Asymptotic Approximation by Regular Languages

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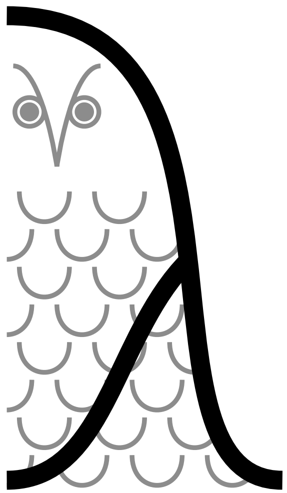


This talk is based on

[S1] Ryoma Sin'ya. Asymptotic Approximation by Regular Languages, *SOFSEM2021* (to appear), draft is available at http://www.math.akita-u.ac.jp/~ryoma

Outline

1. Motivation of this work
2. Set of natural numbers and measure density
3. Density of regular languages and REG-measurability
4. REG-(im)measurability of several languages
5. Open problems

The Primitive Words Conjecture

[Dömösi-Horvath-Ito 1991]

- A non-empty word w is said to be **primitive** if it can not be represented as a power of shorter words, i.e., $w = u^n \Rightarrow u = w$ (and $n = 1$)

  Q_A denotes the set of all primitive words over A.

- The case $\#(A) = 1$ is trivial ($\mathrm{Q}_A = A$). Here after we only consider the case $A = \{a, b\}$ for Q_A, and simply write Q.

Example： $ababa \in \mathrm{Q}$ $ababab = (ab)^3 \notin \mathrm{Q}$

Conjecture: Q is not context-free.

Why is “primitivity” important?

- Primitive words are like prime numbers.
  Fact: For every non-empty word w, there exists a unique primitive word v such that $w = v^k$ for some $k \geq 1$.

- For a word $w = uv$, we denote its *conjugate* (by u) vu by $u^{-1}wu = vu$.
  If u and v are non-empty, $u^{-1}wu$ is called a *proper* conjugate.
  Fact: w is primitive ⇔ $w \neq u^{-1}wu$ for every proper conjugate.

  Note: if we regard a conjugation as a (partial) morphism on words, “w is primitive” means “w has no non-trivial automorphism” (cf. rigid graphs, rigid models in model theory) .

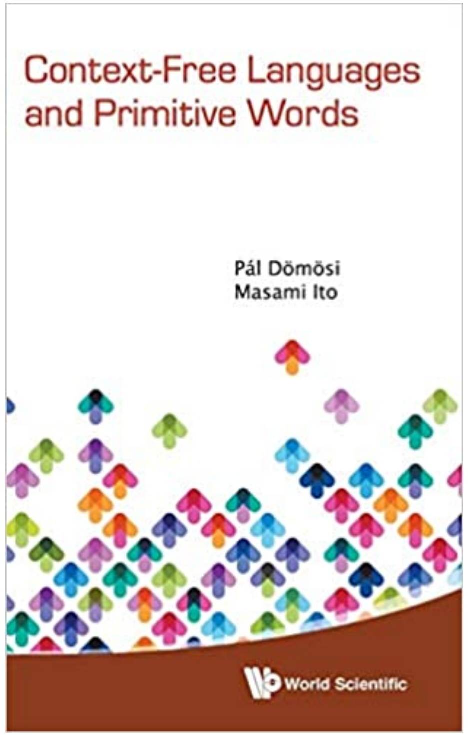
- Primitive words and its special class called *Lyndon words* play a central role in algebraic coding theory and combinatorics on words, also in text compression (cf. Lyndon factorisation, Burrows–Wheeler transformation).

The Primitive Words Conjecture

[Dömösi-Horvath-Ito 1991] **On the Connection between Formal Languages and Primitive Words**

Masami Ito

Pál Dömösi



[Dömösi-Ito 2014]

The Primitive Words Conjecture

[Dömösi-Horvath-Ito 1991] **On the Connection between Formal Languages and Primitive Words**

Masami Ito

Pál Dömösi

Szilárd Fazekas

My motivating intuition

(Intuition 1) Q is "very large" while there is no "good approximation"
by regular languages.

(Intuition 2) Every "very large" context-free language has some
"good approximation" by regular languages.

