Foundations of meta-programming

Martin Berger

Cambridge, 9 August 2016

Based on joint work with Laurie Tratt and Christian Urban.

Research programme

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To set the scene, let's remember why λ-calculus is good, and how meta-programming came about.

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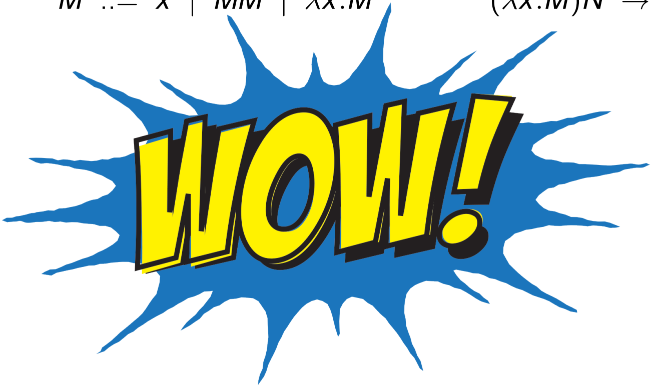

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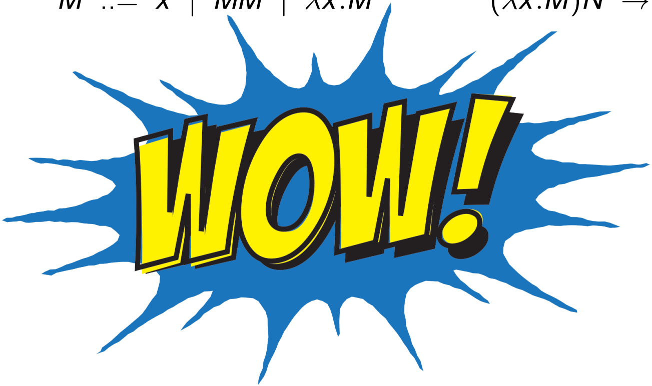

The point of theory is to **simplify**, to focus on essence.

Real-world programming languages:

Real-world programming languages:

- Strings
- Unicode
- FFI
- Backwards compatibility
- Modules
- Performance
- Ergonomics
- ...

In other words a real programming language is:

Let's do theory


META

Meta-programming: L-programs as data in L'.

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Homogeneous meta-programming: MP where $L = L'$.

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Homogeneous meta-programming: MP where $L = L'$.

Homogeneous generative meta-programming (HGMP) is the generation of programs by a program as the latter is being either compiled or executed.

Meta-programming is simple if you don’t care about **convenient** handling of programs as data. Just use strings.

Research on meta-programming is about **convenient**, **principled**, **general purpose** and **safe** handling of programs as data.

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But first ...

History

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Haskell might have been the first typed mainstream language with principled HGMP support (2002).

And then there is ...

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Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I[1]).

Von **Kurt Gödel** in Wien.

1.

Die Entwicklung der Mathematik in der Richtung zu größerer Exaktheit hat bekanntlich dazu geführt, daß weite Gebiete von ihr formalisiert wurden, in der Art, daß das Beweisen nach einigen wenigen mechanischen Regeln vollzogen werden kann. Die umfassendsten derzeit aufgestellten formalen Systeme sind das System der Principia Mathematica (PM)[2]) einerseits, das Zermelo-Fraenkelsche (von J. v. Neumann weiter ausgebildete) Axiomensystem der Mengenlehre[3]) andererseits. Diese beiden Systeme sind so weit, daß alle heute in der Mathematik angewendeten Beweismethoden in ihnen formalisiert, d. h. auf einige wenige Axiome und Schlußregeln zurückgeführt sind. Es liegt daher die Vermutung nahe, daß diese Axiome und Schlußregeln dazu ausreichen, alle mathematischen Fragen, die sich in den betreffenden Systemen überhaupt formal ausdrücken

