", 1),
     ("tree", 0), ("ltree", 1), ("sum", 2), ("one", 0)],
    map mk_termconst
    [("*", (num --> (num --> num))),
     ("+", (num --> (num --> num))),
     (",", ( * --> ( ** --> * ## ** ))),
     ("-", (num --> (num --> num))),
     ("0", (num)),
     ("<=", (num --> (num --> bool))),
     ("<", (num --> (num --> bool))),
     (">=", (num --> (num --> bool))),
     (">", (num --> (num --> bool))),
     ("?!", (( * --> bool) --> bool)),
     ("ABS_list", ((num --> * ) ## num --> list( * ))),
     ("ABS_ltree", (tree ## list( * ) --> ltree( * ))),
     ("ABS_num", (ind --> num)),
     ("ABS_sum", ((bool --> ( * --> ( ** --> bool))) --> * ++ ** )),
     ("ABS_tree", (num --> tree)),
     ("APPEND", (list( * ) --> (list( * ) --> list( * )))),
     ("AP", (list( * --> ** ) --> (list( * ) --> list( ** )))),
     ("ARB", ( * )),
     ("BINDERS", ( * --> bool)),
     ("COND", (bool --> ( * --> ( * --> * )))),
     ("CONS", ( * --> (list( * ) --> list( * )))),
     ("CURRY", (( * ## ** --> *** ) --> ( * --> ( ** --> *** )))),
     ("DIV", (num --> (num --> num))),
     ("EL", (num --> (list( * ) --> * ))),
     ("EVEN", (num --> bool)),
     ("EVERY", (( * --> bool) --> (list( * ) --> bool))),
     ("EXP", (num --> (num --> num))),
     ("FACT", (num --> num)),
     ("FLAT", (list(list( * )) --> list( * ))),
     ("FST", ( * ## ** --> * )),
     ("HD", (list( * ) --> * )),
     ("HOL_DEFINITION", (bool --> bool)),
     ("HT", (tree --> num)),
     ("INL", ( * --> * ++ ** )),
     ("INR", ( ** --> * ++ ** )),
     ("ISL", ( * ++ ** --> bool)),
     ("ISR", ( * ++ ** --> bool)),
     ("IS_ASSUMPTION_OF", (bool --> (bool --> bool))),
     ("IS_NUM_REP", (ind --> bool)),
```

```
("IS_PAIR", (( * --> ( ** --> bool)) --> bool)),
("IS_SUM_REP", ((bool --> ( * --> ( ** --> bool))) --> bool)),
("IS_list_REP", ((num --> * ) ## num --> bool)),
("I", ( * --> * )),
("Is_ltree", (tree ## list( * ) --> bool)),
("Is_tree_REP", (num --> bool)),
("K", ( * --> ( ** --> * ))),
("LENGTH", (list( * ) --> num)),
("LET", (( * --> ** ) --> ( * --> ** ))),
("MAP2", (( * --> ( ** --> *** )) --> (list( * ) -->
          (list( ** )-->list( *** ))))),
("MAP", (( * --> ** ) --> (list( * ) --> list( ** )))),
("MK_PAIR", ( * --> ( ** --> ( * --> ( ** --> bool))))),
("MOD", (num --> (num --> num))),
("NIL", (list( * ))),
("NULL", (list( * ) --> bool)),
("Node", ( * --> (list(ltree( * )) --> ltree( * )))),
("ODD", (num --> bool)),
("OUTL", ( * ++ ** --> * )),
("OUTR", ( * ++ ** --> ** )),
("PART", (list(num) --> (list( * ) --> list(list( * ))))),
("PRE", (num --> num)),
("PRIM_REC_FUN", ( * --> (( * --> (num --> * )) -->
                  (num --> (num --> * ))))),
("PRIM_REC", ( * --> (( * --> (num --> * )) --> (num --> * )))),
("REP_list", (list( * ) --> (num --> * ) ## num)),
("REP_ltree", (ltree( * ) --> tree ## list( * ))),
("REP_num", (num --> ind)),
("REP_prod", ( * ## ** --> ( * --> ( ** --> bool)))),
("REP_sum", ( * ++ ** --> (bool --> ( * --> ( ** --> bool))))),
("REP_tree", (tree --> num)),
("RES_ABSTRACT", (( * --> bool) --> (( * --> ** ) -->
                  ( * --> ** )))),
("RES_EXISTS", (( * --> bool) --> (( * --> bool) --> bool))),
("RES_FORALL", (( * --> bool) --> (( * --> bool) --> bool))),
("RES_SELECT", (( * --> bool) --> (( * --> bool) --> * ))),
("SIMP_REC_FUN", ( * --> (( * --> * ) --> (num --> (num --> * ))))),
("SIMP_REC_REL", ((num --> * ) --> ( * --> (( * --> * )-->
                  (num --> bool))))),
("SIMP_REC", ( * --> (( * --> * ) --> (num --> * )))),
("SND", ( * ## ** --> ** )),
("SPLIT", (num --> (list( * ) --> list( * ) ## list( * )))),
("SUC_REP", (ind --> ind)),
("SUC", (num --> num)),
("SUM", (list(num) --> num)),
("S", (( * --> ( ** --> *** )) -->
       (( * --> ** ) --> ( * --> *** )))),
("Size", (tree --> num)),
("TL", (list( * ) --> list( * ))),
("TRP", (( * --> (list(ltree( * )) --> bool)) -->
         (ltree( * ) --> bool))),
```

```
        ("UNCURRY", (( * --> ( ** --> *** )) --> ( * ## ** --> *** ))),
        ("ZERO_REP", (ind)),
        ("bht", (num --> (tree --> bool))),
        ("dest_node", (tree --> list(tree))),
        ("node_REP", (list(num) --> num)),
        ("node", (list(tree) --> tree)),
        ("o", (( ** --> *** ) --> (( * --> ** ) --> ( * --> *** )))),
        ("one", (one)),
        ("trf", (num --> ((list( ** ) --> ** ) --> (tree --> ** ))))]
       ),
      ("LOG", [],
       map mk_termconst
       [("!", (( * --> bool) --> bool)),
        ("?", (( * --> bool) --> bool)),
        ("T", (bool)),
        ("F", (bool)),
        ("~", (bool --> bool)),
        ("/\\", (bool --> (bool --> bool))),
        ("\\/", (bool --> (bool --> bool))),
        ("ONE_ONE", (( * --> ** ) --> bool)),
        ("ONTO", (( * --> ** ) --> bool)),
        ("TYPE_DEFINITION", (( * --> bool) --> (( ** --> * ) --> bool)))]
       ),
      ("MIN", map mk_typeconst [("bool", 0),  ("ind", 0)],
       map mk_termconst
       [("==>", (bool --> (bool --> bool))),
        ("=", ( * --> ( * --> bool))),
        ("@", (( * --> bool) --> * ))]
       )];
end(* local *)
```

10.2.6 Setting up the environment When an ENVironment expression is found in the input, the function `mk_proof_env` is called. This function sets up the current proof environment. The expression, represented as a triple `(name, tylst, clst)`, is passed to this function as its argument.

If `name` is the name of one of the default environments in the list `envList`, `tylst` and `clst` are ignored. The current environment will consist of all the type and constant structures of the environment(s) in the default list from the first element up to and including the element having the same name as `name`.

If `name` is not in the default environment, it is taken as a new environment built on top of the default environment `HOL`. The type and constant structures in the argument, together with the default structures, are set up as the current environment.

```
local
  fun mk_env ((n,tyl',conl'),(tyl,conl)) =
    ((tyl' @ tyl), (conl' @ conl))
in
  fun mk_proof_env (name, tyl, conl) =
    let val (l1,l2) =
      splitp (fn (n,l1,l2) => (n = name)) envList
    in
      if (null l2) then
```

```
        let val (tys,tms) =
           fold mk_env ((name,tyl,conl)::envList) ([],[])
        in
          current_env := HENV{Name=name, Tys=tys, Tms=tms};
          if ((!debug) > 0)
            then output(std_out, ("Current environment: "^name^"\n"))
          else ()
         end
      else let val (tl,cl) = fold mk_env l2 ([],[])
           in
             current_env := HENV {Name= #1(hd l2), Tys=tl, Tms=cl};
             if ((!debug) > 0)
               then output(std_out,
                           ("Current environment: "^(#1(hd l2))^"\n"))
             else ()
           end
    end
end
```

10.2.7 Well-formedness and well-typedness A type is well-formed under an environment if it is a type variable, or if the name of the type operator appears in the type structure `env` and all its arguments are also well-formed.

A variable is well-typed under the environment $(tysl, constsl)$ if and only if its type is well-formed. A constant is well-typed if and only if it occurs in the constant structure *constsl* and its type is an instance of the type given in the constant structure. Numerical constants are exceptions since they are not in the constant structure. They are well-typed if and only if their type is `num` and their names are strings consists only of digits. Terms which are function applications, are well-typed if and only if the operator is of function type and is well-typed, and the operand is well-typed and in the domain of the operator. For abstractions, the bound variables and the bodies should be well-typed.

The predicate `type_OK` returns `true` if its argument is a well-formed type under the current environment. Similarly, the predicate `well_typed` returns `true` if its argument is a well-typed term under the current environment.

```
local
  fun in_consts cl (name,cty) =
    let val found = findOne (fn (s,ty) => (s = name)) cl
    in
      case found of
        NONE => false
      | SOME(_,ty) => is_type_inst cty ty
    end;
in
  fun type_OK ty =
    if (is_vartype ty) then true
    else
      let
        val {Tyop,Tyargs} = dest_type ty
        val tysl = map dest_typeconst (knownTypes())
        val found = findOne (fn (s,a) => (s = Tyop)) tysl
      in
        case found of
```

```
          NONE =>  false
        | SOME (_,arity) =>
            (arity = length Tyargs) andalso (every type_OK Tyargs)
      end;

  fun well_typed tm =
    let
      fun is_digit_str s =
        every (CType.isDigitOrd o ord) (explode s)
      val constsl = map dest_termconst (knownConstants())
    in
      if (is_var tm) then
        let val ty = #2(dest_var tm) in (type_OK ty) end
      else if (is_const tm) then
        let val (name,ty) = dest_const tm in
         if (is_digit_str name) then (ty_is_num ty)
         else (in_consts constsl (name,ty)) andalso (type_OK ty)
        end
      else if (is_comb tm) then
        let
          val (rator,rand) = dest_comb tm
          val (ranty,rty) = dest_funtype (type_of rator)
        in
          (well_typed rator) andalso (well_typed rand) andalso
          (ranty = (type_of rand))
        end handle CHK_ERR _ => false
      else if (is_abs tm) then
        let val (bv,body) = dest_abs tm
        in
          (well_typed bv) andalso (well_typed body)
        end
      else false
    end

end; (* local *)
```

10.2.8 Augmenting the environment The functions add_type and add_const add a new type and a new constant to the current environment, respectively. We need to check whether the new type or the new constant is already in the environment. When adding constants, we also need to check the well-formedness of its type.

```
fun add_type (tyname, arity) =
  let val HENV {Name=n,Tys=tys,Tms=tms} = !current_env
      val found = findOne (fn (ty,ar) => (ty = tyname))
        (map dest_typeconst tys)
  in
    case found of
      SOME (_, ar) =>
        if (ar = arity)
        then raise (ENV_ERR {function="add_type",
                             message="type already in enrivonment"})
        else raise (ENV_ERR {function="add_type",
```

```
                              message="type name crash"})
      | NONE =>
          (current_env := HENV{Name=n,
                              Tys=((TYCON{Name=tyname,Arity=arity})::tys),
                              Tms=tms};
           ())
    end
  and add_const (name, ty) =
    let val HENV {Name=n,Tys=tys,Tms=tms} = !current_env
        val found = findOne (fn (n,ty) => (n=name)) (map dest_termconst tms)
    in
      case found of
        SOME (_,typ) =>
          if (typ = ty)
            then raise (ENV_ERR {function="add_const",
                              message="constant already in enrivonment"})
          else raise (ENV_ERR {function="add_const",
                              message="constant name crash"})
      | NONE =>
          if (type_OK ty) then
            (current_env := HENV{Name=n, Tys=tys,
                                Tms=((CCON{Name=name,Ty=ty})::tms)};
             ())
          else raise (ENV_ERR {function="add_const",
                              message="type of constant badly formed"})
    end

end;(* functor HenvFun *)
```

11 HOL theorems

The abstraction of HOL theorems is in the module `Hthm`. The signature `Hthm_sig` specifies this abstraction.

11.1 The specification

The type `hthm` represents theorems. A theorem has two parts: the set of hypotheses and the conclusion. These two parts are accessed by the functions `hyp` and `concl`, respectively. The function `mk_thm` creates a theorem when supplied a list of hypotheses and a conclusion. All of these must be boolean terms.

The function `thm_eq` returns `true` when applied to two theorems if the conclusions are identical and their sets of hypotheses are equivalent.

The pretty printer for theorems is `pp_hthm`.

```
signature Hthm_sig =
  sig
    structure Report : Report_sig
    structure Hterm: Hterm_sig
    structure HtermSet: ORD_SET

    type hthm

    val concl : hthm -> Hterm.hterm
    val hyp   : hthm -> HtermSet.set
    val mk_thm: (Hterm.hterm list * Hterm.hterm) -> hthm
    val thm_eq: hthm -> hthm -> bool
    val pr_hthm : hthm -> unit
    val pp_hthm : System.PrettyPrint.ppstream -> hthm -> unit
  end
```

11.2 The implementation

The functor `HthmFUN` generates a structure for theorems when applied to the correct arguments. The argument structure `Hterm` represents terms, and the argument structure `HtermSet` represents sets of terms.

```
functor HthmFUN (structure Report : Report_sig
                 and Hterm: Hterm_sig and HtermSet: ORD_SET
                 sharing type Hterm.hterm = HtermSet.item
                         and Report = Hterm.Report) : Hthm_sig =
  struct
    structure Report = Report;
    structure Hterm = Hterm;
    structure HtermSet = HtermSet;

    fun THM_ERR{function,message} =
```

```
  Exception.CHK_ERR{origin_structure = "Hthm",
                    origin_function = function, message = message};
```

11.2.1 The type `hthm` The type representing theorems consists of a record of two fields. The field `Hyp` is a set of terms representing the hypotheses. The field `Concl` is a term representing the conclusion.

```
datatype hthm = THM of {Hyp: HtermSet.set, Concl: Hterm.hterm};
```

11.2.2 The constructor The function `mk_thm` creates a theorem from a list of terms as assumptions and a term as conclusion. It checks to make sure all terms in the hypotheses and the conclusion are of boolean type.

```
fun mk_thm (hyplst, concl) =
  if (every Hterm.is_bool_ty (concl::hyplst))
    then
      let
        val hyp =
          fold (fn (i,s) => HtermSet.add(s, i)) hyplst (HtermSet.empty)
      in
        (THM {Hyp=hyp, Concl=concl})
      end
  else raise (THM_ERR {function="mk_thm",
                       message="terms are not all of boolean type"});
```

11.2.3 The destructors The functions `concl` and `hyp` extract the conclusion and the hypotheses of their argument theorem, respectively.