My (naive) idea: if we can formalise the above intuition and prove it, then the primitive words conjecture is true!

→ I proved that (the formal statement) of Intuition 1 is true, but Intuition 2 is false.

Approximation of languages

We adopt and extend Buck's *measure density*
to formalise "approximation by regular languages".

- Measure density [Buck 1946]

- Rough set approximation [Păun-Polkowski-Skowron 1996]
- Minimal cover-automata [Câmpeanu-Sânten-Yu 1999]
- Minimal regular cover [Domaratzki-Shallit-Yu 2001]
- Convergent-reliability / Slender-reliability [Kappes-Kintala 2004]
- Bounded-ε-approximation [Eisman-Ravikumar 2005]
- Degree of approximation [Cordy-Salomaa 2007]

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Natural density of a subset of $\mathbb{N}$ $(\ni 0)$

- For an arithmetic progression

$$S = \{cn + d \mid n \in \mathbb{N}\}$$

we define its *natural density* $\delta(S)$ as

- if $c = 0$ (i.e., $S = \{d\}$) then $\delta(S) = 0$
- if $c \neq 0$ (i.e., S is infinite) then $\delta(S) = \dfrac{1}{c}$

Intuitively, $\delta(S)$ represents the "largeness" of S. More formally, it represents the **probability** that a randomly chosen natural number n is in S.

Measure density of a subset of $\mathbb{N}$

[Buck 1946] "The measure theoretic approach to density"

- For a set of numbers $S \subseteq \mathbb{N}$, its *outer measure* $\mu^*(S)$ of S is defined as

$$\mu^*(S) = \inf\left\{ \sum_i \delta(X_i) \mid S \subseteq X, X \text{ is a disjoint union of finitely many arithmetic progressions } X_1, \ldots, X_k \right\}$$

- If a set $S \subseteq \mathbb{N}$ satisfies the condition

$$\mu^*(S) + \mu^*(\overline{S}) = 1 \qquad (☆)$$

then we call $\mu^*(S)$ *the measure density of* S, and we say that "S is *measurable*".

Theorem (Buck)：

$$\mathscr{D}_0 \subsetneq \mathscr{D}_\mu$$

- The class $\mathscr{D}_\mu$ of all subsets of $\mathbb{N}$ satisfying (☆) is the *Carathéodory extension* of

$$\mathscr{D}_0 = \left\{ X \subseteq \mathbb{N} \mid X \text{ is a disjoint union of finitely many arithmetic progresssions} \right\}$$

Observation

- $\mathscr{D}_0 = \left\{X \subseteq \mathbb{N} \mid X \text{ is a finitely many disjoint union of arithemtic progressions}\right\}$ can be seen as the class REG_A of regular languages over a unary alphabet $A = \{a\}$:

$$\mathscr{D}_0 = \{\{\,|w|\mid w \in L\} \mid L \in \mathrm{REG}_A\}$$

The set of lengths of words in a regular language L (i.e., the *Parikh image of* L) is a finite union of arithmetic progressions (i.e., *ultimately periodic set*).

If we can define a "density" notion on REG_A for an arbitrary alphabet A, we can naturally extend Buck's measure density to formal languages!

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Density of formal languages

- The *asymptotic density* $\delta_A(L)$ of a language L over A is defined as

$$\delta_A(L) = \lim_{n\to\infty} \frac{\#(L \cap A^n)}{\#(A^n)}$$

- The *density* $\delta_A^*(L)$ is defined as

$$\delta_A^*(L) = \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \frac{\#(L \cap A^i)}{\#(A^i)}$$

Fact: if $\delta_A(L)$ converges then $\delta_A^*(L)$ also converges, and moreover $\delta_A(L) = \delta_A^*(L)$.

But the converse is not true!

trivial example: $L = (AA)^*$

$\delta_A(L) = \bot$ (diverges) but

$\delta_A^*(L) = 1/2$

Density of formal languages

- The *asymptotic density* $\delta_A(L)$ of a language L over A is defined as

$$\delta_A(L) = \lim_{n\to\infty} \frac{\#(L \cap A^n)}{\#(A^n)}$$

- The *density* $\delta^*_A(L)$ is defined as

$$\delta^*_A(L) = \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \frac{\#(L \cap A^i)}{\#(A^i)}$$

Fact1 (cf. [Salomaa-Soittla 1978]): for any regular language L over A, $\delta^*_A(L)$ converges to a rational number.