Arithmetisation of syntax

„0" . . . 1	„$\vee$" . . . 7	„(" . . . 11
„f" . . . 3	„Π" . . . 9	„)" . . . 13
„$\sim$" . . . 5		

ferner den Variablen n-ten Typs die Zahlen der Form p^n (wo p eine Primzahl > 13 ist). Dadurch entspricht jeder endlichen Reihe von Grundzeichen (also auch jeder Formel) in eineindeutiger Weise eine endliche Reihe natürlicher Zahlen. Die endlichen Reihen natürlicher Zahlen bilden wir nun (wieder eineindeutig) auf natürliche Zahlen ab, indem wir der Reihe $n_1, n_2, \ldots n_k$ die Zahl $2^{n_1} . 3^{n_2} \ldots p_k{}^{n_k}$ entsprechen lassen, wo p_k die k-te Primzahl (der Größe nach) bedeutet. Dadurch ist nicht nur jedem Grundzeichen, sondern auch jeder endlichen Reihe von solchen in eineindeutiger Weise eine natürliche Zahl zugeordnet. Die dem Grundzeichen (bzw. der Grundzeichenreihe) a zugeordnete Zahl bezeichnen wir mit $\Phi(a)$. Sei nun irgend eine Klasse oder Relation $R(a_1, a_2 \ldots a_n)$ zwischen Grundzeichen oder Reihen von solchen gegeben. Wir ordnen ihr diejenige Klasse (Relation) $R'(x_1, x_2 \ldots x_n)$ zwischen natürlichen Zahlen zu, welche dann und nur dann zwischen $x_1, x_2 \ldots x_n$ besteht, wenn es solche $a_1, a_2 \ldots a_n$ gibt, daß $x_i = \Phi(a_i)$ $(i = 1, 2, \ldots n)$ und $R(a_1, a_2 \ldots a_n)$ gilt. Diejenigen Klassen und Relationen natürlicher

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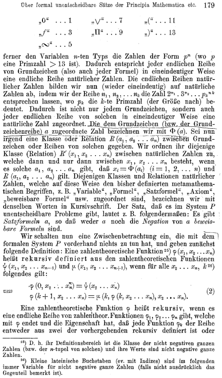

Über formal unentscheidbare Sätze der Principia Mathematica etc. 179

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Wir schalten nun eine Zwischenbetrachtung ein, die mit dem formalen System P vorderhand nichts zu tun hat, und geben zunächst folgende Definition: Eine zahlentheoretische Funktion[25] $\varphi(x_1, x_2 \ldots x_n)$ heißt **rekursiv definiert** aus den zahlentheoretischen Funktionen $\psi(x_1, x_2 \ldots x_{n-1})$ und $\mu(x_1, x_2 \ldots x_{n+1})$, wenn für alle $x_2 \ldots x_n$, k[26] folgendes gilt:

$$\begin{aligned} \varphi(0, x_2 \ldots x_n) &= \psi(x_2 \ldots x_n) \\ \varphi(k+1, x_2 \ldots x_n) &= \mu(k, \varphi(k, x_2 \ldots x_n), x_2 \ldots x_n). \end{aligned} \quad (2)$$

Eine zahlentheoretische Funktion φ heißt **rekursiv**, wenn es eine endliche Reihe von zahlentheor. Funktionen $\varphi_1, \varphi_2 \ldots \varphi_n$ gibt, welche mit φ endet und die Eigenschaft hat, daß jede Funktion φ_k der Reihe entweder aus zwei der vorhergehenden rekursiv definiert ist oder

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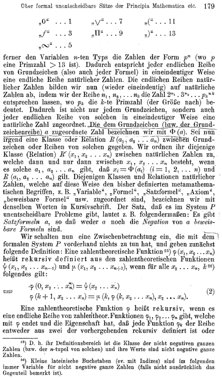

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Much work done by proof theorists and the dependent types community, connection with 'our' meta-programming not clear to me. See *J. Harrison*, *Metatheory and Reflection in Theorem Proving: A Survey and Critique* (1995) for more.

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In summary, meta-programming enables abstractions without run-time penalty. Thus MP resolves the tension between abstraction and performance, albeit **at the cost of increasing language complexity**.

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Tooling hard, e.g. debugging.

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No convincing way of deriving semantics of embedded DSL from embedding

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Good news 2: good solutions are immediately relevant for industry.

Good news 3: problems look fairly tractable, no $P \stackrel{?}{=} NP$-like difficulties. Lot's of theory ready to go, e.g. nominal techniques, proof assistants.