```
fun concl (THM {Concl,...}) = Concl
and hyp (THM {Hyp,...}) = Hyp
```

11.2.4 The comparator The theorems are equal if and only if their conclusions are identical and their sets of hypotheses are equivalent. As `equal` compares the elements of the sets, corresponding hypotheses must be identical. α-equivalent theorems are considered not equal by this function.

```
fun thm_eq (THM{Concl=con1,Hyp=hyp1}) (THM{Concl=con2,Hyp=hyp2}) =
  (con1 = con2) andalso (HtermSet.equal(hyp1, hyp2))
```

11.2.5 Output function The function `pr_hthm` outputs a theorem to the default output stream in the proof file format.

```
local
  open Report
in
  fun pr_hthm (THM {Hyp, Concl}) =
    (write_opening(Keyword.THM);
     write_list Hterm.pr_hterm (HtermSet.listItems Hyp);
     Hterm.pr_hterm Concl;
     write_closing_line())
end
```

11.2.6 Pretty Printer The function `pp_hthm` is a pretty printer for theorems. It outputs its second argument to the pretty printing stream given as the first argument. The conclusion of a theorem is always prefixed by the turnstile `|-`. The hypotheses are enclosed in square brackets and are printed before the turnstile if they occur. The individual hypotheses and the conclusion are printed by the term pretty printer.

```
  local
    open System.PrettyPrint;
    fun with_ppstream ppstrm =
      {add_string = add_string ppstrm,
       add_break = add_break ppstrm,
       begin_block = begin_block ppstrm,
       end_block = fn () => end_block ppstrm,
       flush_ppstream =fn () => flush_ppstream ppstrm};
  in
    fun pp_hthm ppstrm (THM {Hyp,Concl}) =
      let
        val {add_string, add_break,
             begin_block, end_block, flush_ppstream} =
          with_ppstream ppstrm;
        val pp_hterm = Hterm.pp_hterm ppstrm
        and pp_qterm = Hterm.pp_qterm ppstrm
        fun pp_hyp [] = ()
          | pp_hyp terml =
            let
              fun p_hyp [] = ()
                | p_hyp [tm] = pp_qterm tm
                | p_hyp (tm::(tml as (x::l))) =
                  (pp_qterm tm;
                   add_string ";"; add_break (1,1);
                   p_hyp tml)
            in
              (begin_block CONSISTENT 0;
               add_string"["; p_hyp terml;
               add_string"]";
               end_block ())
            end
        val hyps = HtermSet.listItems Hyp
      in
        (begin_block CONSISTENT 0;
         pp_hyp hyps;
         add_break (1,1); add_string "|- ";
         pp_hterm Concl;
         end_block ())
      end
  end(* local *)
end; (* functor HthmFUN *)
```

12 HOL terms

The `Hterm` module is the abstraction of HOL terms. It is created by applying the functor `HtermFUN`.

12.1 The specification

The signature `Hterm_sig` is the specification of the term structure. It contains the type structure `Htype` and the report generation module `Report`.

```
signature Hterm_sig =
  sig
    structure Report : Report_sig
    structure Htype : Htype_sig
```

12.1.1 Types The type `hterm` represents HOL terms. It admits equality so two terms are considered equal if their representations are the same. There are four kinds of primitive terms: variables, constants, λ-abstractions and combinations (also known as function applications). The actual representation of different kinds of terms within the type `hterm` is local to this module.

```
    eqtype hterm
```

12.1.2 Constructors and destructors The following functions are provided for constructing, destructing and testing primitive terms. All constructors have prefix `mk_`. They all take a pair as their argument. All destructors have prefix `dest_`. They all return a pair. All term testers have prefix `is_`.

```
    val mk_var : string * Htype.htype -> hterm
    val mk_const : string * Htype.htype -> hterm
    val mk_comb : hterm * hterm -> hterm
    val mk_abs : hterm * hterm -> hterm

    val dest_var : hterm -> string * Htype.htype
    val dest_const : hterm -> string * Htype.htype
    val dest_comb : hterm -> hterm * hterm
    val dest_abs : hterm -> hterm * hterm

    val is_var : hterm -> bool
    val is_const : hterm -> bool
    val is_comb : hterm -> bool
    val is_abs : hterm -> bool
```

12.1.3 More destructors and testers These are destructors for higher level terms, i.e., for terms of certain common structures, for instance equalities. They follow the naming convention described above.

```
    val dest_eq : hterm -> hterm * hterm
    val dest_forall : hterm -> hterm * hterm
```

```
val dest_exists : hterm -> hterm * hterm
val dest_imp : hterm -> hterm * hterm
val dest_not : hterm -> hterm
val dest_conj : hterm -> hterm * hterm
val is_eq : hterm -> bool
val is_conj : hterm -> bool
```

Listed below are special destructors. `rev_dest_eq` is the same as `dest_eq` except the order of the fields returned is reversed. The destructors whose names have the prefix `strip_` are the repetitive version of their simple counterparts.

```
val rev_dest_eq : hterm -> (hterm * hterm)
val strip_comb : hterm -> (hterm * hterm list)
val strip_forall : hterm -> (hterm list * hterm)
val strip_exists : hterm -> (hterm list * hterm)
val strip_conj : hterm -> hterm list
val lhs : hterm -> hterm
val rhs : hterm -> hterm
```

12.1.4 Term comparison The function `cmpTerm` is an ordering function for terms. The expression `cmpTerm` $tm_1 tm_2$ evaluates to `Equal` if tm_1 and tm_2 are exactly the same. Otherwise, it is either `Less` or `Greater` depending on their kinds and names.

```
val cmpTerm :(hterm * hterm) -> LibBase.relation
```

12.1.5 Type of a term This group of functions deal with types of a term. When applied to a term tm, the function `type_of` returns the type of tm. The expression `tyvars` tm evaluates to a list of distinct type variables occured in tm. The function `tyvarsl` is similar to `tyvars` but it takes a list of terms.

The expression `type_in` ty tm evaluates to `true` if tm or any of its subterms has the type ty.

The expression `vty_occurs` vty tm evaluates to `true` if the type variable vty occurs in any subtype of any subterm of tm. This is a combination of the functions `type_in` and `type_in_type` in `Htype` with the restriction that the type to be checked must be a type variable.

```
val type_of : hterm -> Htype.htype
val tyvars : hterm -> Htype.htype list
val tyvarsl : hterm list -> Htype.htype list
val type_in : Htype.htype -> hterm -> bool
val vty_occurs : Htype.htype -> hterm -> bool
```

12.1.6 α-conversion The predicate `aconv` takes two terms as its arguments. It returns `true` if they are α convertible from each other.

```
val aconv : hterm -> hterm -> bool
```

12.1.7 Type instantiation The function `term_is_type_inst` tests two terms to see whether one is the result of type-instantiating the other. The first argument is a list of type pairs specifying the type instantiations.

```
val term_is_type_inst : (Htype.htype * Htype.htype)list ->
  hterm -> hterm -> bool
```

The function `term_is_tyty_inst` is a bounded version which takes an extra argument of a list of variable pairs. This list specifies the variables renamed when the term is instantiated.

```
val term_is_tyty_inst : (Htype.htype * Htype.htype)list ->
  (hterm * hterm) list -> hterm -> hterm -> bool
```

The function `term_inst_renames` returns the list of variable pairs which are renamed in the instantiation.

```
val term_inst_renames : (Htype.htype * Htype.htype)list ->
  hterm -> hterm -> (hterm * hterm)list
```

The function `term_inst_chk` takes an extra list of variables. These are assumed to be the free variables in the assumptions of a theorem. Any variables occurring in this list should not be renamed. The new variables should not be the same as any of these variables.

```
val term_inst_chk : (Htype.htype * Htype.htype)list ->
  (hterm)list -> hterm -> hterm -> bool
```

12.1.8 Free and bound variables The function `frees` returns a list of the free variables in the input term. The function `freesl` is similar but takes a list of terms. The resulting list is treated as a set, i.e., every free variable appears only once in the list if it occurs one or more times in the term.

The expression `free_in` $tm_1\ tm_2$ evaluates to true if tm_1 occurs free in tm_2 where tm_1 must be a variable. (Note that this is different from the function having the same name in HOL88.)

The expression `variant` $vl\ x$ returns a variable which is a variant of x and is different from any of the variables in the list vl. The variant is created by appending a prime($'$) to the name of x.

```
val frees : hterm ->hterm list
val freesl : hterm list -> hterm list
val free_in : hterm -> hterm -> bool

val variant : hterm list -> hterm -> hterm
```

12.1.9 Substitutions A substitution of a term $tm[t_1, \ldots, t_n]$ is a term

$$tm[t'_1, \ldots, t'_n/t_1, \ldots, t_n]$$

in which free occurrences of t_i are replaced by t'_i for $i = 1, \ldots, n$. The type of t'_i must be the same as t_i. If free occurrences of variables in t'_i will be captured, the corresponding bound variables will be renamed. A substitution can be specified as a pair (t'_i, t_i).

A list of substitutions $[(t'_1, t_1), \ldots, (t'_n, t_n)]$ is consistent if and only if

1. all the t_i's are distinct, or
2. if $t_i = t_j$, then $t'_i = t'_j$.

The function `term_subst_chk` checks the correctness of a substitution. It takes four arguments in the following order:

- a *substitution list* whose elements are triples, $((t_i, x_i), v_i)$, specifying the substitutions;
- a *template* which specifies the positions where substitutions are performed;
- a *substituted term* which is the result of the substitutions;
- an *original term* which is to be substituted.

Positions of substitutions are indicated in the template by the occurrences of dummy variable v_i. The corresponding positions of the original term should be free occurrences of the variable x_i. They are replaced by the term t_i in the substituted term. The function `term_subst_chk` returns `true` if the substituted term is a consistent substitution according to the above description. Otherwise, it returns `false`.

```
val term_subst_chk : ((hterm * hterm) * hterm)list ->
    hterm -> hterm -> hterm -> bool
```

The function `mk_subs_templ` is used in the `check` module. They generate a substitution template..

```
val mk_subs_templ : ((hterm * hterm) * hterm) list -> hterm -> hterm
val mk_subs_occu_templ : hterm -> hterm -> hterm -> int -> int list
  -> (hterm * int * int list)
```

12.1.10 Generating unique variables The function `mk_gen_var` creates a unique variable every time it is called. This is used when the checker needs an internal general variable, e.g., for renaming.

```
val mk_gen_var : Htype.htype -> hterm
```

12.1.11 Special types Since it is frequently required to check whether a term has boolean type, the function `is_bool_ty` is defined for this purpose. Similarly, the function `is_num_ty` returns `true` if applied to a term whose type is `num`.

```
val is_bool_ty : hterm -> bool
val is_num_ty : hterm -> bool
```

12.1.12 Output function This function prints a term to the default output stream in the `prf` format.

```
val pr_hterm : hterm -> unit
```

12.1.13 Pretty Printer The function `pp_qterm` is a pretty printer for quoted terms. The function `pp_hterm` is a pretty printer for terms without being enclosed in double quotes. They use the system pretty-printer module to print terms.

```
  val pp_qterm : System.PrettyPrint.ppstream -> hterm -> unit
  val pp_hterm : System.PrettyPrint.ppstream -> hterm -> unit

end; (* signature Hterm_sig *)
```

12.2 The implementation

The functor `HtermFUN` takes three structures: `Report` the report generation module, `Htype` the abstraction of HOL types and their operations and `HtypeCmp` the ordering of HOL types. The last two have to share the type representing HOL types. The `Report` structure should be the same as the `Report` structure in the type module.

```
functor HtermFUN (structure Report : Report_sig
                  and Htype: Htype_sig and HtypeCmp: ORD_KEY
                  sharing type Htype.htype = HtypeCmp.ord_key
                          and Report = Htype.Report) : Hterm_sig =
  struct
    structure Report = Report;
    structure Htype = Htype;
    structure HtypeCmp = HtypeCmp;

    val debug = ref 0;
    fun write_out s = (output(std_err, s); flush_out std_err);
```

```
fun init () = (
               debug := Debug.get_debug "Hterm";
               ());
```

12.2.1 Type for HOL terms The type representing HOL terms is `hterm`. In addition to the four kinds of primitive terms, an extra kind of term `BVAR` is allowed. It represents the bound variable in an abstraction. This follows the de Bruijn's name-free representation[3]. The integer in a `BVAR` is the depth (or level) of λ's between the bound variable and its binder.

```
type atom = {Name: string, Ty: Htype.htype};

datatype hterm = BVAR of int
  | VAR of atom
  | CONST of atom
  | COMB of {Rator: hterm, Rand: hterm}
  | ABS of {Bvar: hterm, Body: hterm};
```

12.2.2 Exception This function generates exception for this module.

```
fun TERM_ERR{function,message} =
  Exception.CHK_ERR{origin_structure = "Hterm",
          origin_function = function, message = message};
```

12.2.3 Ordering function The function `cmpTerm` compares two terms. By convention, variables are `Less` than constants, which are in turn `Less` than function applications, and which are in turn `Less` than λ-abstraction. If two terms are both variables, or constants, their names are compared in alphabetical order. If two terms are both function applications, the operators are compared first. If they are the same, the operands are compared. If two terms are both λ-abstractions, the bound variables are compared first, then the bodies. This function is required to create the `HtermCmp` structure which is used by the ordered-set of terms structure.

```
local open LibBase
in
  fun cmpTerm (tm1, tm2) =
    case tm1 of
      (BVAR n1) =>
        (case tm2 of
           (BVAR n2) =>
             if (n1 = n2) then Equal
             else if (n1 < n2) then Less else Greater
          | _ => Less)
    | (VAR {Name=name, Ty=ty}) =>
        (case tm2 of
           (VAR {Name=name2, Ty=ty2}) =>
             if (name < name2) then Less
             else if (name > name2) then Greater
                  else Htype.cmpType (ty,ty2)
          | _ => Less)
    | (CONST {Name=name, Ty=ty}) =>
        (case tm2 of
           (VAR _) => Greater
          | (CONST {Name=name2, Ty=ty2}) =>
             if (name < name2) then Less
             else if (name > name2) then Greater
```

```
                    else Htype.cmpType (ty,ty2)
           | _ => Less)
      | (COMB {Rator=rat, Rand=rnd}) =>
          (case tm2 of
             (ABS _) => Less
           | (COMB {Rator=rat2, Rand=rnd2}) =>
               let val c = cmpTerm(rat,rat2)
               in
                 if not(c = Equal) then c else cmpTerm(rnd,rnd2)
               end
           |  _ => Greater)
      | (ABS {Bvar=var, Body=body}) =>
          (case tm2 of
             (ABS {Bvar=var2, Body=body2}) =>
               let val c = cmpTerm(var,var2)
               in
                 if not(c = Equal) then c else cmpTerm(body,body2)
               end
           |  _ => Greater)
end ;
```

12.2.4 Term constructors The term constructors create terms like their counterparts in HOL, namely taking the appropriate arguments and returning a term. A special case is the λ term constructor since de Bruijn's name-free representation is used. The function `mk_bind` binds the variable in the body of an abstraction. It is called when a λ term is created. It recursively descends into the body of the abstraction to change the free occurrences of the variable into `BVAR`s. A variable is considered to be the same as the bound variable if its name and its type are identical to those of the bound variable. The integer argument n is incremented when an abstraction is entered.