Fact2 (cf. [S2]): A regular language L is *not null* (i.e., $\delta^*_A(L) \neq 0$) if and only if L is *dense* (i.e., $L \cap A^*wA^* \neq \emptyset$ for any $w \in A^*$).

Not null: measure theoretic "largeness"
Dense: topological "largeness"

Note: "L is not null $\Rightarrow$ L is dense" is true for any language L, but
"L is dense $\Rightarrow$ L is not null" is false for general non-regular languages.

Density of formal languages

Note: "L is not null $\Rightarrow$ L is dense" is true for any language L, but
"L is dense $\Rightarrow$ L is not null" is false for general non-regular languages.

Infinite Monkey Theorem (cf. [Borel 1913]): $\delta_A(A^*wA^*) = 1$ for any $w \in A^*$.

L is not dense means that there exists w such that $L \cap A^*wA^* = \varnothing$
(such word is called a *forbidden word* of L),
thus $\delta_A(L) \leq 1 - \delta_A(A^*wA^*) = 0$ by the infinite monkey theorem.

The *semi-Dyck* language $\mathrm{D} = \{\varepsilon, (), (()), ()(), ((())), \ldots\}$ over $A = \{(,)\}$
is dense, but actually null.

$($ $)(()($ $))$

Density of formal languages

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Fact1 (cf. [Salomaa-Soittla 1978]): for any regular language L over A, $\delta_A^*(L)$ converges to a rational number.

Fact2 (cf. [S2]): A regular language L is *not null* (i.e., $\delta_A^*(L) \neq 0$) if and only if L is *dense* (i.e., $\forall w \in A^*\ L \cap A^*wA^* \neq \emptyset$).

Measure density of languages

- We now consider the Carathéodory extension of the class of regular languages:

For $L \subseteq A^*$, its *outer measure* is defined as
$\overline{\mu}_{\mathrm{REG}}(L) = \inf\{\delta_A^*(R) \mid L \subseteq R \in \mathrm{REG}_A\}$.

We say that L is *REG-measurable* if $\overline{\mu}_{\mathrm{REG}}(L) + \overline{\mu}_{\mathrm{REG}}(\overline{L}) = 1$ holds.