Why are these problems still open?

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Even humor can be based on this difficulty:

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All PL researchers are liars.

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Adding HGMP is deceptively easy to the untrained eye ... just add a data type representing programs ...

How hard can it be?

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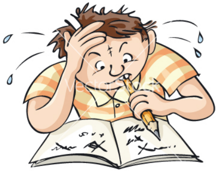
Remember: real programming languages are a mess

How hard can it be?

Remember: real programming languages are a mess

Adding HGMP to mess, means we need to create meta-mess, meta-meta-mess ... and think about how mess, meta-mess, meta-meta-mess ... relate.



Time for theory

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... that means we focus on the **essential features** of HGMP, and **nothing else.**

- Language representation (code as data)
- Language levels (base, meta, meta-meta ...)
- Navigation between language levels
- Computation is driven by the base-language

Let's simplify

We ignore:

- Hygiene
- Types
- Notions of equality
- Beauty of syntax
- Efficiency, performance
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But: **What base language?**

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What we really want is to add HGMP to **arbitrary** base languages. I.e. an explicit **function** $HGMP(\cdot)$:

$$L \mapsto L'$$

taking a programming language L as input and returning as output a language L' that is the HGMPified version of L.

Why arbitrary base language?

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Remember real-world programming languages:

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automatised ...

Why arbitrary base language?

Remember real-world programming languages:

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automatised ...

... so we can experiment with languages without being drowned in uninteresting minutiae.

Research hypothesis

The foundation of meta-programming is the **function** *HGMP*$(\cdot)$:

$$\frac{\lambda\text{-calculus}}{\text{Functional programming}} = \frac{\color{red}{\mathit{HGMP}(\cdot)}}{\text{Meta-programming}}$$

Note that $HGMP(\cdot)$ is a theoretical tool, it's not intended to give nice result, e.g. good looking syntax.

But ...

Does *HGMP*$(\cdot)$ exist at all (as a computable algorithm)?

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But first: PL empiricism: the HGMP design space

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- What kind of MP?
- When is MP executed?
- How are programs represented as data?

HGMP design space: What kind of MP?

- **Homogeneous MP:** the object- and the meta-language are identical (examples: Racket, Template Haskell, MetaOcaml, Converge).
- In **heterogeneous MP** object- and the meta-language are different (example: compiler written in C from Java to x86).

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We restrict our attention to homogeneous meta-programming.

HGMP design space: What kind of MP?

- **Generative MP:** where an object-program is generated (put together) by a meta-program.
- **Intensional MP:** where an object-program is analysed (taken apart) by a meta-program, e.g. reflection.

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HGMP design space: How are programs represented as data?

- Using strings.
- ADTs (algebraic data types).
- Higher-level language support, e.g. upMLs, downMLs, quasi-quotes, inserts and splices.

HGMP design space: How are programs represented as data?

- Using strings.
- ADTs (algebraic data types).
- Higher-level language support, e.g. upMLs, downMLs, quasi-quotes, inserts and splices.

Let's look at them briefly. Consider the following criteria:

- Syntactic overhead
- Support for generating only 'valid' programs
- Expressivity

Strings

Strings

Example, selector function that chooses the *i*-the component from an *n*-tuple.

```
let pi_1_0 =  fun ( x ) -> x;;
let pi_2_0 =  fun ( x, _  ) -> x;;
let pi_2_1 =  fun ( _, x ) -> x;;
let pi_3_0 =  fun ( x, _, _  ) -> x;;
let pi_3_1 =  fun ( _, x, _  ) -> x;;
let pi_3_2 =  fun ( _, _, x ) -> x;;
let pi_4_0 =  fun ( x, _, _, _  ) -> x;;
let pi_4_1 =  fun ( _, x, _, _  ) -> x;;
let pi_4_2 =  fun ( _, _, x, _  ) -> x;;
let pi_4_3 =  fun ( _, _, _, x ) -> x;;
let pi_5_0 =  fun ( x, _, _, _, _  ) -> x;;
let pi_5_1 =  fun ( _, x, _, _, _  ) -> x;;
let pi_5_2 =  fun ( _, _, x, _, _  ) -> x;;
let pi_5_3 =  fun ( _, _, _, x, _  ) -> x;;
let pi_5_4 =  fun ( _, _, _, _, x ) -> x;;
```

selector.ml

Advantages of string-based MP

- Flexible, expressive.
- Easy to do for basic MP.
- Ubiquitous support.
- Not restricted to a single target language.