```
fun mk_var (name, ty) = VAR {Name = name, Ty = ty};
fun mk_const (name, ty) = CONST {Name = name, Ty = ty};
fun mk_comb (rator, rand) = COMB {Rator = rator, Rand = rand};

fun mk_bind (n, (bv as VAR{Name=bvname,Ty=bvty}),
             (v as VAR{Name=name,Ty=ty})) =
    if ((bvname = name) andalso (bvty = ty)) then (BVAR n) else v
  | mk_bind (n, (bv as VAR{Name=bvname,Ty=bvty}),
             (COMB{Rator=rator,Rand=rand})) =
    (COMB{Rator=(mk_bind(n,bv,rator)), Rand=(mk_bind(n,bv,rand))})
  | mk_bind (n, (bv as VAR{Name=bvname,Ty=bvty}),
             (ABS {Bvar=v,Body=body})) =
    (ABS {Bvar=v,Body=(mk_bind((n+1),bv,body))})
  | mk_bind (n, (bv as VAR{Name=bvname,Ty=bvty}), tm as _) = tm
  | mk_bind (n,_,_) =
    raise (TERM_ERR {function="mk_bind",
                     message="not a variable"});;

fun mk_abs (bv as VAR _, body) =
  ABS {Bvar = bv, Body = (mk_bind(0,bv,body))}
  | mk_abs _ = raise TERM_ERR {function = "mk_abs",
                               message = "first term not a variable"};
```

12.2.5 System generated variables The names of all system generated unique variables begin with the string `gv%%` and are followed by a number. The variable `gen_var_count` keeps a count of the number of variable generated.

```
val gen_var_prefix = "gv%%"
and gen_var_count = ref 0
fun mk_gen_var ty =
  (gen_var_count := (!gen_var_count) + 1;
   mk_var((gen_var_prefix^(makestring (!gen_var_count))), ty))
```

12.2.6 Term destructors The function `dest_bind` performs the inverse process of `mk_bind`. It is called by `dest_abs`. It recursively descends into the body of the abstraction and converts the bound variable into a free variable. The integer argument n indicates the depth of the nested λs. If it encounters a bound variable of the same level, it changes it to free variable.

```
fun dest_var (VAR {Name=s, Ty=ty}) = (s,ty)
  | dest_var _ = raise TERM_ERR {function = "dest_var",
                                 message = "not a variable"};
fun dest_const (CONST {Name=s, Ty=ty}) = (s,ty)
  | dest_const _ = raise TERM_ERR {function = "dest_const",
                                   message = "not a constant"};
fun dest_comb (COMB {Rator=ra,Rand=rnd}) = (ra,rnd)
  | dest_comb _ = raise TERM_ERR {function = "dest_comb",
                                  message = "not an application"};

fun dest_bind (n, (bv as VAR{Name=bvname,Ty=bvty}),
               (BVAR n')) =
  if (n = n') then bv else (BVAR n')
  | dest_bind (n, (bv as VAR{Name=bvname,Ty=bvty}),
              (COMB{Rator=rator,Rand=rand})) =
    (COMB{Rator=(dest_bind(n,bv,rator)), Rand=(dest_bind(n,bv,rand))})
  | dest_bind (n, (bv as VAR{Name=bvname,Ty=bvty}),
               (ABS {Bvar=v,Body=body})) =
    (ABS {Bvar=v,Body=(dest_bind((n+1),bv,body))})
  | dest_bind (n, (bv as VAR{Name=bvname,Ty=bvty}), tm as _) = tm
  | dest_bind (n,_,_) =
    raise (TERM_ERR {function="dest_bind",
                     message="not a variable"});

fun dest_abs (ABS {Bvar=v, Body=b}) = (v, (dest_bind(0,v,b)))
  | dest_abs _ = raise TERM_ERR {function = "dest_abs",
                                 message = "not an abstraction"};
```

12.2.7 Term testers The primitive term testers are implemented using the primitive term destructors in the previous section.

```
val is_var = can dest_var;
val is_const = can dest_const;
val is_comb = can dest_comb;
val is_abs = can dest_abs;
```

12.2.8 More destructors and testers There are also destructors for special terms, such as equations, negations, universal and existential quantifiers, implications and conjunctions.

```
fun dest_eq (COMB {Rator=(COMB {Rator=eq,Rand=lhs}), Rand=rhs}) =
```

```
  let
    val (eqname,ety) = dest_const eq
      handle CHK_ERR _ => ("",(Htype.mk_vartype"*"))
  in
    if (eqname = "=") then (lhs,rhs)
    else raise (TERM_ERR {function="dest_eq",
                          message="not an equality"})
  end
  | dest_eq  _ =
    raise (TERM_ERR {function="dest_eq",
                          message="not an equality"});

fun rev_dest_eq tm =
  let val (l,r) = dest_eq tm in (r,l) end
and lhs tm =
  let val (l,r) = dest_eq tm in l end
and rhs tm =
  let val (l,r) = dest_eq tm in r end;

val is_eq = can dest_eq;

fun dest_forall (COMB{Rator=(CONST u),Rand=abs as (ABS{Bvar,Body})}) =
  if ((#Name u) = "!") then dest_abs abs
  else raise (TERM_ERR {function="dest_forall",
                        message="Not a universal quantification"})
  | dest_forall _ =
    raise (TERM_ERR {function="dest_forall",
                     message="Not a universal quantification"});

fun dest_exists (COMB {Rator=(CONST u), Rand=abs as (ABS {Bvar,Body})}) =
  if ((#Name u) = "?") then dest_abs abs
  else raise (TERM_ERR {function="dest_exists",
                        message="Not an existential quantification"})
  | dest_exists _ =
     raise (TERM_ERR {function="dest_exists",
                        message="Not an existential quantification"});

fun dest_imp (COMB {Rator=(COMB {Rator=eq,Rand=ante}), Rand=conc}) =
  let
    val (eqname,ety) = dest_const eq
      handle CHK_ERR _ => ("",(Htype.mk_vartype"*"))
  in
    if (eqname = "==>") then (ante,conc)
    else raise (TERM_ERR {function="dest_imp",
                          message="not an implication"})
  end
  | dest_imp _ =
     raise (TERM_ERR {function="dest_imp",
                          message="not an implication"});

fun dest_not (COMB {Rator=(CONST{Name=name,...}), Rand=body}) =
```

```
  if (name = "~") then body
  else raise (TERM_ERR {function="dest_not",
                        message="not a negation"})
  | dest_not _ =
     raise (TERM_ERR {function="dest_not",
                        message="not a negation"});

local
  fun str_comb (tm,l) =
    if is_comb tm then
      let val (rator,rand) = dest_comb tm
      in
        str_comb (rator, (rand::l))
      end
    else (tm,l)
in
  fun strip_comb tm = str_comb (tm,[])
end;

local
  fun it_right f (l,tm) =
    if (can f tm) then
      let val (fst,snd) = f tm
      in
        it_right f ((fst::l), snd)
      end
    else (l,tm)
in
  fun strip_forall tm =
    let val(vs,b) = it_right dest_forall ([],tm) in ((rev vs),b) end
  fun strip_exists tm =
    let val (vs,b) = it_right dest_exists ([],tm) in ((rev vs),b) end
end;

fun dest_conj (COMB {Rator=(COMB {Rator=conj,Rand=lhs}), Rand=rhs}) =
  let
    val (cname,cty) = dest_const conj
      handle CHK_ERR _ => ("",(Htype.mk_vartype"*"))
  in
    if (cname = "/\\") then (lhs,rhs)
    else raise (TERM_ERR {function="dest_conj",
                          message="not a conjunction"})
  end
  | dest_conj  _ =
    raise (TERM_ERR {function="dest_conj",
                          message="not a conjunction"})

val is_conj = can dest_conj

fun strip_conj term =
  let
```

```
    fun stripc tm lrl =
      if (is_conj tm) then
        let
          val (l,r) = dest_conj tm
        in
          stripc l (stripc r lrl)
        end
      else (tm :: lrl)
  in
    stripc term []
  end
```

12.2.9 Free variables The task of finding free occurrences of variables is made simple by the name-free representation. All we need to do is to recursively descend into the subterms to pick up all the `VAR` nodes since bound occurrences are represented by `BVAR`.

The local function `freevars` does the actual job of finding free variables. Its second argument is used to cumulate the free variables found in the term. This improves the performance.

```
local
  fun freevars ((tm as VAR _), l) = tm :: l
    | freevars ((COMB{Rator,Rand}), l) =
        (freevars (Rator, (freevars (Rand, l))))
    | freevars ((ABS{Body,...}), l) = freevars (Body, l)
    | freevars (_, l) = l
in
  fun frees tm = setify (freevars (tm, []))
  and freesl tms = setify (fold freevars tms [])
  and free_in (v as VAR _) tm = mem v (freevars (tm, []))
    | free_in _ tm =
      raise (TERM_ERR{function="freein",
                      message="argument not a variable"})
end;
```

12.2.10 Variant of variable This function generates a variant of the given variable by appending prime($'$) on to it. The new variant is tested against the list vl. More primes are added if required.

```
fun variant [] x = x
  | variant vl (x as VAR{Name,Ty}) =
    if (mem x vl) then variant vl (VAR{Name=(Name^"'"),Ty=Ty})
    else x
  | variant vl _ = raise (TERM_ERR {function="variant",
                                    message="not a variable"});
```

12.2.11 Type of a term The types of variables and constants are carried in them explicitly. The types of combinations and abstractions are worked out dynamically. The operator of a combination must be of a function type. If it is not, `dest_funtype` will fail. The type of the operand must be the same as the domain of the operator. An exception is raised if this is not the case. The type of an abstraction is guaranteed to be a function type. Its domain is the type of the bound variable, and its range is the type of the body. `dest_abs` is called to convert the bound occurrences of the variable into free occurrences before the type of the body can be worked out.

```
fun type_of (VAR {Ty,...}) = Ty
```

```
  | type_of (CONST {Ty,...}) = Ty
  | type_of (COMB {Rator,Rand}) =
    let
      val (randty,rty) = (Htype.dest_funtype (type_of Rator))
    in
      if (randty = type_of Rand) then rty
      else raise (TERM_ERR {function="type_of",
                            message="incompatible types in application"})
    end
  | type_of (tm as (ABS _)) =
    let val (bv,body) = dest_abs tm
    in
      Htype.mk_funtype ((type_of bv), (type_of body))
    end
  | type_of _ = raise (TERM_ERR {function="type_of",
                                 message="type_of bound variable"});
```

12.2.12 Special types testers These functions return `true` if the term is of type *bool* or *num*, respectively.

```
fun is_bool_ty tm = ((type_of tm) = Htype.bool_ty)
and is_num_ty tm = ((type_of tm) = Htype.num_ty)
```

12.2.13 Type variables in term The function `tyvars`, when applied to a term, returns a list of the distinct type variables occurring in the term. These include the type variables occurring in any subterm of the input term. The resulting list is like a set, i.e., no repeated variables.

The function `tyvarsl` is similar but taking a list of terms.

```
local
  fun ty_vars ((VAR {Ty,...}),vl) = union (Htype.type_tyvars Ty) vl
    | ty_vars ((CONST {Ty,...}),vl) = union (Htype.type_tyvars Ty) vl
    | ty_vars ((ABS {Bvar,Body}),vl) =
      union (Htype.type_tyvars (type_of Bvar)) (ty_vars(Body,vl))
    | ty_vars ((COMB {Rator,Rand}),vl) = ty_vars(Rand,(ty_vars(Rator,vl)))
    | ty_vars (_,vl) = vl
in
  fun tyvars tm = ty_vars(tm, [])
  and tyvarsl tml = fold ty_vars tml []
end;
```

12.2.14 Type in a term The function `type_in` returns true if its first argument *ty* is the type of the term or the type of any of its proper subterm. If the term is a combination, its type is the range of the operator. We find the types of the operator and the operand separately. If *ty* is equal to the range of the operator, we require the type of the operand to be the same as the domain of the operator. We also find the types of the bound variable and the body separately if the term is an abstraction.

```
fun type_in ty (VAR {Ty,...}) = (ty = Ty)
  | type_in ty (CONST {Ty,...}) = (ty = Ty)
  | type_in ty (COMB {Rator, Rand}) =
    let val (aty,rty) = Htype.dest_funtype (type_of Rator)
        and ranty = type_of Rand
    in
```

```
          ((ty = rty) andalso (rty = ranty)) orelse
          (type_in ty Rator) orelse (type_in ty Rand)
        end
      | type_in ty (tm as ABS _) =
        let val (bv,body) = dest_abs tm
            val bty = type_of body and vty = type_of bv
        in
          (ty = Htype.mk_funtype(vty,bty)) orelse
          (ty = vty) orelse (ty = bty) orelse
          (type_in ty body)
        end
      | type_in ty _ = false;
```

12.2.15 Occurrence of type variables The expression `vty_occurs` *vty tm* evaluates to true if the type variable *vty* occurs in any subtype of any subterm of *tm*. This is implemented by first finding all type variables occurring in the type of the term or the types of any subterms using `tyvars`; and then checking whether *vty* is one of these type variables.

```
fun vty_occurs vty tm =
  if (Htype.is_vartype vty) then mem vty (tyvars tm)
  else raise (TERM_ERR {function="vty_occurs",
                        message="not a type variable"});
```

12.2.16 Checking type instantiation The function `term_is_type_inst` calls the local function `is_ty_inst` to perform the type instantiation checking. The local function takes, as its arguments, two terms tm_1 and tm_2, a list of type pairs *tytyl* and a list of pairs variables *nl*. It checks to see if tm_1 is an instance of tm_2 according to *tytyl*. The pairs of variables in *nl* are the renaming found in the checking. The function will raise an exception if inconsistent renaming is found, i.e., two occurrences of a variable are renamed differently or two distinct variables become identical.

The checking uses the fact that the structure of term trees of tm_1 and tm_2 must be the same if one is a type instantiation of the other, i.e., the corresponding nodes are of the same kind. The complication due to renaming only occurs at the nodes which are free variables. There are five different cases corresponding to the five kinds of primitive terms:

- For bound variable nodes, they must be at the same level of nested abstraction.
- For constant nodes, they must be the same constant and their types must be a consistent instantiation according to the list *tytyl*.
- For free variable nodes, their names must be the same and their types must be a consistent instantiation according to the list *tytyl* if renaming does not occur. Otherwise, the renaming is checked against the list *nl*. This is performed by the local function `chk_rename`. It takes, as arguments, the pair of variables (v_1, v_2) which are the current nodes, and the list *nl* which contains all the renaming found so far. If v_2 has not been seen before, then v_1 must not have been seen either. Then, the pair (v_1, v_2) is added into *nl*. If v_2 is not seen before but v_1 is found in the list *nl*, we have a renaming that makes two variables identical. The function returns `false`. If v_2 is found in *nl*, then v_1 must be the same as the first field of the pair found in the list, otherwise, we find an inconsistent renaming. In the latter case, the function returns `false`.
- For `COMB` nodes, we check the `rator` (left) subtree first. If it is OK, we check the `rand` (right) subtree.

- For ABS nodes, the type $ty1$ of a bound variable in tm_1 must be a consistent instantiation of $ty2$. This variable may be renamed, but it does not matter what it is renamed to as long as the name is not the name of any free variable in the body. The renaming does not need to be recorded in nl.