Lemma: the followings are equivalent

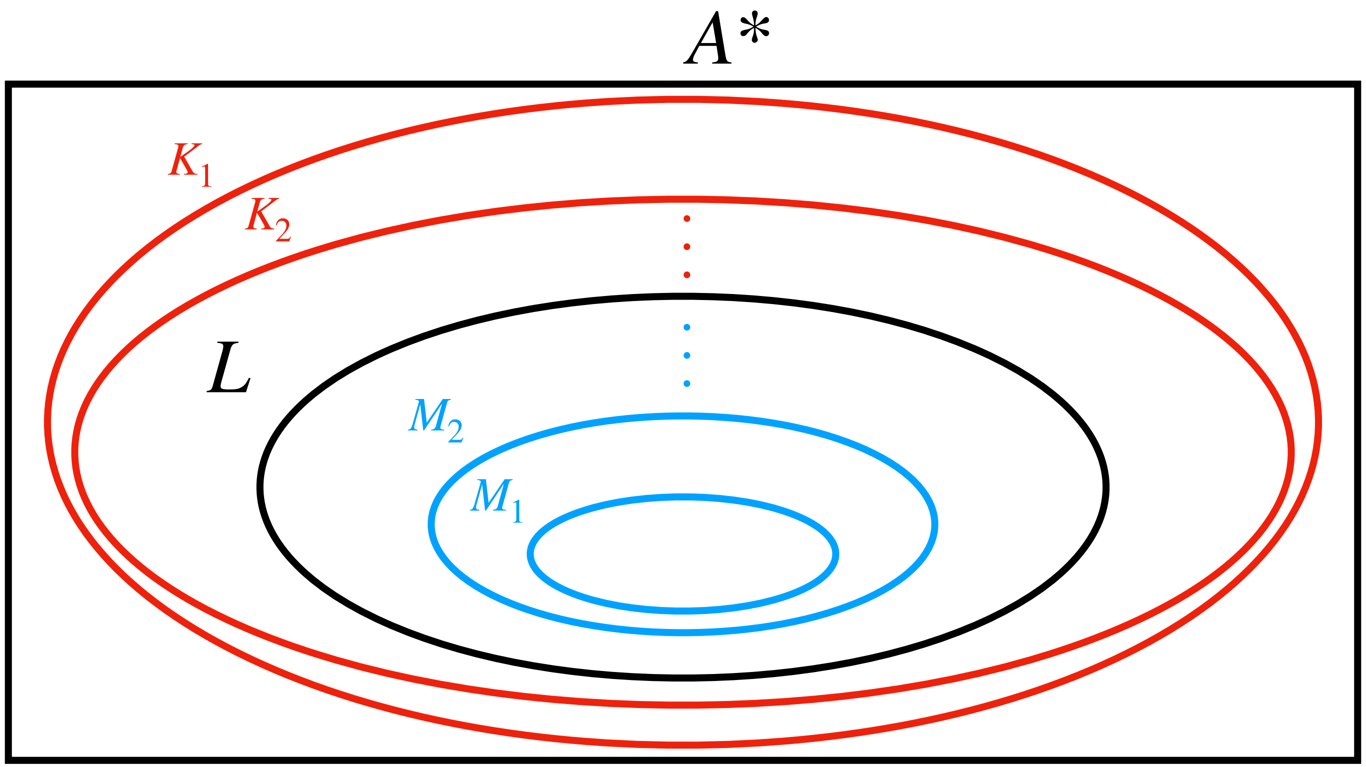
(1) L is REG-measurable

(2) $\overline{\mu}_{\mathrm{REG}}(L) = \underline{\mu}_{\mathrm{REG}}(L) = \sup\{\delta_A^*(R) \mid L \supseteq R \in \mathrm{REG}_A\}$

the *inner measure* of L

Note: $\underline{\mu}_{\mathrm{REG}}(L) \leq \delta_A^*(L) \leq \overline{\mu}_{\mathrm{REG}}(L)$ always holds (if $\delta_A^*(L)$ is defined).

Measure density of languages



L is REG-measurable if we can take an infinite sequence of pairs or regular languages $(M_n \subseteq L \subseteq K_n)_n$ such that $\lim_{n\to\infty} \delta_A^*(K_n \backslash M_n) = 0$.

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Example of REG-measurable CFLs

Theorem:

The semi-Dyck language $\mathsf{D} = \{\varepsilon, ab, aabb, abab, \ldots\}$ over $A = \{a, b\}$ is REG-measurable.

Note: D is null, but there does not exist a null regular superset $\mathsf{D} \subseteq L$.

(D is dense implies $\mathsf{D} \subseteq L$ is dense, and thus L is not null by Fact2)

Proof: Let $L_k = \{w \in A^* \mid |w|_a = |w|_b \mod k\}$ for each $k \geq 1$.

the # of occurrences of a in w

Then, for each $k \geq 1$, $\mathsf{D} \subseteq L_k$ and $\delta_A^*(L_k) = \dfrac{1}{k} \to 0$ (if $k \to \infty$) .

Thus the infinite sequence $(\emptyset, L_k)_{k \geq 1}$ converges to D .

Example of REG-measurable CFLs

Theorem: The following languages are all REG-measurable.

1. $\mathrm{O}_3 = \{w \in \{a, b, c\}^* \mid |w|_a = |w|_b$ or $|w|_a = |w|_c\}$
2. $\mathrm{O}_4 = \{w \in \{x, \bar{x}, y, \bar{y}\}^* \mid |w|_x = |w|_{\bar{x}}$ or $|w|_y = |w|_{\bar{y}}\}$
3. $\mathrm{P} = \{w \in \{a, b\}^* \mid w = \mathrm{reverse}(w)\}$ (the set of all *palindromes*)
4. $\mathrm{G} = \{a^{n_1}ba^{n_2}b\cdots a^{n_k}b \mid k \geq 1, n_i \neq i$ for some $i\}$ (the *Goldstine* language)

Note:

(1) and (2) are inherently ambiguous context-free languages [Flajolet 1985].