Disadvantages of string-based MP

- No language support for constructing syntactically correct programs.
- No support for “hygiene”, i.e. sane management of free and bound variables. Hygiene is hard to add for strings.

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NB Lack of hygiene is sometimes useful, especially in large-scale MP.

We reject strings in our foundational approach.

ADTs

ADTs

```
sealed abstract class Binop
   case class Add () extends Binop
   case class Sub () extends Binop
   case class Mul () extends Binop
   case class Div () extends Binop
   case class Eq () extends Binop

sealed abstract class Term
   case class CInt ( n : Int ) extends Term
   case class Op2 ( m : Term, op : Binop, n : Term ) ext
   case class Var ( x : Int ) extends Term
   case class App ( m : Term, n : Term ) extends Term
   case class Lam ( x : Int, m : Term ) extends Term
   case class Rec ( f : Int, x : Int, m : Term ) extends
   case class If ( c : Term, m : Term, n : Term ) extend
```

We call this representation AST (abstract syntax tree), the workhorse of HGMP.

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Disadvantages:

- Verbose.
- No support for “hygiene”, i.e. sane management of free and bound variables. But hygiene is easy to add.

What we really want is ...

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... to combine the terseness of strings with the guarantees of syntactic correctness that ASTs offer.


META

UpMLs and downML

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Enter **upMLs** and **downMLs**, another good idea from logic, first introduced in Lisp.

UpMLs (AKA quasi-quotes, backquotes) are quotes with holes. In the holes we can execute arbitrary programs that produces code.

DownMLs (AKA splices or inserts) are the corresponding un-quotation mechanism.

UpMLs and downML

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UpML (Up MetaLevel) represent AST (or AST-like) structures by quoted chunks of normal program syntax. We have chosen the term 'upMLs' to highlight an important relationship with downMLs.

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UpMLs are “syntactic sugar” for ASTs.

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NB: DownMLs make sense only within the process of AST generation.

UpML and DownML in Racket

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```
Welcome to Racket v6.5.
> (quasiquote (+ 2 3))
'(+ 2 3)

> (eval (quasiquote (+ 2 3)))
5

> (define f  (lambda (x) (quasiquote (* (unquote x) (unquote x)))))

> (f 5)
'(* 5 5)

> (eval (f 5))
25

>  (define g (lambda (x) (quasiquote (* 8 (unquote (f x))))))

> (g 3)
'(* 8 (* 3 3))