```
local
  open Htype ListUtil
  fun chk_rename (v1,v2) nl =
    let
      val found1 = ListUtil.findOne (fn (v,u)=> (v2 = u)) nl
    in
      case found1 of
        NONE =>
          let val found2 = ListUtil.findOne (fn (v,u)=> (v1 = v)) nl
          in
            case found2 of
              NONE => (true, ((v1,v2)::nl))
            | SOME _ => (* renaming makes two variables identical *)
                (false, nl)
          end
      | SOME(v,u) => (* variable already seen *)
          ((v1 = v), nl)
    end

  fun is_ty_inst (BVAR n1) (BVAR n2) tytyl nl =
      ((n1 = n2), tytyl, nl)
    | is_ty_inst (CONST{Name=n1,Ty=ty1}) (CONST{Name=n2,Ty=ty2}) tytyl nl =
      (((n1 = n2) andalso (is_tyty_inst tytyl ty1 ty2)), tytyl, nl)
    | is_ty_inst (v1 as VAR{Name=n1,Ty=ty1})
                 (v2 as VAR{Name=n2,Ty=ty2}) tytyl nl =
      if (is_tyty_inst tytyl ty1 ty2) then
        let val (nameok, nl') = chk_rename (v1,v2) nl
        in
          (nameok, tytyl, nl')
        end
      else (false, tytyl, nl)
    | is_ty_inst (COMB{Rator=rat1,Rand=ran1})
                 (COMB{Rator=rat2,Rand=ran2}) tytyl nl =
      let val (rat,tytyl',nl') = is_ty_inst rat1 rat2 tytyl nl
      in
         if rat then is_ty_inst ran1 ran2 tytyl' nl'
         else (false, tytyl, nl)
      end
    | is_ty_inst (ABS{Bvar=(v1 as VAR{Name=bv1,Ty=ty1}),Body=bo1})
                 (ABS{Bvar=(VAR{Name=bv2,Ty=ty2}),Body=bo2}) tytyl nl =
      if ((is_tyty_inst tytyl ty1 ty2) andalso
          ((bv1 = bv2) orelse (not (free_in v1 bo1))))
        then  (is_ty_inst bo1 bo2 tytyl nl)
      else (false, tytyl, nl)
    | is_ty_inst tm1 tm2 tytyl nl =
      raise (TERM_ERR{function="is_ty_inst",
```

```
                        message="term structures are different"})
in
```

Here comes the type instantiation checking function and its variants.

```
  fun term_is_type_inst tytyl tm1 tm2 =
    let
      val (OK, ttl, nnl) = is_ty_inst tm1 tm2 tytyl []
    in
      OK
    end
  fun term_is_tyty_inst tytyl nl tm1 tm2 =
    let
      val (OK, ttl, nnl) = is_ty_inst tm1 tm2 tytyl []
    in
      OK andalso ((length nl) = (length nnl))
    end
  fun term_inst_renames tytyl tm1 tm2 =
    let
      val (OK, ttl, nnl) = is_ty_inst tm1 tm2 tytyl []
    in
      ListUtil.remove (fn (v,u) => (v = u)) nnl
    end
  fun term_inst_chk tytyl vl tm1 tm2 =
    let
      val (OK, ttl, nnl) = is_ty_inst tm1 tm2 tytyl []
      val (nl,vl') = unzip(remove (fn (v,u) => (v = u)) nnl)
    in
      (OK andalso (null(intersect vl vl')) andalso
       (null(intersect vl nl)))
    end
end
```

12.2.17 **α-conversion** Two terms are α equivalent if and only if

1. they are of the same kind of term; and
2. all corresponding subterms are α equivalent. This can be divided into the following cases:
   - if they are combinations the operators and the operands are α equivalent, respectively;
   - if they are abstractions, the bound variables must be of the same type and the bodies must be α equivalent;
   - otherwise they must be the same constant, or the same free variable (the same in name and type), or they must both be bound variables associated with the bounder of the same level, i.e., with the same depth index number.

```
fun aconv (COMB{Rator = M1, Rand = M2}) (COMB{Rator=N1,Rand=N2}) =
  aconv M1 N1 andalso aconv M2 N2
  | aconv (ABS{Bvar=VAR{Ty=ty1,...}, Body = body1})
          (ABS{Bvar=VAR{Ty=ty2,...}, Body = body2}) =
          (ty1=ty2) andalso (aconv body1 body2)
  | aconv tm1 tm2 = (tm1=tm2);
```

12.2.18 Pretty Printer The pretty printer prints a term to the pretty printing stream given as the first argument. No special syntactic treatment is performed on the term since the checker knows nothing about special syntactic status of constants, such as infix or binder status. Although it is entirely primitive, a term printed by the pretty printer is a legal HOL input term, i.e., it is accepted by the HOL system parser. Therefore, one can cut a pretty printed term from the checker and paste it to HOL. The latter will print the term in the proper infix and binder form.

```
local
  open System.PrettyPrint;
  fun with_ppstream ppstrm =
    {add_string = add_string ppstrm,
     add_break = add_break ppstrm,
     begin_block = begin_block ppstrm,
     end_block = fn () => end_block ppstrm,
     flush_ppstream =fn () => flush_ppstream ppstrm};
in
  fun pp_hterm ppstrm tm =
    let
      val {add_string, add_break,
           begin_block, end_block, flush_ppstream} =
        with_ppstream ppstrm;
      val pp_htype = Htype.pp_htype ppstrm;
      fun pp_term (VAR {Name,...}) = add_string Name
        | pp_term (CONST {Name,...}) =
          (if not(CType.isAlphaNum(Name,0))
             then add_string"$" else ();
           add_string Name)
        | pp_term (COMB {Rator,Rand}) =
          (begin_block CONSISTENT 0;
           add_string "(";
           pp_term Rator; add_break (1,1);
           pp_term Rand;
           add_string ")";
           end_block ())
        | pp_term (ABS {Bvar,Body}) =
          let val btm = dest_bind(0,Bvar,Body)
          in
            (begin_block CONSISTENT 0;
             add_string "(\\";
             pp_term Bvar;
             add_string "."; add_break(1,2);
             pp_term btm;
             add_string")";
             end_block ())
          end
        | pp_term _ =
          raise (TERM_ERR {function="pp_term",
                           message="Unknown term"})
    in
      (begin_block CONSISTENT 0;
       pp_term tm; add_break(1,1);
```

```
          pp_htype (type_of tm);
          end_block ()) handle e => (flush_ppstream();
                                     clear_ppstream ppstrm; raise e)
      end
    and pp_qterm ppstrm tm =
      let
        val {add_string, add_break,
             begin_block, end_block, flush_ppstream} =
          with_ppstream ppstrm;
      in
        (begin_block CONSISTENT 0;
         add_string "\"";
         pp_hterm ppstrm tm;
         add_string "\"";
         end_block ()) handle e => (flush_ppstream();
                                     clear_ppstream ppstrm; raise e)
      end

  end(* local *)
```

12.2.19 Substitutions The local function `sublst_chk` checks the consistency of the substitution list *subl* which is in the following form: $[\ldots,((t_i,t'_i),v_i),\ldots]$. The v_i's must all be variables. Their types must be the same as the corresponding t_i and t'_i.

```
  local
    open ListUtil

    fun sublst_chk [] = true
      | sublst_chk (((t, t'), (VAR{Ty,...}))::l) =
        (Ty = (type_of t)) andalso (Ty = (type_of t')) andalso (sublst_chk l)

      | sublst_chk (((t, t'), _)::l) =
        raise (TERM_ERR{function="sublst_chk",
                        message="substitution list contains non-variable"})
```

The local function `mk_sublst` converts the substitution list *subl* into a pair of lists of pairs. The second fields of the list elements are the dummy variables v_i. The first fields of the first list are the terms or variables to be replaced t_i. The first fields of the second list are the new terms t'_i.

```
    and mk_sublst ttvl =
      let val (l,vl) = unzip ttvl
        val (tl, tl') = unzip l
      in
        (zip(tl',vl), zip(tl,vl))
      end
```

The local function `is_tm_subst` checks two terms tm_1 and tm_2 to see if the first is a substitution of the second according to the list *sublst*. The elements of the list are pairs of the form (tm_i, v_i) where the v_i's are variables and the tm_i's are terms to replace the free occurrences of the corresponding v_i in tm_2.

The term trees should be the same except where substitutions occur and bound variables are renamed. Substitutions only occur at the `VAR` nodes of tm_2. For any `VAR` nodes v in tm_2, if there is a pair (t, v) in the list *sublst*, then the corresponding node in tm_1 must be the same as t. For bound variables and constants, the corresponding nodes in the two terms must be the same. For function applications, the left subtrees are checked, then the right subtrees. For abstraction, only the bodies are checked against each other. The bound variable may have been renamed. Their names can be different, but their types should be identical.

```
fun is_tm_subst tm1 (v as VAR{Name,Ty}) sublst =
  let
    val found = findOne (fn (t,v') => (v = v')) sublst
  in
    case found of
      NONE => (tm1 = v)
    | SOME (t,v') => (aconv tm1 t)
  end
  | is_tm_subst (BVAR n1) (BVAR n2) sublst = (n1 = n2)
  | is_tm_subst (c1 as CONST{...})
                (c2 as CONST{...}) sublst = (c1 = c2)
  | is_tm_subst (COMB{Rator=rat1,Rand=ran1})
                (COMB{Rator=rat2,Rand=ran2}) sublst =
      (is_tm_subst rat1 rat2 sublst) andalso
      (is_tm_subst ran1 ran2 sublst)
  | is_tm_subst (ABS{Bvar=(VAR{Ty=ty1,...}),Body=bo1})
                (ABS{Bvar=(VAR{Ty=ty2,...}),Body=bo2}) sublst =
      (ty1 = ty2) andalso (is_tm_subst bo1 bo2 sublst)
  | is_tm_subst tm1 tm2 sublst = false
```

The user function `term_subst_chk` returns `true` if tm_1 is a substitution of tm_2 according to the template *tmpl* and the substitution list *ttvl*. It first verifies the substitution list by calling the local function `sublst_chk`. It then calls `is_tm_subst` to check whether tm_1 is a substitution of the template and also tm_2 is a substitution of the template.

```
in
  fun term_subst_chk ttvl tmpl tm1 tm2 =
    if (sublst_chk ttvl) then
      let
        val (sublst1,sublst2) = mk_sublst ttvl
        fun out_ppstream outstrm n =
          System.PrettyPrint.mk_ppstream{linewidth = n,
              flush = fn () => flush_out outstrm,
              consumer = outputc outstrm}
        val pstrm = out_ppstream std_err 78
      in
       if (is_tm_subst tm1 tmpl sublst1)
       then if (is_tm_subst tm2 tmpl sublst2)
            then true
            else raise (TERM_ERR{function="term_subst_chk",
                                 message="second check failed"})
       else(pp_qterm pstrm (#1(hd sublst1));
            pp_qterm pstrm (#2(hd sublst1));
            pp_qterm pstrm tm1;
            pp_qterm pstrm tmpl;
            System.PrettyPrint.flush_ppstream pstrm;
            raise (TERM_ERR{function="term_subst_chk",
                            message="first check failed"}))
```

```
        end
      else false
  end
```

12.2.20 Create template for substitution The function `mk_subs_occu_templ` builds a template for use in the substitution checking function. It takes five arguments: t is the original term to which the substitution is done. x is the variable to be replaced. v is a new variable which indicates where the required substitutions are. It should not occur anywhere in t. *seen* is the number of times x is encountered during the left-to-right scan of t. l is a list of numbers indicating the occurrences of x in t where the substitutions are required. This list should be sorted in ascending order.

It returns a triple. The first is the template. The second is the number of times x is found. The third is the list of occurrences.

The algorithm of this function is to perform a preorder scan on the term t while recording the number of times x is found. When the occurrence matches the required one in the list l, x is replaced by the new variable v. This should only occur at a `VAR` node.

```
fun mk_subs_occu_templ t x v seen [] = (t, seen, [])
  | mk_subs_occu_templ (COMB {Rator,Rand}) x v seen l =
    let
      val (tm',s, nl) = mk_subs_occu_templ Rator x v seen l
      val (tm'', s', nl') = mk_subs_occu_templ Rand x v s nl
    in
      (COMB{Rator=tm',Rand=tm''}, s', nl')
    end
  | mk_subs_occu_templ (ABS{Bvar,Body}) x v seen l =
    let
      val (tm,s,nl) = mk_subs_occu_templ Body x v seen l
    in
      (ABS{Bvar=Bvar,Body=tm}, s, nl)
    end
  | mk_subs_occu_templ (t as VAR{Name,Ty}) x v seen l =
    if (t = x) then
      let val seen' = seen + 1
      in
        if (seen' = (hd l)) then (v, seen', (tl l))
        else (t, seen', l)
      end
    else (t, seen, l)
  | mk_subs_occu_templ t x v seen l = (t, seen, l)
```

The function `mk_subs_templ` is similar to the previous one except it scans the term to find all free occurrences of variables specified in the substitution list `sub_list`.

```
fun mk_subs_templ [] tm = tm
  | mk_subs_templ sub_list tm =
    (
    case ListUtil.findOne (fn ((t,t'),v) => (t = tm)) sub_list of
      SOME((t,t'),v) => ( v)
    | NONE =>
        case tm of
          (COMB{Rand,Rator}) =>
            let
```

```
              val rand  = mk_subs_templ sub_list Rand
              and rator = mk_subs_templ sub_list Rator
            in
              (COMB{Rand=rand,Rator=rator})
            end
        | (ABS{Bvar,Body}) =>
            (ABS{Bvar=Bvar,Body=mk_subs_templ sub_list Body})
        | _ => tm)
```

12.2.21 Output function The function `pr_hterm` outputs a term to the default output stream in the `prf` format.