The generating function of (4) is transcendental (i.e., not algebraic) [Flajolet 1987], thus (4) is also inherently ambiguous by Chomsky-Schützenberger theorem.

Example of REG-measurable CFLs

Theorem: The following languages are all REG-measurable.

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5. $\mathsf{K} = S_1\{c\}A^* \cup S_2\{c\}A^*$ where $A = \{a,b,c\}$,
   $S_1 = \{a\}\{b^ia^i \mid i \geq 1\}^*$ and $S_2 = \{a^ib^{2i} \mid i \geq 1\}^*\{a\}^+$.

Note: the density of (5) is transcendental [Kemp 1980], thus it is inherently ambiguous by the fact [Berstel 1972] that the density of every unambiguous context-free language is algebraic.

Example of REG-measurable CFLs

Theorem:

For every alphabet A and a language $L \subseteq A$, its *suffix extension by* $c \notin A$

$L' = L\{c\}(A \cup \{c\})^*$ is REG-measurable.

Corollary: $\mathrm{K} = (S_1 \cup S_2)\{c\}A^*$ is REG-measurable (because $S_1, S_2 \subseteq A \backslash \{c\}$).

Corollary: There exist **uncountably many** REG-measurable languages.

REG-gap: complexity of immeasurable sets

- For a language $L \subseteq A^*$ the difference $\overline{\mu}_{\mathrm{REG}}(L) - \underline{\mu}_{\mathrm{REG}}(L)$ of outer and inner measure is called the *REG-gap* of L.

REG-gap represents how a given language is “hard to approximate”.

(Intuition 1) Q is “very large” while there is no “good approximation” by regular languages.

Formal statement: Q is co-null (i.e., $\delta_A^*(\mathrm{Q}) = 1$) but $\underline{\mu}_{\mathrm{REG}}(\mathrm{Q}) = 0$.

(Intuition 2) Every “very large” context-free language has some “good approximation” by regular languages.

Formal statement: Every co-null context-free language L satisfies $\underline{\mu}_{\mathrm{REG}}(L) > 0$.

REG-immesurability of Q

(Intuition 1) Q is "very large" while there is no "good approximation"
by regular languages.

Formal statement: Q is co-null (i.e., $\delta_A^*(\mathrm{Q}) = 1$) but $\underline{\mu}_{\mathrm{REG}}(\mathrm{Q}) = 0$.

Theorem (1): Q is co-null.

Theorem (2): Every regular subset of Q is null. In particular, every non-null regular language contains infinitely many non-primitive words.

Note: The proof of Theorem (2) uses basic semigroup theory
(Green's relation and Green's theorem)

REG-immesurability of context-free langugaes

(Intuition 2) Every "very large" context-free language has some "good approximation" by regular languages.

Formal statement: Every co-null context-free language L satisfies $\underline{\mu}_{\mathrm{REG}}(L) > 0$.

Theorem: A deterministic context-free language $\mathsf{M}_2 = \{w \in \{a,b\}^* \mid |w|_a > 2|w|_b\}$ over $A = \{a,b\}$ is null but $\overline{\mu}_{\mathrm{REG}}(\mathsf{M}_2) = 1$, i.e., whose REG-gap is 1.

Corollary: $\overline{\mathsf{M}_2}$ is co-null (deterministic) context-free language with $\underline{\mu}_{\mathrm{REG}}(\overline{\mathsf{M}_2}) = 0$.

Note: This counter-example is inspired by a result of [Eisman-Ravikumar 2011]. They showed that the *majority language* $\mathsf{M} = \{w \in \{a,b\}^* \mid |w|_a > |w|_b\}$ is "hard to approximate".

REG-immesurability of context-free langugaes

Theorem: A deterministic context-free language

$\mathsf{M}_2 = \{w \in \{a,b\}^* \mid |w|_a > 2|w|_b\}$ over $A = \{a,b\}$ is null

but $\overline{\mu}_{\mathsf{REG}}(\mathsf{M}_2) = 1$, i.e., whose REG-gap is 1.