> (eval (g 3))
72
```

racket.r

UpMLs and hygiene

UpMLs and hygiene

By default upMLs in Racket, MetaOCaml, Converge and other languages are hygienic, i.e. prevent capture of free variables. This is an implementation choice, but not necessary. Advanced MP languages like Racket or Converge offer both, capturing and non-capturing behaviour.

UpMLs and hygiene

By default upMLs in Racket, MetaOCaml, Converge and other languages are hygienic, i.e. prevent capture of free variables. This is an implementation choice, but not necessary. Advanced MP languages like Racket or Converge offer both, capturing and non-capturing behaviour.

We will strictly separate both concepts, i.e. our upMLs are not hygienic.

HGMP design space: When is MP executed?

- At **compile-time:** e.g. the Lisp family, Template Haskell, Converge, C++. We call this **CTMP**.
- At **run-time:** e.g. the MetaML family, Javascript, printf-based MP. We call this **RTMP**.

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The difference is subtle. The result of CTMP is '**frozen**' (e.g. by saving the produced executable), multiple evaluations of a CTMP'ed program can be done with one compilation. RTMP'ed programs are **regenerated on every run**. Whether that leads to observable differences depends on the available language features.

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```
print(1);
print(2 + eval(print(3); ASTInt(4)))
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```
print(1);
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```

yields ... `3 1 6`

The `3` is printed during compilation, the `1 6` is printed every time the compiled code is run.

Modern languages and HGMP

Modern languages and HGMP

Language	Strings	ASTs	UpMLs	CT-HGMP	RT-HGMP
Converge	●	●	●	●	●
JavaScript	●	○	○	○	●
Lisp	●	●	●	●	●
MetaML	○	○	●	○	●
Haskell	○	●	●	●	○
Scala	○	●	●	●	●

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$$M \quad ::= \quad x \mid MN \mid \lambda x.M \mid c \mid M + N \mid \ldots$$

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$$\begin{array}{rcl} M & ::= & \ldots \mid \mathrm{ast}_{\mathrm{t}}(\tilde{M}) \\ t & ::= & \mathrm{var} \mid \mathrm{app} \mid \mathrm{lam} \mid \mathrm{int} \mid \mathrm{string} \mid \mathrm{add} \mid \ldots \end{array}$$

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An AST constructor $\mathsf{ast}_{\mathsf{t}}(\tilde{M})$ takes $|M|+1$ arguments. **Tag** t specifies the specific AST datatype. The rest is relative to that datatype.

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$\text{ast}_{\text{lam}}(\text{ast}_{\text{string}}(''x''), \text{ast}_{\text{var}}(''x''))$ is the AST of $\lambda x.x$

Notice something?

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Adding ASTs **mirrors** the syntax of the language. We make a 'copy' of the base language.

This is not λ-specific, we'd do the same for any other base.

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- AST gets 'spliced into' the rest of the AST the compiler is constructing
- Compilation proceeds

Operational semantics of the foundational calculus

We keep the usual $\Downarrow_\lambda$ from λ-calculus, but now add a second phase:

$$\underbrace{M \Downarrow_{ct}}_{\text{compile-time}} \quad A \quad \underbrace{\Downarrow_\lambda \; V}_{\text{run-time}}$$

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  - then M' is evaluated using $\Downarrow_\lambda$, the usual CBV evaluation of λ-calculus, giving an AST A
  - Then $\Downarrow_{dl}$ de-ASTifies A, and splice into rest of program