```
    local
      open Report
    in
      fun pr_hterm (VAR {Name,...}) = write_sitem(Keyword.VAR,Name)
        | pr_hterm (CONST {Name,...}) = write_sitem(Keyword.CONST,Name)
        | pr_hterm (COMB {Rator,Rand}) =
          (Io.write_output_string(Keyword.LP ^ Keyword.APP ^" ");
           pr_hterm Rator; pr_hterm Rand;
           Io.write_output_string Keyword.RP)
        | pr_hterm (ABS {Bvar,Body}) =
          let
            val btm = dest_bind(0,Bvar,Body)
          in
            Io.write_output_string(Keyword.LP ^ Keyword.APP ^" ");
            pr_hterm Bvar; pr_hterm  btm;
            Io.write_output_string Keyword.RP
          end
    end

  end; (* functor Hterm *)
```

12.3 Structure HtermCmp

This structure encapsulates the term ordering function `cmpTerm`. It is used to create ordered sets of terms.

```
functor HtermCmpFUN(Hterm : Hterm_sig) : ORD_KEY =
  struct

    type ord_key = Hterm.hterm

    fun cmpKey (tm1,tm2) = Hterm.cmpTerm (tm1, tm2)
  end;
```

13 HOL types

The structure `Htype` is the abstraction for HOL types. It defines a type `htype` for representing HOL types. There are a number of constructors and destructors. Their names are the same as their counterpart in the HOL system if they perform the same operation.

13.1 The specification

The signature `Htype_sig` is the specification of the type structure.

```
signature Htype_sig =
 sig
   structure Report : Report_sig
```

13.1.1 Types The type `htype` represents HOL types. It admits equality as two types are equal if and only if their representations are the same. There are two kind of HOL types: type variables and type operators. Although atomic types and function types are treated separately in Section 15.2 of [5], they are treated as special cases of type operators. Atomic types are type operators taking no arguments, i.e., having an arity of 0. Function types are type operators having name `fun` and arity of 2.

```
  eqtype htype
```

13.1.2 Constructors and destructor The functions `mk_type` and `mk_vartype` return a type operator and a type variable, respectively. The string argument to these functions is the name of the type. The type list argument of `mk_type` is the list of argument types of the type operator.

```
   val mk_type: {Tyargs:htype list, Tyop:string} -> htype
   val mk_vartype: string -> htype
```

The function `dest_type` and `dest_vartype` return the constituent part(s) of the type. If an incorrect kind of type is supplied to these functions, they will raise an exception.

```
   val dest_type : htype -> {Tyargs:htype list, Tyop:string}
   val dest_vartype: htype -> string
```

As function types are used very often, a constructor and a destructor are provided for creating and taking apart function types. Their names are `mk_funtype` and `dest_funtype`, respectively. The function `domain_of` takes a function type and returns the type of its domain.

```
   val mk_funtype: (htype * htype) -> htype
   val dest_funtype : htype -> (htype * htype)
   val domain_of : htype -> htype
```

Two predicates `is_vartype` and `is_funtype` are provided for testing whether a type is a type variable or a function type respectively.

```
   val is_vartype : htype -> bool
   val is_funtype : htype -> bool
```

13.1.3 Type comparison The function `cmpType` is an ordering function for types. The expression `cmpType` $ty_1\,ty_2$ evaluates to `Equal` if ty_1 and ty_2 are exactly the same. Otherwise, it is either `Less` or `Greater` depending on their kinds and names. Type variables are `Less` than type operators. Types of the same kind are ordered according to their names. If they are of the same type operator, the order depends on their arguments.

```
val cmpType : (htype * htype) -> LibBase.relation
```

13.1.4 Getting type variables The expression `type_tyvars` ty evaluates to a list of all distinct type variables occurring in the type ty. The function `ty_tyvarsl` is similar except it takes a list of types.

```
val type_tyvars: htype -> htype list
val type_tyvarsl: htype list -> htype list
```

13.1.5 Test for sub-types The expression `type_in_type` $ty_1\,ty_2$ evaluates to `true` if ty_1 is equal to ty_2 or any sub-type of it.

```
val type_in_type : htype -> htype -> bool
```

13.1.6 Special types Some types are used very frequently. Symbolic names are bound to these types for convenience.

```
val bool_ty : htype
val ty_is_bool : htype -> bool
val num_ty : htype
val ty_is_num : htype -> bool
```

13.1.7 Type instantiation The expression `is_type_inst` $ty_1\,ty_2$ evaluates to `true` if ty_1 is an instance of ty_2. An instantiation of a type σ is defined as

$$\sigma' = \sigma[\tau_1, \ldots, \tau_n/\beta_1, \ldots, \beta_n]$$

where τ_i for $i = 1, \ldots, n$ are types and β_i are type variables in σ. σ' is the result of simultaneous substitution of τ_i for β_i in σ.

The function `is_tyty_inst` is a bounded version of `is_type_inst`. It takes an extra argument of type `(htype * htype) list`. This specifies the type variables to be instantiated and the corresponding types. The type instantiations must be compatible with this list.

```
val is_type_inst : htype -> htype -> bool
val is_tyty_inst : (htype * htype)list -> htype -> htype -> bool
```

When applied to two types ty_1 and ty_2, the function `type_instl` returns a list of type instantiations in the form

$$[\ldots, (\tau_i, \sigma_i), \ldots]$$

if ty_1 is an instance of ty_2. Otherwise it fails. If the list is null, no instantiation is necessary to unify the two types, i.e., they are exactly the same. The first fields of the list elements τ_i are types. The second fields σ_i are type variables. If τ_i for all i are substituted into ty_2 for corresponding σ_i, the result is ty_1. This Substitution can be performed by the function `inst_type`.

The expression `type_compat` $ty_1 ty_2$ evaluates to `true` if the types ty_1 and ty_2 are compatible. The definition of compatible is that they have similar structure, i.e., the shape of the tree representing the two are the same. A type variable is compatible to any type. Two type operators are compatible if and only if they are the same operator and their arguments are pair-wise compatible.

```
val type_instl : htype -> htype -> (htype * htype) list
```

```
    val inst_type : (htype * htype) list -> htype -> htype
    val type_replace : (htype * string) list -> htype -> htype
    val type_compat : htype -> htype -> bool
```

13.1.8 Output function The function `pr_htype` outputs a type to the default output stream in the proof file format `prf`.

```
  val pr_htype : htype -> unit
```

13.1.9 Pretty Printer The function `pp_htype` is a pretty printer for types. It uses the system pretty-printer module to print types.

```
    val pp_htype : System.PrettyPrint.ppstream -> htype -> unit
  end;
```

13.2 The implementation

The functor `HtypeFUN` implements HOL types which are represented by the `datatype htype`. It has two cases:

1. `TV` represents type variables whose name is the argument string;
2. `TO` represents type operators.

The type operators consist of a record. The `Tyop` field is the name of the operator, and the `Tyargs` is the arguments of the operator which is a list of types. The length of this list is the arity of the operator. The atomic types in HOL are treated as type operators with 0-arity.

Since this representation has the same semantics as the abstract HOL type under equality, it admits equality. However, the type constructors are hidden.

```
functor HtypeFUN (structure Report : Report_sig) : Htype_sig =
  struct
    structure Report = Report

    datatype htype = TV of string
        | TO of {Tyop: string, Tyargs: htype list};
```

13.2.1 Exception This function generates exceptions for this module.

```
    fun TYPE_ERR{function,message} =
      Exception.CHK_ERR{message = message,
                        origin_function = function,
                        origin_structure = "HType"};
```

13.2.2 `cmpType` This function is the ordering function for `htype`. The convention is that if two types are both variables, their names are compared. If two types are of the same type operator, their arguments are paired up and compared by calling the function `cmp_Type_list` which recursively calls `cmpType`. We can stop as soon as a non-equal pair is found.

```
        open LibBase ListUtil
        fun cmpType ((TV s), ty2) =
          (case ty2 of
             (TV s2) =>
               if (s < s2) then Less
               else if (s > s2) then Greater
                    else Equal
           | _ => Less)
```

```
      | cmpType ((TO {Tyop=tyop, Tyargs=tyargs}), ty2) =
        (case ty2 of
           (TO {Tyop=tyop2, Tyargs=tyargs2}) =>
             if (tyop < tyop2) then Less
             else if (tyop > tyop2) then Greater
                  else cmp_type_list (zip(tyargs,tyargs2))
         | _ => Greater)
    and cmp_type_list [] = Equal
      | cmp_type_list ((ty1,ty2)::tys) =
        let val c = cmpType (ty1,ty2)
        in
          if not(c = Equal) then c else cmp_type_list tys
        end
```

13.2.3 `type_tyvars` and `type_tyvarsl` These functions return a list of the type variables occurring in their arguments. The local function `tyvars` does the actual work. The argument `vlist` is used to cumulate the type variables.

```
local
  fun tyvars (v as (TV _)) vlist =
    if (mem v vlist) then vlist else v::vlist
    | tyvars (TO{Tyargs,...}) vlist = tyvarsl Tyargs vlist
  and tyvarsl tylist vlist = rev_itlist tyvars tylist vlist
in
  fun type_tyvars ty = rev(tyvars ty [])
  fun type_tyvarsl tyl = rev(tyvarsl tyl [])
end;
```

13.2.4 Type constructors, destructors and testers These are the interface functions to the datatype `htype`. The real constructors are hidden so that the type appears as an abstract type and its implementation may be changed without affecting the user. Note that an exception is raised if a destructor is applied to the wrong kind of argument.

```
fun mk_type {Tyop, Tyargs} = TO {Tyop=Tyop, Tyargs=Tyargs};
fun mk_vartype s = TV s;
fun mk_funtype (ratty, randty) =
  TO {Tyop = "fun", Tyargs=[ratty, randty]};

fun dest_type (TO r) = r
  | dest_type _ = raise TYPE_ERR{function="dest_type",
                                 message="Not compound type"};
fun dest_vartype (TV v) = v
  | dest_vartype _ = raise TYPE_ERR{function="dest_vartype",
                                    message="Not type variable"};
fun dest_funtype (TO {Tyop="fun", Tyargs=[ratty, randty]}) =
    (ratty, randty)
  | dest_funtype _ =
    raise (TYPE_ERR {function="dest_funtype",
                     message="Not a function type"});

val is_vartype = can dest_vartype;
fun is_funtype (TO {Tyop,Tyargs}) =
  (Tyop = "fun") andalso (length Tyargs = 2)
```

```
  | is_funtype _ = false;

fun domain_of ty = #1(dest_funtype ty);
```

13.2.5 type_in_type We use `cmpType` to test whether two types are equal. If they are type operators, their arguments are compared.

```
fun type_in_type ty (TV v2) = (cmpType (ty, (TV v2)) = Equal)
  | type_in_type ty (TO {Tyargs=tyargs2,Tyop=tyop2}) =
  (case ty of
     (TV v) => exists (type_in_type ty) tyargs2
   | (TO {Tyargs,Tyop}) =>
       ((Tyop = tyop2) andalso
        ((cmp_type_list (zip(Tyargs,tyargs2))) = Equal))
       orelse
       (exists (type_in_type ty) tyargs2));
```

13.2.6 Special type constants Since *:bool* and *:num* types are used very frequently, we define the following constructors and predicates for them.

```
val bool_ty = (TO {Tyop="bool",Tyargs=[]});
val num_ty = (TO {Tyop="num",Tyargs=[]});
fun ty_is_bool (TO {Tyop="bool",Tyargs=[]}) = true
  | ty_is_bool _ = false
and ty_is_num (TO {Tyop="num",Tyargs=[]}) = true
  | ty_is_num _ = false
```

13.2.7 Type instantiation The local function `inst_compat` checks a type instantiation, specified as a pair (ty, vty) against an instantiation list, *instl*. *vty* should be a type variable. If no such variable appears in the list, then this is a new instantiation, it can be added to the list. If it does appear in the list, the type *ty* must be equal to the corresponding type in the list. Otherwise, it is incompatible and the function fails.

The local function `is_ty_inst` takes a pair of types (ty_1, ty_2) and a list of type pairs. It does the actual checking of the type instantiation. It returns `true` if ty_1 is an instance of ty_2. The list *instl* is used to cumulate the type instantiations found so far. It calls `inst_compat` to check the compatibility of instantiation. It fails if ty_1 is not an instance of ty_2. The function `is_ty_instl` checks a list of type pairs.

```
local
  exception TY_INST
  fun inst_compat instl (ty,vty) =
    case (findOne (fn (ty',v')=> (vty = v')) instl) of
      NONE => (true, ((ty,vty)::instl))
    | SOME (ty',v') =>
        if (ty = ty') then (true, instl)
        else raise TY_INST
             (* TYPE_ERR {function="inst_compat",
              message="incompatible instantiation"} *)

  fun is_ty_inst (ty, v as (TV s)) instl =
    (inst_compat instl (ty,v))
    | is_ty_inst (ty2,(TO {Tyop=tyop1,Tyargs=tyargs1})) instl =
      (case ty2 of
         (TO {Tyop=tyop2,Tyargs=tyargs2}) =>
```

```
            if not (tyop1 = tyop2)
              then raise TY_INST
                   (* TYPE_ERR {function="is_ty_inst",
                    message="different type op"} *)
            else is_ty_instl instl (zip(tyargs2,tyargs1))
        | _ => raise TY_INST)
              (* TYPE_ERR {function="is_ty_inst",
               message="different types"} *)
  and is_ty_instl instl [] = (true, instl)
    | is_ty_instl instl (tt::ttl) =
      let
        val (f,l) = is_ty_inst tt instl
      in
        if f then is_ty_instl l ttl
        else raise TY_INST
             (* TYPE_ERR {function="is_ty_instl",
              message="different type argument"} *)
      end
in
  fun is_type_inst ty1 ty2 = can (is_ty_inst (ty1,ty2)) []
  and is_tyty_inst tytyl ty1 ty2 =
    let val (OK,ilst) = (is_ty_inst (ty1,ty2) tytyl)
      handle TY_INST => (false,tytyl) | e => raise e
    in
      OK andalso (ilst = tytyl)
    end
  and type_instl ty1 ty2 = #2(is_ty_inst (ty1,ty2) [])
end
```

13.2.8 Perform type instantiation The function `inst_type` performs the substitution of types in its second argument. Its first argument is a type-pair list specifying the substitution. In the case that its second argument is a type variable, the function looks for the same variable in the second fields of type pairs in the substitution list. If this is found, the corresponding type is returned. Otherwise, no substitution is performed. In the case the second argument is a type operator, the function recurses down to its arguments.

```
fun inst_type tytyl (v as (TV s)) =
  let
    val typ = findOne (fn (ty,tv) => (s = dest_vartype tv)) tytyl
  in
    case typ of
      NONE => v
    | SOME (ty,tv) => ty
  end
  | inst_type tytyl (TO {Tyop=tyop,Tyargs=tyargs}) =
    (TO {Tyop=tyop, Tyargs=(map (inst_type tytyl) tyargs)});
```

The function `type_replace` is the same as `inst_type` except it takes a `(type, string)`-pair as its first argument. The string is the variable name.