Proof: $\delta_A^*(\mathsf{M}_2) = 0$ can be shown by using the law of large numbers.

For a regular language L with $\delta_A^*(L) < 1$, we show that $\mathsf{M}_2 \subsetneq L$ (i.e., $\overline{L} \cap \mathsf{M}_2 \neq \emptyset$).

Let $\eta : A^* \to M = A^*/\simeq_{\overline{L}}$ be the syntactic morphism of $\overline{L}$.

$$c = \max_{m \in M} \min_{w \in \eta^{-1}(m)} |w| \qquad a^{4c+1}$$

$\overline{L}$ is non-null implies $\overline{L}$ is dense (infinite monkey theorem)

$\exists x, y$ such that $|x|, |y| \leq c$ and $xa^{4c+1}y \in \overline{L}$

$$|xa^{4c+1}y|_b \leq |x| + |y| \leq 2c < \frac{1}{2}|xa^{4c+1}y|_a$$

Thus $xa^{4c+1}y \in \mathsf{M}_2$ and $\mathsf{M}_2 \subsetneq L$

REG-immesurability of context-free langugaes

(Intuition 2) Every "very large" context-free language has some "good approximation" by regular languages.

Formal statement: Every co-null context-free language L satisfies $\underline{\mu}_{\mathrm{REG}}(L) > 0$.

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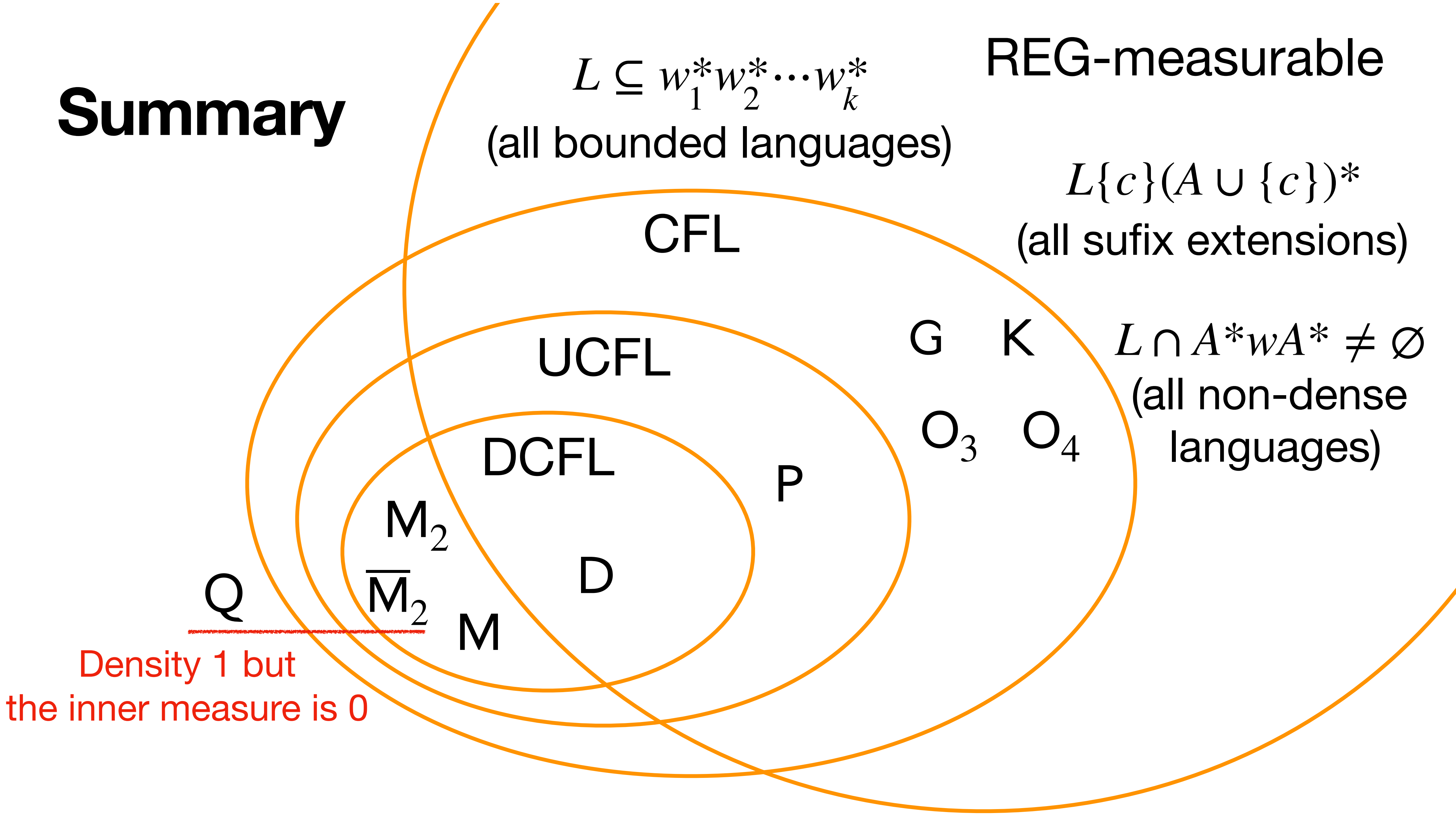
Corollary: $\overline{\mathsf{M}_2}$ is co-null (deterministic) context-free language with $\underline{\mu}_{\mathsf{REG}}(\overline{\mathsf{M}_2}) = 0$.