Idea: $\Downarrow_{ct}$ scans for $\downarrow\{\cdot\}$ and eliminates them by evaluation and splicing.

$$\frac{}{x \Downarrow_{ct} x}\ \textsc{Var ct} \qquad \frac{M \Downarrow_{ct} A \quad N \Downarrow_{ct} B}{MN \Downarrow_{ct} AB}\ \textsc{App ct} \qquad \frac{M \Downarrow_{ct} N}{\lambda x.M \Downarrow_{ct} \lambda x.N}\ \textsc{Lam ct}$$

$$\frac{}{c \Downarrow_{ct} c}\ \textsc{Const ct} \qquad \frac{M \Downarrow_{ct} A \quad N \Downarrow_{ct} B}{M + N \Downarrow_{ct} A + B}\ \textsc{Add ct}$$

$$\frac{M_i \Downarrow_{ct} N_i}{\mathrm{ast_t}(\tilde{M}) \Downarrow_{ct} \mathrm{ast_t}(\tilde{N})}\ \textsc{Ast}_c\ \textsc{ct} \qquad \frac{M \Downarrow_{ct} A \quad A \Downarrow_{\lambda} B \quad B \Downarrow_{dl} C}{\downarrow\{M\} \Downarrow_{ct} C}\ \textsc{DownML ct}$$

$\Downarrow_{dl}$

Idea: $\Downarrow_{dl}$ removes one layer of ASTs, i.e. goes down a meta-level.

$$\frac{}{\mathsf{ast}_{\mathsf{var}}(''x'') \Downarrow_{dl} x}\ \textsc{Var dl} \qquad \frac{M \Downarrow_{dl} M' \quad N \Downarrow_{dl} N'}{\mathsf{ast}_{\mathsf{app}}(M, N) \Downarrow_{dl} M'N'}\ \textsc{App dl}$$

$$\frac{M \Downarrow_{dl} ''x'' \quad N \Downarrow_{dl} N'}{\mathsf{ast}_{\mathsf{lam}}(M, N) \Downarrow_{dl} \lambda x.N'}\ \textsc{Lam dl} \qquad \frac{}{\mathsf{ast}_{\mathsf{int}}(n) \Downarrow_{dl} n}\ \textsc{Int dl}$$

$$\frac{}{\mathsf{ast}_{\mathsf{string}}(''x'') \Downarrow_{dl} ''x''}\ \textsc{String dl} \qquad \frac{M \Downarrow_{dl} M' \quad N \Downarrow_{dl} N'}{\mathsf{ast}_{\mathsf{add}}(M, N) \Downarrow_{dl} M' + N'}\ \textsc{Add dl}$$

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Note that non-ASTs have no $\Downarrow_{dl}$ rules, they are stuck.

Scoping

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$$\frac{L \Downarrow_{\lambda} M \quad M \Downarrow_{dl} N \quad N \Downarrow_{\lambda} N'}{\mathrm{eval}(L) \Downarrow_{\lambda} N'}\ \textsc{Eval rt}$$

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$$\frac{M \Downarrow_{ct} N}{\mathrm{eval}(M) \Downarrow_{ct} \mathrm{eval}(N)}\ \textsc{Eval ct} \qquad \frac{M \Downarrow_{dl} N}{\mathrm{ast}_{\mathrm{eval}}(M) \Downarrow_{dl} \mathrm{eval}(N)}\ \textsc{Eval dl}$$

$$\frac{L \Downarrow_\lambda M \quad M \Downarrow_{dl} N \quad N \Downarrow_\lambda N'}{\mathrm{eval}(L) \Downarrow_\lambda N'}\ \textsc{Eval rt}$$

Note that eval is **not** 'disappeared' at compile-time.

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$$M ::= \ldots \mid \mathsf{tag}_t \qquad\qquad t ::= \ldots \mid \mathsf{promote}$$

Hence AST datatype $\mathsf{ast}_{\mathsf{promote}}(M, \tilde{N})$ which allows an arbitrary AST with a tag M and parameters $\tilde{N}$ to be promoted up a meta-level. Promoted ASTs can then be reduced one meta-level with the existing $\Downarrow_{dl}$ relation. E.g.:

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$$\mathsf{ast}_{\mathsf{promote}}(\mathsf{string}, \mathsf{ast}_{\mathsf{string}}("x")) \Downarrow_{dl} \mathsf{ast}_{\mathsf{string}}("x")$$

Operational semantics of higher-order ASTs

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$$\frac{}{\mathsf{tag}_{\mathsf{t}} \Downarrow_{dl} \mathsf{tag}_{\mathsf{t}}}\ \textsc{Promote Tag}$$

$$\frac{L \Downarrow_{dl} \mathsf{tag}_{\mathsf{t}} \quad t \neq \mathsf{promote} \quad \ldots \quad M_i \Downarrow_{dl} N_i \quad \ldots}{\mathsf{ast}_{\mathsf{promote}}(L, \tilde{M}) \Downarrow_{dl} \mathsf{ast}_{\mathsf{t}}(\tilde{N})}\ \textsc{Promote dl 1}$$