```
fun type_replace tytynamel (v as (TV s)) =
  let
    val typ = findOne (fn (ty,tvname) => (s = tvname)) tytynamel
```

```
  in
    case typ of
      NONE => v
    | SOME (ty,tv) => ty
  end
  | type_replace tytynamel (TO {Tyop=tyop,Tyargs=tyargs}) =
    (TO {Tyop=tyop, Tyargs=(map (type_replace tytynamel) tyargs)});
```

13.2.9 Compatibility of types This implements the function having the same name in JVW's theory. The local function `ty_compat` compares the structure of the two types.

```
local
  fun ty_compat (ty, (TV s)) = true
    | ty_compat ((TO {Tyop=tyop, Tyargs=tyargs}),
                 (TO {Tyop=tyop2, Tyargs=tyargs2})) =
      (tyop = tyop2) andalso (length tyargs = length tyargs2) andalso
      (every ty_compat (zip(tyargs,tyargs2)))
    | ty_compat _ = false
in
  fun type_compat ty1 ty2 = ty_compat(ty1,ty2)
end
```

13.2.10 Output function The function `pr_htype` prints its argument to the default output stream in the same format as in the proof file.

```
local
  open Report
in
  fun pr_htype (TV name) = write_sitem (Keyword.TYVAR, name)
    | pr_htype (TO{Tyop,Tyargs}) =
      if (null Tyargs) then write_sitem (Keyword.TYCONST, Tyop)
      else write_item_list (Keyword.TYOP, Tyop, (pr_htype, Tyargs))
end
```

13.2.11 Pretty Printer The function `pp_htype` is a pretty printer for `htypes`. It outputs its second argument to the pretty printing stream specified by the first argument. A type is always prefixed by a colon (`:`). If the type is a type variable, its name is printed. If the type is an atomic type, the name of the type operator is printed. If a type is a type operator having one or more arguments, its arguments are printed first, which are enclosed in parentheses and separated by commas, then the name of the type operator are printed. The exceptions are that if the type operator is a function type, a product type or a disjoint sum type, their name become infix, for instance, a function type taking a `num` to `bool` is printed as `:num -> bool` instead of `:(num,bool)fun`.

```
local
  open System.PrettyPrint;
  fun with_ppstream ppstrm =
    {add_string = add_string ppstrm,
     add_break = add_break ppstrm,
     begin_block = begin_block ppstrm,
     end_block = fn () => end_block ppstrm,
     flush_ppstream =fn () => flush_ppstream ppstrm};
in
```

```
fun pp_htype ppstrm ty =
  let
    val {add_string, add_break,
         begin_block, end_block, flush_ppstream} =
      with_ppstream ppstrm;

    fun pp_type (TV s) = add_string s
      | pp_type (TO {Tyop="fun",Tyargs=[ty1,ty2]}) =
        (begin_block CONSISTENT 0;
         add_string "(";
         pp_type ty1;
         add_string " ->"; add_break(1,1);
         pp_type ty2;
         add_string ")";
         end_block ())
      | pp_type (TO {Tyop="prod",Tyargs=[ty1,ty2]}) =
        (begin_block CONSISTENT 0;
         add_string "(";
         pp_type ty1;
         add_string ","; add_break(1,1);
         pp_type ty2;
         add_string ")";
         end_block ())
      | pp_type (TO {Tyop="sum",Tyargs=[ty1,ty2]}) =
        (begin_block CONSISTENT 0;
         add_string "(";
         pp_type ty1;
         add_string " +"; add_break(1,1);
         pp_type ty2;
         add_string ")";
         end_block ())
      | pp_type (TO {Tyop,Tyargs}) =
        (begin_block CONSISTENT 0;
         pp_type_list Tyargs;
         add_string Tyop;
         end_block ())
    and pp_type_list [] = ()
      | pp_type_list tyl =
        (begin_block CONSISTENT 0;
         add_string"(";
         pp_tyl tyl;
         add_string")";
         end_block())
    and pp_tyl [] = ()
      | pp_tyl [ty] = pp_type ty
      | pp_tyl (ty::tyl) =
        (pp_type ty;
         add_string ",";
         add_break(1,2);
         pp_tyl tyl)
  in
```

```
          (begin_block CONSISTENT 0;
           add_string ":";
           pp_type ty;
           end_block ()) handle e => (flush_ppstream();
                                      clear_ppstream ppstrm; raise e)
       end
   end

 end; (* functor HtypeFUN *)
```

13.3 Structure HtypeCmp

This structure encapsulates the type ordering function `cmpType`. It is used to create ordered sets of types.

```
functor HtypeCmpFUN (Htype:Htype_sig) : ORD_KEY =
  struct

    type ord_key = Htype.htype

    fun cmpKey (ty1,ty2) = Htype.cmpType (ty1,ty2)

  end(* functor HtypeCmpFUN *)
```

14 Error handling

Error handling can be divided into two parts: *debugging* and *exception handling*. Debugging is for use while developing the program. Exception handling is used to trap abnormal inputs while the program is in operation. They are implemented as two separate structures.

14.1 Debugging utilities

The debugging utilities consist of a centralised database for storing debugging settings and three functions to access the data. Each individual module in the program has an entry in the debugging database. An entry is keyed by module name, and the associated data is an integer. The higher the value, the more debugging information is printed. The default value is 0 which means no debugging information is printed. This provides a flexible way of controlling the amount of information.

There are 3 levels of debugging settings. Level 0 suppresses any messages. Level 1 will show informative messages, such as the name of the inference rule of the current proof line. Level 2 provides more messages including the names of the functions.

The function `set_debug` sets a new debugging level for a module. It takes the module name and the new value as arguments. The function `get_debug` returns the current setting of a module. The function `print_flags` shows all the entries in the database.

However, a new value stored into the database will not take effect immediately. It is up to each individual module to decide when to update its own private copy of the setting. Generally, this happens when the module's initialisation function is called, or the structure is created by applying the functor. Otherwise, the user has no way of updating the private copies of the settings in individual modules directly.

```
signature Debug_sig =
  sig
    (* structure Dict : DICT *)
    val set_debug : (string * int) -> unit
    val get_debug : string -> int
    val print_flags : unit -> unit
  end
```

14.1.1 The implementation The debugging setting database is implemented as a dictionary whose signature is `DICT` (from the SML/NJ library). It requires a structure implementing a key comparison function. The `StrCmp` structure is for this purpose. It has the signature `ORD_KEY` as specified in the library. It has a function for comparing two strings.

```
structure StrCmp : ORD_KEY =
  struct
    open LibBase
    type ord_key = string
    fun cmpKey (s1:ord_key,s2:ord_key) =
      if (s1 < s2) then Less
      else if (s1 = s2) then Equal else Greater;
  end;
```

The structure Debug implements the debugging database in its private structure Dict and the data accessing functions. An empty dictionary is created initially.

```
structure Debug :Debug_sig =
  struct
    structure Dict =  BinaryDict(StrCmp);

    val status_dict = ref ((Dict.mkDict()):(int) Dict.dict)
```

Since the dictionary entries cannot be updated in place, to assign a new value to an entry, one has to remove the entry first and then insert a new entry with the new value.

```
    fun set_debug (name, value) =
      let
        val dic = #1(Dict.remove((!status_dict), name))
                   handle NotFound => (!status_dict)
      in
        (status_dict := Dict.insert(dic, name, value); ())
      end
```

If no entry having the key name is found in the dictionary, the default value of 0 is returned.

```
    and get_debug name =
      case Dict.peek((!status_dict), name) of
        NONE => 0
      | SOME n => n
```

The function print_flags simply lists all the entries in the dictionary.

```
    and print_flags () =
      let
        val lst = Dict.listItems(!status_dict)
      in
        map (fn (s,n) => output(std_err,(s^": "^(makestring n)^"\n"))) lst;
        ()
      end
  end;(* struct *)
```

14.1.2 Initialisation The database is initialised with the following values when this file is loaded.

```
map Debug.set_debug
  [("Io", 0), ("Htype", 0), ("Hterm", 0), ("Hthm", 0), ("Henv", 1),
   ("Proof", 0), ("Parsing", 0), ("Check",2), ("Pass1", 0), ("Pass2", 1)];
```

14.2 Exception handling

This is the same as the HOL90 exception handling system[1]. It defines a single exception format for the entire checker. There is one exception constructor:

```
signature Exception_sig =
  sig
    exception CHK_ERR of {message:string,
                          origin_function:string,
                          origin_structure:string}
```

[1]Thanks to Elsa Gunter who wrote the HOL90 exception handling structure.

It takes three strings: the structure name, the routine name, and the message. There is an (assignable) function for printing `CHK_ERR`s, plus a function `Raise` that will print out exceptions at the site of occurrence.

The function `Raise` can be used to raise an exception instead of the SML operation `raise`. By default, it prints the three fields of a `CHK_ERR` exception in a readable form. Suppose that the evaluation of the expression *exp* raises an exception, this can be handled as

exp `handle` *e* `=> Raise` *e*

which will print the fields of the exception.

```
    val Raise : exn -> 'a
```

The boolean value `print_exceptions` is for controlling the display of exceptions. The (assignable) function `output_CHK_ERR` is for printing the exception messages. Its default value is to print the three fields of `CHK_ERR`. The function `print_CHK_ERR` prints a checker exception package.

```
    val print_exceptions : bool ref
    val output_CHK_ERR : ({message:string,
                           origin_function:string,
                           origin_structure:string} -> unit) ref
    val print_CHK_ERR : exn -> unit
  end;
```

14.2.1 The structure `Exception` The implementation of the structure `Exception` is as follows:

```
structure Exception : Exception_sig =
  struct

    exception CHK_ERR of {origin_structure:string,
                          origin_function:string,
                          message:string}

    val print_exceptions = ref true;

    (* Assignable function for printing errors *)
    val output_CHK_ERR =
      ref (fn {message,origin_function,origin_structure} =>
         ( output(std_err, ("\nException raised at "^origin_structure^"."^
                            origin_function^
                            (if (origin_structure = "")
                                 then ""
                             else ":\n")^
                                 message^"\n"));
           flush_out std_err));

    fun print_CHK_ERR (CHK_ERR sss) = !output_CHK_ERR sss
      | print_CHK_ERR _ = print_CHK_ERR(CHK_ERR{origin_structure="Exception",
                                               origin_function="print_CHK_ERR",
                                               message="not a checker error"})

    fun Raise (e as CHK_ERR sss) =
      ( if (!print_exceptions)
```

```
          then !output_CHK_ERR sss
        else ();
          raise e)
      | Raise (e as _) = raise e

end  (* Exception *)
```

15 Input and output management

The input and output of the checker is managed by the `Io` module. This provides a simple interface between the file system and the core of the checker. Conceptually, the checker accepts input from a single input stream which is connected to a named proof file. The checker outputs to a single output stream connected to a log file. The connections are made via sockets.

15.1 The specification

The signature `IO_sig` specifies the interface of the input/output manager. It consists of a small set of functions for opening/closing a file, reading from an opened file via an input socket or writing to an opened file via an output socket.

This arrangement provides a convenient way of filtering the input and output text. Since the proof file is expected to be stored in compressed form, instead of decompressing the file then reading the decompressed file directly, a decompresser can be run as a filter which outputs the decompressed text into a socket, and the checker gets its input from the socket.

The IO manager maintains a pair of sockets, as the current input/output sockets, one to input from and one to output to. After the sockets are opened, all subsequent input is read from the input socket, and output is written to the output socket.

The functions `open_input_socket` and `open_output_socket` open a socket to input from and output to, respectively. They take two strings as their arguments. The first string is the name of the file. The second string is the name of the filter. In the case that no filter is required, the second argument should be a null string. The function `close_io_socket` closes both the input and output sockets.

```
signature IO_sig =
  sig
    val open_input_socket : string -> string -> unit
    val open_output_socket : string -> string -> unit
    val close_io_socket : unit -> unit
```

There are two functions for reading the input file. The function `read_bytes` reads a fixed number of bytes from the input socket. It returns a pair whose first field is an integer giving the actual number of bytes read and whose second field is a `ByteArray` containing the input bytes. The function `read_input_line` reads a line (terminated by carriage return or newline) from the input socket, and returns it as a string. The function `write_output_string` writes its argument string to the default output socket.

```
    val read_bytes : unit -> (int * ByteArray.bytearray)
    val read_input_line : unit -> string
    val write_output_string : string -> unit
```

The function `mk_command` takes a file name as its sole argument. It creates and returns three strings: the name of the log file, and the command strings of the input and output filters. The rules for deriving the output file name are as follows:

- if the input file name ends with suffix `.gz`, it is assumed to be a compressed file. The output file will be compressed. The decompression command is `gzcat`, and the compression command is `gzip`. The output file name will also end with the suffix `.gz`.

- if the input input name ends with suffix .prf after deleting the .gz suffix if it occurs, then the output file name is the same as the input file name with the string prf replaced by the string clg.
- otherwise, the suffix .clg is appended to the input name to obtain the output name.

The rules are applied in the order listed.

```
    val mk_command : string -> string * string * string
  end;
```

15.2 The structure Io

This structure implements the input/output manager. It uses the low level input/output functions in the structure System.Unsafe.SysIO in the SML/NJ library because we need to create a separate filter process and to manage sockets.

```
structure Io : IO_sig =
  struct

    fun IO_ERR{function,message} =
      Exception.CHK_ERR{message = message,
                        origin_function = function,
                        origin_structure = "Io"};
```

15.2.1 Local variables A set of internal variables is used by the IO manager. The variables in_sname and out_sname are assigned the names of the input and output sockets, respectively.

```
    val in_sname = ref ""
    and out_sname = ref ""
```

The sockets are accessed through file descriptors. After the sockets are opened, the file descriptors are assigned to the identifiers default_ifp and default_ofp. When the sockets are not opened, the names are assigned a null string and the default file descriptors are the invalid descriptor which has the symbolic name NULL.

```
    val NULL = ((~1):System.Unsafe.SysIO.fd)
    val default_ifp = ref NULL
    and default_ofp = ref NULL
```

The name of the socket server is bound to the identifier server. This server is a separate program which creates the sockets and the filter process, and connects them together. Refer to Appendix C for detailed description of the server.

```
    val server = "so_server"
```

str_buf is the input buffer. The default input block size is bound to default_block_size.

```
    val str_buf = ref ""
    and default_block_size = 80
```

The local variable debug holds the current debugging setting for this module. The function write_out simply writes a message to the standard error stream. It is used for displaying debugging messages.

```
    val debug = Debug.get_debug("Io");
    fun write_out s = (output(std_err,s);flush_out std_err);
```

15.2.2 Data conversions Because the low level system IO functions work with `bytearray`s rather than strings, conversion functions are needed. The function `str_to_barray` takes a string and its length as its arguments. It returns a byte array containing a sequence of ASCII codes representing the string. Since the element type of a byte array must by integer, and the elements can only be updated one-by-one, the input string is converted into a list of integers (character codes) first; then, they fill the byte array one-by-one.

```
local
  fun str_to_barray (s,n) =
    let
      val len = ref n
      val si = (map ord (explode s));
      val barray = ByteArray.array(n, 0)
    in
      (while ((!len) > 0) do
        (len := (!len) - 1;
         ByteArray.update(barray, (!len), (nth(si, (!len))))));
         barray
    end
```

The function `barray_to_string` does the reverse conversion. It takes a byte array and returns a string. The conversion from byte array to string is simpler since the SML/NJ library has a function for this task.

```
  fun barray_to_string ba =
    ByteArray.extract(ba, 0, ByteArray.length(ba))
```

15.2.3 Reading input file The actual reading of a file is carried out by the function `read_string`. It takes a file descriptor and an integer indicating the required number of bytes. It returns a pair whose first field *len* is an integer giving the actual number of characters read, and whose second field is a string containing the input characters. On reaching the end of file, *len* is 0.

```
  fun read_string fp n =
    let
      val barray = ByteArray.array(n, 0)
      val len = System.Unsafe.SysIO.read(fp, barray, n)
    in
      (len, ByteArray.extract(barray, 0, len))
    end
```

15.2.4 Input buffering Because the logical input unit of the lexical analyser is the line, but the system input function is byte oriented, a local string buffer, namely `str_buf`, is managed. It can hold at most one string. When a block of characters is read, it is split into two parts just after the first end of line character (`\n`) if such a character occurs. The first part is returned whilst the second part is saved in the buffer `str_buf`.

When an input line is needed, the function `get_string` is called. It checks the string buffer first. If it is not empty, the string is returned. Otherwise, the function `read_string` is called to read a block of characters from the input file.

```
  fun get_string fp =
    if (!str_buf) = "" then (read_string fp default_block_size)
    else
      let
        val s = !str_buf
```

```
    in
      (str_buf := ""; ((size s),s))
    end;
```

The function `unget_string` puts a string back in the string buffer provided the latter is empty. Otherwise, it raises an exception.

```
fun unget_string s =
  if (!str_buf) = "" then str_buf := s
  else raise
    (IO_ERR{function="unget_string", message="Buffer not empty"});
```

15.2.5 Splitting a string This function `split_string` takes two strings (cs, s) as its arguments. It returns a triple $(found, s_1, s_2)$. If one of the characters in cs, say c, occurs in s, then s is split into two sub-strings s_1 and s_2. The last character of s_1 is c. $found$ is set to `true`. If none of the characters in cs occurs in s, $found$ is set to `false`, s_1 becomes s and s_2 is a null string.

```
  fun split_string chrs s =
    let
      val n = size s
      val (found,ix) = ((true, StringUtil.index chrs (s,0) )
                 handle StringUtil.NotFound => (false,n))
      val (s1,s2) = if (ix = n) then (s, "")
                     else (substring(s,0,ix+1), substring(s,ix+1, n-ix-1))
    in
      (found, s1, s2)
    end;
in
```

15.2.6 Input functions The function `read_line` reads a line from the file pointed to by the file descriptor fp. A line is a string whose last character is the 'end of line' character ("`\n`" by convention). If the end of file is reached, `get_string` returns 0 to indicate that no character is read. This function returns a null string. Otherwise, the input string is checked for an end of line by calling `split_string`. If none is found, another string is read, the process is repeated until an end of line or end of file is found. If an end of line is found, the characters after it are pushed back to the string buffer.

```
fun read_line fp =
  let
    val outstr = ref ""
    val done = ref false
  in
    while not (!done) do
      let
        val (n,s) = get_string fp
      in
        if (n = 0) then (done := true)
        else
          let val (found, s1,s2) = split_string "\n" s
          in
            outstr := !outstr ^ s1;
            if found
              then (done := true;
```

```
                      if not(size s2 = 0) then unget_string s2 else ())
              else done := false
            end
        end;
      (!outstr)
    end;
```

The function `read_input_line` reads an input line from the default input file. If no default input file is opened, it raises an exception.

```
fun read_input_line () =
  if ((!default_ifp) = NULL)
    then raise (IO_ERR{function="read_input_line",
                       message="No default input stream"})
  else read_line (!default_ifp);
```

The function `read_bytes` reads a block of bytes from the default input stream. The size of the block is determined by the value in `default_block_size`. The checker now works directly on bytes, i.e., ASCII codes, therefore, the local functions for converting bytes to strings and buffering are not needed. They have not been deleted in case they are needed in the future. See Chapter 6 for details of how the checker recognises ASCII codes.

```
fun read_bytes () =
  if ((!default_ifp) = NULL)
    then raise (IO_ERR{function="read_bytes",
                       message="No default input stream"})
  else
    let
      val barray = ByteArray.array(default_block_size,0)
      val len = System.Unsafe.SysIO.read
        ((!default_ifp), barray, default_block_size)
    in
      (len, barray)
    end
```

15.2.7 Output functions The function `write_string` writes its second argument *str* to the file pointed by the file descriptor *fp*.

```
fun write_string fp str =
  let val len = size str
    val ba = str_to_barray(str,len)
    val n = makestring(ByteArray.length ba) and n0 = makestring len
  in
    if (debug > 3) then
      write_out("write_string:"^n0^"`"^str^"`"^n)
    else ();
    System.Unsafe.SysIO.write(fp, ba, len)
  end;
```

The function `write_output_string` writes its argument *str* to the default output file. If no default output file is opened, it raises an exception.

```
fun write_output_string str =
  if ((!default_ofp) = NULL)
    then raise (IO_ERR{function="read_output_string",
                       message="No default output stream"})
```

```
      else write_string (!default_ofp) str;
  end(* local *)
```

15.2.8 Closing sockets The function `close_socket` takes the file descriptor `fp` and the file name `sname` of a socket as its argument. It closes the socket and removes the socket file without any checking.

```
fun close_socket (fp,sname) =
  (System.Unsafe.SysIO.closef fp;
   System.Unsafe.SysIO.unlink sname)
  handle e => raise IO_ERR {function="close_socket",
                            message= "Error in closing socket"};
```

The function `close_io_socket` closes the default IO sockets. It checks to ensure at least one socket is currently opened. If no socket is opened, it raises an exception.

```
fun close_io_socket () =
  if ((!default_ifp) = NULL) andalso ((!default_ofp) = NULL)
    then raise (IO_ERR {function="close_io_socket",
                        message= "No open socket"})
  else
    (if not(!default_ifp = NULL)
       then (close_socket ((!default_ifp), (!in_sname));
             default_ifp := NULL;
             in_sname := ""; ())
     else ();
     if not(!default_ofp = NULL)
       then (close_socket ((!default_ofp), (!out_sname));
             default_ofp := NULL;
             out_sname := ""; ())
     else ());
```

15.2.9 File access checking The function `check_file` takes a file name and an access mode. Both arguments are strings. It performs the following actions according to the value of the access mode string:

- `mode = "R"` — return true if the file *fname* exists and can be read;
- `mode = "W"` — return true if the file *fname* exists and can be written or if the file *fname* can be created with write permission;
- `mode` = anything else — return true if the file *fname* exists.

```
fun check_file (fname, mode) =
  case mode of
    "R" =>
      (System.Unsafe.SysIO.access(fname,[System.Unsafe.SysIO.A_READ]))
  | "W" =>
      if (System.Unsafe.SysIO.access(fname,[System.Unsafe.SysIO.A_WRITE]))
        then true
      else
        let
          val fp =
            System.Unsafe.SysIO.openf(fname,System.Unsafe.SysIO.O_WRITE)
            handle e => (~1)
```

```
          in
            if (fp = ~1) then false
            else
              (System.Unsafe.SysIO.closef fp;
               System.Unsafe.SysIO.unlink fname; true)
          end
      | _ =>
          (System.Unsafe.SysIO.access(fname,[]));
```

15.2.10 Delay function This function is used to delay the program in retrying to open a socket. It sleeps for *sec* seconds before it returns.

```
fun sleep sec =
  let val cmd = System.Unsafe.CInterface.c_string("sleep "^sec) in
    System.Unsafe.CInterface.system cmd
  end;
```

15.2.11 Opening sockets There are four functions for opening sockets arranged into two tiers. All of these take two strings as their arguments. The first string is the name of the file to be opened. The second string is the filter to be run. The lower tier functions `open_in_socket` and `open_out_socket` return a file descriptor to the socket and the name of the socket as a pair. The higher tier functions `open_input_socket` and `open_output_socket` return a unit on successful opening of a socket and make it the default input or output file, respectively.

Conceptually, the input file whose name is given as the first argument to `open_input_socket` becomes the default input file. In fact, the file is opened for reading and connected to the input end of the filter specified as the second argument. The filter sends its output to the socket. The socket is created with a unique name *sname* by a socket server *server*. The default socket server is an auxiliary program whose name is bound to the identifier `server`. The new socket is connected by calling the system function `connect_unix` which returns a file descriptor to the socket. This becomes the default input file descriptor and is used to read all inputs. The process of creating the filter process and socket and connecting them up is carried out by the lower tier function `open_in_socket`. The output channel is managed in the same way as the input.

```
fun open_in_socket fname proc =
  let
    val pid = System.Unsafe.CInterface.getpid()
    val sname = "RTMP"^(makestring pid)
    val cmd' = server ^ " R "^sname
    val cmd = if (proc = "") then (cmd' ^" < "^fname^" &")
              else (proc ^" < "^fname^ " | "^cmd' ^ " &")
    val ifp = ref NULL
    val n = ref 10
  in
    if not(check_file (fname, "R"))
      then raise (IO_ERR{function="open_in_socket",
                         message = "Cannot read file "^fname})
    else
      (System.Unsafe.CInterface.system
       (System.Unsafe.CInterface.c_string cmd)
       handle e => raise (IO_ERR{function="open_in_socket",
                                 message= "Error in creating socket"});
```

```
          while ((!n > 0) andalso (!ifp = NULL)) do
            (sleep " 2 ";
             n := !n - 1;
             ifp := (System.Unsafe.SysIO.connect_unix sname handle e=> NULL));


          if (!ifp = NULL)
            then raise (IO_ERR {function="open_in_socket",
                                message= "Error in connecting socket"})
          else
            (!ifp, sname)
            )
      end;

    fun open_out_socket fname proc =
      let
        val pid = System.Unsafe.CInterface.getpid()
        val sname = "WTMP"^(makestring pid)
        val cmd' = server ^ " W "^sname
        val cmd = if (proc = "") then (cmd' ^" > "^fname^" &")
                  else (cmd' ^ " | " ^ proc ^" > "^fname^" &")
        val NULL = ((~1):System.Unsafe.SysIO.fd)
        val ofp = ref NULL
        val n = ref 10
      in
        if not(check_file (fname, "W"))
          then raise (IO_ERR{function="open_out_socket",
                             message = "Cannot create file "^fname})
        else
          (System.Unsafe.CInterface.system
           (System.Unsafe.CInterface.c_string cmd)
           handle e => raise (IO_ERR{function="open_out_socket",
                                     message= "Error in creating socket"});

          while ((!n > 0) andalso (!ofp = NULL)) do
            (sleep " 2 ";
             n := !n - 1;
             ofp := (System.Unsafe.SysIO.connect_unix sname handle e => NULL))
;

          if (!ofp = NULL)
            then raise (IO_ERR {function="open_out_socket",
                                message= "Error in connecting socket"})
          else
            (!ofp, sname)
          )
      end;

    fun open_input_socket fname proc =
      if not((!default_ifp) = NULL)
        then raise (IO_ERR{function="open_input_socket",
```

```
                      message="socket already opened"})
      else
        let
          val (fp, sname) = (open_in_socket fname proc
           handle e => raise (IO_ERR{function="open_input_socket",
                                    message="error in opening socket"}))
        in
          (default_ifp := fp;
           in_sname := sname;
           str_buf := ""; ())
        end;

    fun open_output_socket fname proc =
      if not((!default_ofp) = NULL)
        then raise (IO_ERR{function="open_output_socket",
                      message="socket already opened"})
      else
        let
          val (fp, sname) = (open_out_socket fname proc
           handle e => raise (IO_ERR{function="open_output_socket",
                                    message="error in opening socket"}))
        in
          (default_ofp := fp;
           out_sname := sname; ())
        end;
```

15.2.12 Local variables for file name parsing The names `compr_cmd` and `decompr_cmd` are bound to the shell commands for compressing and decompressing files, respectively. They are the filtering processes connecting the sockets to the files.

```
    val compr_cmd = "gzip - "
    and decompr_cmd = "gzcat"
```

The names `prf_suff`, `log_suff` and `compr_suff` are bound to strings which are the file name suffixes of proof files, log files and compressed files, respectively. The name `suf_sep` is the suffix separator.

```
    val prf_suff = "prf"
    and log_suff = "clg"
    and compr_suff = "gz"
    and suf_sep = "."
```

The names `log_suf`, `compr_suf` and `log_compr_suf` are bound to the suffixes with separators of the log files, compressed files and compressed log files, respectively.

```
    val log_suf = (suf_sep ^ log_suff)
    and compr_suf = (suf_sep ^ compr_suff)
    and log_compr_suf = (suf_sep ^ log_suff ^ suf_sep ^ compr_suff)
```

15.2.13 File name parsing The function `mk_command` parses its string argument and returns a triple `(outname, incmd, outcmd)` where `outname` is a string to be used as the output file name, `incmd` and `outcmd` are strings to be used to invoke the input and output filter processes, respectively. These filters are decompression and compression programs. The rules for generating these strings are described in the signature section.