$$\frac{L \Downarrow_{dl} \mathsf{tag}_{\mathsf{promote}} \quad M \Downarrow_{dl} \mathsf{tag}_{\mathsf{t}} \quad \ldots \quad N_i \Downarrow_{dl} N_i' \quad \ldots}{\mathsf{ast}_{\mathsf{promote}}(L, M, \tilde{N}) \Downarrow_{dl} \mathsf{ast}_{\mathsf{promote}}(\mathsf{tag}_{\mathsf{t}}, \tilde{N}')}\ \textsc{Promote dl 2}$$

$\Downarrow_{\lambda}$ and $\Downarrow_{ct}$ are unchanged as rules, but work on larger set of programs.

UpMLs (aka quasi-quotes)

We have now finished, and obtained a λ-calculus with CTMP and RTMP.

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$$\uparrow\{2\} \quad \Downarrow_{ct} \quad \mathsf{ast}_{\mathsf{int}}(2)$$

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Like downMLs, upMLs are disappeared by the compile-time stage.

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Let's reuse $\downarrow\{\cdot\}$!

A downML $\downarrow\{\cdot\}$ inside $\uparrow\{...\downarrow\{M\}...\}$ is a 'hole' where arbitrary computation can be executed to produce an AST. This AST is then used as is. For example:

$$\uparrow\{2+\downarrow\{\mathrm{ast}_{\mathrm{int}}(7)\}\} \quad \Downarrow_{ct} \quad \mathrm{ast}_{\mathrm{add}}(\mathrm{ast}_{\mathrm{int}}(2), \mathrm{ast}_{\mathrm{int}}(7))$$

$$\uparrow\{2+\downarrow\{\uparrow\{3+4\}\}\} \quad \Downarrow_{ct} \quad \mathrm{ast}_{\mathrm{add}}(\mathrm{ast}_{\mathrm{int}}(2), \mathrm{ast}_{\mathrm{add}}(\mathrm{ast}_{\mathrm{int}}(3), \mathrm{ast}_{\mathrm{int}}(4)))$$

Operational semantics for $\uparrow\{M\}$

We introduce a new reduction relation $\Downarrow_{ul}$:

$$\frac{M \Downarrow_{ul} A}{\uparrow\{M\} \Downarrow_{ct} A}\ \textsc{UpML ct} \qquad \frac{M \Downarrow_{ct} A}{\downarrow\{M\} \Downarrow_{ul} A}\ \textsc{DownML ul}$$

$$\frac{}{''x'' \Downarrow_{ul} \mathsf{ast}_{\mathsf{string}}(''x'')}\ \textsc{String ul} \qquad \frac{M \Downarrow_{ul} A \quad N \Downarrow_{ul} B}{MN \Downarrow_{ul} \mathsf{ast}_{\mathsf{app}}(A, B)}\ \textsc{App ul}$$

$$\frac{M \Downarrow_{ul} A}{\lambda x.M \Downarrow_{ul} \mathsf{ast}_{\mathsf{lam}}(\mathsf{ast}_{\mathsf{string}}(''x''), A)}\ \textsc{Lam ul} \qquad \frac{}{\mathsf{tag}_{\mathsf{t}} \Downarrow_{ul} \mathsf{tag}_{\mathsf{t}}}\ \textsc{Tag ul}$$

$$\frac{M \Downarrow_{ul} A}{\mathsf{eval}(M) \Downarrow_{ul} \mathsf{ast}_{\mathsf{eval}}(A)}\ \textsc{Eval ul} \qquad \frac{M \Downarrow_{ul} A \quad A \Downarrow_{ul} B}{\uparrow\{M\} \Downarrow_{ul} B}\ \textsc{UpML ul}$$

$$\frac{}{x \Downarrow_{ul} \mathsf{ast}_{\mathsf{var}}(''x'')}\ \textsc{Var ul} \qquad \frac{\ldots \quad M_i \Downarrow_{ul} A_i \quad \ldots}{\mathsf{ast}_{\mathsf{t}}(\tilde{M}) \Downarrow_{ul} \mathsf{ast}_{\mathsf{promote}}(\mathsf{tag}_{\mathsf{t}}, \tilde{A})}\ \textsc{Ast ul}$$

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Thus RT-HGMP and CT-HGMP are neatly connected as two facets of the same AST-coin.

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- Lifting, where semi-arbitrary **run-time** values to be lifted up a meta-level, e.g. $\text{lift}(3) \Downarrow_\lambda \text{ast}_{\text{int}}(3)$.
- Cross-level variable scoping.

Staged power function $\lambda nx.x^n$

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How do we prove this, when existing approaches to HGMP diverge from our proposals?

Reflective equilibrium, balance or coherence between model and PL reality.

If you have HGMP phenomena that don't agree with our calculus, please contact us.

HGMP$(\cdot)$: mechanical HGMPification of languages

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Nothing in the HGMPification of λ-calculus depended on λ-calculus being the source language. The process was completely generic.