```
    fun mk_command in_name =
```

```
    let
      fun imploded l =
        let fun f(x,s) = x^"."^s in fold f (butlast l) (last l) end
      fun do_path name =
        let
          val i = StringUtil.revindex "/" (name,(size name) -1)
            handle NotFound => ~1
                 | e => raise e
        in
          if (i < 0) then ("",name)
          else (substring(name,0,i+1), StringUtil.suffix(name,i+1))
        end

      val (path,fname) = do_path in_name
      val len = String.size fname
      val l = StringUtil.tokenize suf_sep (fname,len-1)
      val n = length l
      val compressed = ((nth(l,n-1)) = compr_suff)
      val (outname,incmd,outcmd) =
        case n of
          1 => ((fname ^ log_suf), "", "")
        | 2 =>
            if compressed then
              (((hd l)^log_compr_suf), decompr_cmd, compr_cmd)
            else if ((nth(l,n-1)) = prf_suff) then
                   (((hd l) ^ log_suf), "", "")
                 else (((imploded l) ^ log_suf), "", "")
        | 0 => ("checker.clg", "", "")
        | _ =>
            if compressed then
              if (last(butlast l) = prf_suff) then
                ((imploded(butlast(butlast l))) ^ log_compr_suf,
                 decompr_cmd, compr_cmd)
              else ((imploded(butlast l)) ^ log_compr_suf,
                    decompr_cmd, compr_cmd)
            else if ((last l) = prf_suff) then
                   ((imploded(butlast l)) ^ log_suf, "", "")
                 else ((imploded l) ^ log_suf, "", "")
    in
      if (path = "") then (outname,incmd,outcmd)
      else (path ^ outname,incmd,outcmd)
    end

end; (* end of structure Io *)
```

16 Report generation

The `Report` module contains several utility functions for outputting various objects to the log file. The log file is assumed connected to the default output stream managed by the `Io` module. All writings targeted to this file are performed by the function `write_output_string` in Page 15.2.7.

The format of the log file is essentially the same as the proof file. For each proof, all the hypotheses, theorems and definitions which have been used are written into the log file.

The complete syntax of the log file format in an augmented BNF is as below:

```
log_file ::= version time stamp proof_logs ;;

version ::= LP VERSION STRING RP ;;

timestamp ::= LP TIMESTAMP STRING RP ;;

proof_logs ::= proof_log | proof_logs proof_log ;;

proof_log ::= LP PROOF STRING used proved unsolve RP ;;

used ::= LP USED just_list RP ;;

proved ::= LP PROVED thm_list RP ;;

unsolved ::= LP UNSOLVED thm_list RP ;;

just ::= LP justi RP ;;

justi ::= HYPOTHESIS thm
        | DEFINITION STRING STRING thm
        | AXIOM STRING STRING thm
        | THEOREM STRING STRING thm
        | STOREDEFINITION STRING STRING thm
        | NEWAXIOM STRING STRING thm ;;
```

Below is a log file generated by the checker when checking a simple proof named `ap_term`. In this proof, one hypothesis is used and one theorem is proved.

```
(VERSION HOL CHECKER 0.1)
(TIMESTAMP Thu Nov 23 14:39:55 HKT 1995 )

(PROOF ap_term
(USED [(HYPOTHESIS  (THM [](A (A (C =)(A (A (C +)(V m))(V n)))
   (A (A (C +)(V n))(V m))))
)
])
```

```
(PROVED [(THM [](A (A (C =)(A (C SUC)(A (A (C +)(V m))(V n))))
   (A (C SUC)(A (A (C +)(V n))(V m)))))
])
)
```

16.1 The specification

The signature `Report_sig` is the specification of the report generation module.

```
signature Report_sig =
  sig
    structure Io : IO_sig
    structure Keyword : Keyword_sig

    val write_tok : string -> unit
    val write_opening : string -> unit
    val write_line_opening : string -> unit
    val write_opening2 : string * string -> unit
    val write_line_opening2 : string * string -> unit
    val write_closing : unit -> unit
    val write_closing_line : unit -> unit

    val write_item_list : string * string * (('a -> 'b) * 'a list) -> unit
    val write_list : ('a -> 'b) -> 'a list -> unit
    val write_sitem : string * string -> unit
```

The function `timeofday` returns a string containing the current date and time in the format as returned by the UNIX shell command `date`. It is used by the function `write_log_preamble` which writes the `version` and `timestamp` to the log file.

```
    val timeofday : unit -> string
    val write_log_preamble : unit -> unit
  end
```

16.2 The implementation

The functor takes two structures: `Io`, the input/output manager and `Keyword` which specifies the concrete syntax of the file format.

```
functor ReportFUN (structure Io : IO_sig and Keyword : Keyword_sig
                  ) : Report_sig =
  struct
    structure Io = Io
    structure Keyword = Keyword

    open Io Keyword
```

16.2.1 Simple items The function `write_sitem` outputs a simple item to the log file. A simple item is an item which has a tag and a string as its only fields. The tag and the string is enclosed in a pair of parentheses and separated by a blank.

```
    fun write_sitem(tag,s) =
        Io.write_output_string (Keyword.LP ^ tag ^" "^ s ^ Keyword.RP)
```

16.2.2 A list of items The function `write_list` outputs a list of items. Each individual item is output by the function *f* which is the first argument to this function. The entire list is enclosed in a pair of brackets.

```
and write_list f xs = (write_output_string LB;
                        map f xs;
                        write_output_string RB)
```

16.2.3 Item having a list The function `write_item_list` outputs an item which contains a list. It take a triple whose first and second field are the tag and a string, respectively. The tag identifies the type of the item. The last field of the triple is itself a pair whose first field *f* is an output function and whose second field is the list of items which are output by the function *f*.

```
and write_item_list (tag,name,(f,args)) =
  (write_output_string (LP ^ tag ^ " " ^ name);
   write_list f args;
   write_output_string RP)
```

16.2.4 Item opening and closing The function `write_opening` outputs the open parenthesis and the tag of an item. The function `write_opening2` outputs the open parenthesis and the first two fields of an item. The first field of an item is always a tag. The second field accepted by this function is a string.

The function `write_closing` simply outputs a closing parenthesis. The function `write_closing_line` is a variant of `write_closing`. It closes the item and adds a new line. There are also variants of opening functions which begin a new line before writing the opening parenthesis.

There should always be a matching call to one of the closing functions for every call to an opening function.

```
and write_line_opening tag =
  write_output_string ("\n" ^ LP ^ tag ^ " ")
and write_opening tag =
  write_output_string (LP ^ tag ^ " ")
and write_line_opening2(tag, s) =
  write_output_string ("\n" ^ LP ^ tag ^ " " ^ s)
and write_opening2(tag, s) =
  write_output_string (LP ^ tag ^ " " ^ s)
and write_closing () =
  write_output_string  RP
and write_closing_line () =
  write_output_string  (RP ^ "\n")
val write_tok  = write_output_string
```

16.2.5 Time of day The function `timeofday` is implemented by calling the UNIX command `date`. The output of the command is written in a temporary file whose name consists of the current process number (so it is unique). The output is a string showing the current date and time, and its terminated by a newline character. This string is read from the file. The newline character is replaced by a space character. The string is returned as the value of the function.

```
fun timeofday () =
  let
    val fname = "CHK"^(makestring(System.Unsafe.CInterface.getpid()))
    val n = System.Unsafe.CInterface.system
      (System.Unsafe.CInterface.c_string("date > "^fname))
```

```
    val inf = open_in fname
    val date = input_line inf
  in
    close_in inf;
    System.Unsafe.SysIO.unlink (System.Unsafe.CInterface.c_string fname);
    StringUtil.stringTrans("\n"," ") date
  end
```

16.2.6 Writing the preamble This function simply writes out the version and the time stamp.

```
  fun write_log_preamble () =
    (write_opening2 (VERSION, log_versionName);
     write_closing_line ();
     write_opening2 (TIMESTAMP, (timeofday()));
     write_closing_line ())

end (* functor Report *)
```

A Static arrays

The functor StaticArrayFUN is an implementation of the signature STATIC_ARRAY in the SML/NJ library. This is in turn used by the library functor DynamicArray to implement dynamic arrays.

A static array is a data structure whose size is fixed at the time the array is created. An attempt to access an element using an out-of-bound index raises the exception Subscript. In contrast, a dynamic array is a data structure whose size can grow when required. Its elements can be accessed randomly using their indices like an ordinary static array. When an out-of-bound element is accessed, the array grows to a size larger than the given index so that the element will be in the array. This feature may be unsafe, but provides an unbounded data structure so there is no hard limit imposed on the size of data.

Dynamic arrays are used in the checker to hold theorem reference information. See Chapter 9 for more detail.

A.1 The static array functor

The static array module is similar to the Array structure in the pervasive environment. Actually, it is implemented using the Array structure.

```
functor StaticArrayFUN (type elemType) : STATIC_ARRAY =
  struct
    type elem = elemType
    type array = elem Array.array

    exception Size
    exception Subscript

    fun array (n,e:elem) =
      (Array.array (n,e)
      handle Array.Size => raise Size | exn  => raise exn);

    fun sub (a:array,n) =
      (Array.sub (a,n)
      handle Array.Subscript => raise Subscript | exn  => raise exn);

    fun update (a:array,n,e) =
      (Array.update (a,n,e)
      handle Array.Subscript => raise Subscript | exn  => raise exn);

    fun length (a:array) = Array.length a;
    fun tabulate (n,f:int -> elem) = Array.tabulate (n,f);
    fun arrayoflist (l:elem list) = Array.arrayoflist l;

  end
```

B Linking the modules

This appendix contains the SML file `join1.sml` for loading and linking the modules. To use the checker, one simply loads this file in an SML session by typing the command

```
use "join1.sml";
```

Afer SML processes this file, a function `check_proof` will be defined at the top level as the entry point to the checker. This function takes a single string as its argument. The sting is the name of a proof file.

```
(* loading some libraries from the SML/NJ library *)

val libpath = "/u/compstaff/wwong/lib/sml/smlnj-lib/";

fun load_libs libs = app (fn lib => use (libpath^lib)) libs;

load_libs
 ["lib-base-sig.sml",         "lib-base.sml",
  "charset-sig.sml",          "charset.sml",
  "ctype-sig.sml",            "ctype.sml",
  "makestring-sig.sml",       "makestring.sml",
  "list-util-sig.sml",      "list-util.sml",
  "listsort-sig.sml",         "list-mergesort.sml",
  "string-util-sig.sml",      "string-util.sml",
  "string-cvt-sig.sml",       "string-cvt.sml",
  "format-sig.sml",           "format.sml",
  "static-array-sig.sml",     "dynamic-array-sig.sml",
  "dynamic-array.sml",
  "ord-key-sig.sml",
  "dict-sig.sml",             "binary-dict.sml",
  "ord-set-sig.sml",        "binary-set.sml"];

use "array.sml";
use "genfuns.sml";
use "debug.sml";
open Exception;

use "io.sml";
use "keyword.sml";
use "report.sml";
use "parsing.sml";
use "type.sml";
use "term.sml";
use "thm.sml";
use "proof.sml";
```

```
use "env.sml";
use "check.sml";
use "pass1.sml";
use "pass2.sml";

structure Report =
  ReportFUN(structure Io = Io and Keyword = HOLProofKey);

structure Parsing =
  ParsingFUN(structure Io = Io and Keyword = HOLProofKey);

structure Htype = HtypeFUN(structure Report = Report);
structure HtypeCmp = HtypeCmpFUN(Htype);

structure Hterm = HtermFUN(structure Report = Report
   and Htype = Htype and HtypeCmp = HtypeCmp);
structure Termcmp = HtermCmpFUN(Hterm);
structure HtermSet = BinarySet(Termcmp);

structure Hthm = HthmFUN(structure Report = Report
 and Hterm = Hterm and HtermSet = HtermSet);

structure Hproof = ProofFUN(structure Hthm = Hthm
     and Hterm = Hterm and Htype = Htype);

structure Henv = HenvFUN(structure Htype = Htype and Hterm = Hterm);

structure Check = CheckFUN(structure Report = Report and Htype = Htype
   and Hterm = Hterm and Hthm = Hthm
   and Proof = Hproof and Henv = Henv
   and HtermSet = HtermSet);

structure Pass1 =
  Pass1FUN(structure Parsing = Parsing and Henv = Henv
   and Proof = Hproof and Hthm = Hthm
   and Hterm = Hterm and Htype = Htype);

structure Pass2 =
  Pass2FUN(structure Parsing = Parsing and Henv = Henv
   and Proof = Hproof and Hthm = Hthm
   and Hterm = Hterm and Htype = Htype
   and Check = Check and Report = Report);

fun check_proof fname =
  (Pass1.parse_file fname; Pass2.parse_file fname);
```

C Socket server

This appendix contains a small C program which manages sockets and spawns external processes for the checker. It should be invoked with two arguments: the first is a single character which can be either 'R' for reading or 'W' for writing; the second is the name for the socket.

C.1 The program

This is a rather simple program. It starts by including some standard header files and defining a number of symbolic constants.

```
#include <sys/types.h>
#include <sys/socket.h>
#include <sys/un.h>
#include <stdio.h>

#define STDIN 0
#define STDOUT 1

#define READ 0
#define WRITE 1

#define NAME "mysocket"
```

C.1.1 The main function Here comes the beginning of the `main` function.

```
main(argc, argv)
int argc;
char *argv[];
{
    int sock, msgsock, rval, rc;
    int rw, nchars;
    struct sockaddr_un server;
    char buf[BUFSIZ], *bp;
```

The command line arguments are checked first.

```
    if (argc != 3)
        {
            fprintf(stderr,"(so_server) command line: number of arguments\n");
            exit(1);
        }
    if (*argv[1] == 'R') rw = READ;
    else if(*argv[1] == 'W') rw = WRITE;
    else
        {
            fprintf(stderr,"(so_server) command line: direction\n");
            exit(1);
        }
```

The system function **socket** is called to open a socket. If an error occurs, a negative value is returned.

```
/* create a socket */
sock = socket(AF_UNIX, SOCK_STREAM, 0);
if(sock < 0)
    {
        perror("Opening stream socket");
        exit(1);
    }
```

The name specified by the command line is bound to the socket. The socket appears to the program just as a UNIX file.

```
server.sun_family = AF_UNIX;
strcpy(server.sun_path, argv[2]);
if(bind(sock, (struct sockaddr *)&server,
        sizeof(struct sockaddr_un)) < 0)
    {
        perror("Binding stream socket");
        exit(1);
    }

fprintf(stderr,"socket name:%s\n", server.sun_path);
```

Now, we are ready to accept a connection to the socket. When an incoming request is processed, **accept** returns a value to indicate the outcome.

```
listen(sock, 1);

msgsock = accept(sock, (struct sockaddr *)0, (int *)0);
fprintf(stderr,"SERVER:accept socket No.%d\n", msgsock);
```

If the connection is successful, we initialise the input/output buffer **buf** and start passing characters. If the socket was opened for reading, we pass everything from the standard input to the socket until the end of the file is found, in this case, **read** returns 0. If the socket was opened for writing, we pass anything from the socket to the standard output until **read** returns 0.

```
if(msgsock == -1) perror("accept");
else
    {
        bzero(buf, sizeof buf);

        if (rw == READ)
            {
                while((nchars = read(STDIN, buf, BUFSIZ)) > 0)
                    {
                        if((rval = write(msgsock, buf, nchars)) < 0)
                            {
                                perror("Writing stream message");
                                rc = 1;
                                break;
                            }

                    }
```

```
                }
            else
                {
                    do
                        {
                            if((rval = read(msgsock, buf, (BUFSIZ-1))) < 0)
                                perror("Reading stream message");
                            else if (rval == 0)
                                /* printf("Ending connection\n"); */
                                rc = 1;
                            else{
                                *(buf + rval) = '\0';
                                write(STDOUT, buf, rval);
                            }
                        }while (rval > 0);
                }

        }
```

All sockets are closed before exit.

```
    close(msgsock);
    close(sock);
/*     unlink(NAME); */
    exit(rc);
}
```

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