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Add appropriate reduction rules for ASTs, upMLs and downMLs with computation driven by the base language. Note that $\mathit{HGMP}(\lambda)$ does not change the reduction rules of λ-calculus itself. **Only adds rules.**

$HGMP(\cdot)$ semi-formally

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Assume C is the set of L's program constructors, L_{mp}'s constructors and tags then:

$$T = C \cup \{\mathrm{eval}, \mathrm{promote}\}$$

$$C_{mp} = C \cup \{\mathrm{eval}, \downarrow\{\_\}, \uparrow\{\_\}\} \cup \{\mathrm{ast}_{\mathrm{t}} \mid t \in T\} \cup \{\mathrm{tag}_{\mathrm{t}} \mid t \in T\}$$

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The arities and binders of the new syntax are as follows:

- If $c \in C$ then its arity and binders are unchanged in C_{mp}.
- $\mathrm{ast_c}$ has the same arity as $c \in C$ and no binders.
- $\mathrm{ast_{promote}}$ has variable arity, or, equivalently has arity 2, with the second argument being of type list. There are no binders.
- $\mathrm{ast_{eval}}$ has arity 1 and no binders.
- $\mathrm{tag_t}$ has arity 0 and no binders for $t \in T$.
- eval, $\downarrow\{\_\}$, and $\uparrow\{\_\}$ have arity 1 and no binders.

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We add the following rules to the operations rules of L (omitting rules for upML for simplicity).

$$\frac{t \in T}{t \Downarrow_\lambda t} \qquad \frac{L \Downarrow_\lambda M \quad M \Downarrow_{dl} N \quad N \Downarrow_\lambda N'}{\mathrm{eval}(L) \Downarrow_\lambda N'}$$

$$\frac{\ldots \quad M_i \Downarrow_\lambda N_i \quad \ldots \quad t \in T}{\mathrm{ast}_{\mathrm{t}}(\tilde{M}) \Downarrow_\lambda \mathrm{ast}_{\mathrm{t}}(\tilde{N})}$$

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Constructors with binders are most easily explained by example. If c has arity 2, with the first argument being a binder, the following rule must be added:

$$\frac{M \Downarrow_{dl} \text{''}x\text{''} \quad N \Downarrow_{dl} N'}{\mathrm{ast}_{\mathrm{c}}(M, N) \Downarrow_{dl} c(x, N')}$$

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$$\frac{L \Downarrow_{dl} \mathrm{tag}_{\mathrm{t}} \quad M_i \Downarrow_{dl} N_i \quad t \in T}{\mathrm{ast}_{\mathrm{promote}}(L, \tilde{M}) \Downarrow_{dl} \mathrm{ast}_{\mathrm{c}}(\tilde{N})}$$

$$\frac{L \Downarrow_{dl} \mathrm{tag}_{\mathrm{promote}} \quad M \Downarrow_{dl} \mathrm{tag}_{\mathrm{t}} \quad N_i \Downarrow_{dl} R_i}{\mathrm{ast}_{\mathrm{promote}}(L, M, \tilde{N}) \Downarrow_{dl} \mathrm{ast}_{\mathrm{promote}}(\mathrm{tag}_{\mathrm{t}}, \tilde{R})} \qquad \frac{t \in T}{\mathrm{tag}_{\mathrm{t}} \Downarrow_{dl} \mathrm{tag}_{\mathrm{t}}}$$

$HGMP(\cdot)$

Assuming we wish to enable compile-time HGMP, a $\Downarrow_{ct}$ relation must be added:

$$\frac{M \in \{x, ``x"\} \cup \{\mathrm{tag_t} \mid t \in T\}}{M \Downarrow_{ct} M} \qquad \frac{M \Downarrow_{ct} N}{\mathrm{eval}(M) \Downarrow_{ct} \mathrm{eval}(N)}$$

$$\frac{M_i \Downarrow_{ct} N_i \quad c \in C}{c(\tilde{M}) \Downarrow_{ct} c(\tilde{N})} \qquad \frac{M_i \Downarrow_{ct} N_i \quad t \in T}{\mathrm{ast_t}(\tilde{M}) \Downarrow_{ct} \mathrm{ast_t}(\tilde{N})}$$

$$\frac{t \in T}{\mathrm{tag_t} \Downarrow_{ct} \mathrm{tag_t}} \qquad \frac{M \Downarrow_{ct} A \quad A \Downarrow_{\lambda} B \quad B \Downarrow_{dl} C}{\downarrow\{M\} \Downarrow_{ct} C}$$

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- What languages or language features **cannot** be handled satisfactorily by $HGMP(.)$?

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