Summary



$L \subseteq w_1^* w_2^* \cdots w_k^*$
(all bounded languages)
REG-measurable
$L\{c\}(A \cup \{c\})^*$
(all sufix extensions)
CFL
UCFL
DCFL
G
K
$L \cap A^*wA^* \neq \emptyset$
(all non-dense languages)
O_3
O_4
P
M_2
D
Q
$\overline{M}_2$
M
Density 1 but
the inner measure is 0

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Open problems

1. Can we give an alternative characterisation of the class of null (resp. co-null) context-free languages?

   Note: it is **undecidable** whether a given CFG generates null (resp. co-null) CFL
   [Nakamura 2019].

2. Can we give an alternative characterisation of REG-measurable (context-free) languages?

   Note: it is **undecidable** whether a given CFG generates REG-measurable CFL,
   because REG-measurability is preserved under left/right quotients
   thus we can apply Greibach's metatheorem.

Open problems

3. Can we find a language class that “separates” Q and CFLs? i.e., is there a language class $\mathscr{C}$ such that
   - Q has full $\mathscr{C}$-gap but no co-null context-free language has full $\mathscr{C}$-gap, or
   - Q is $\mathscr{C}$-immeasurable but every co-null context-free language is $\mathscr{C}$-measurable?

Note: measurability can be parameterised by a language class $\mathscr{C}$:

Define the outer measure of L over A as

$$\overline{\mu}_{\mathscr{C}} = \{\delta_A^*(K) \mid L \subseteq K \in \mathscr{C}\}$$

and L is said to be $\mathscr{C}$-measurable if $\overline{\mu}_{\mathscr{C}}(L) + \overline{\mu}_{\mathscr{C}}(\overline{L}) = 1$.

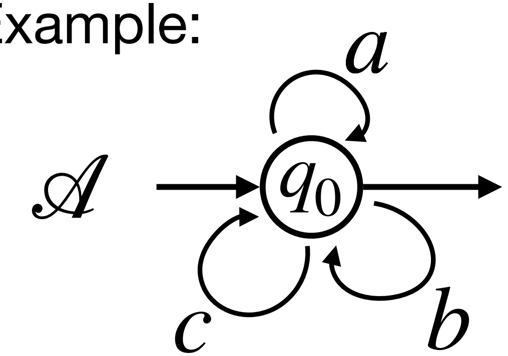
What’s happen if we consider $\mathscr{C} =$ DCFL, UCFL, CFL or UnCA?

Digression: constrained automata

- A constrained automaton is a pair $(\mathscr{A}, S)$ of a finite automaton $\mathscr{A}$ and a semi-linear set $S \subseteq \mathbb{N}^d$ whose dimension d is the # of transition rules of $\mathscr{A}$.
(i.e., Presburger definable set)

$(\mathscr{A}, S)$ accepts a word w iff there exists an accepting run ρ labeled by w and the vector $(n_1, n_2, \ldots, n_d)$ is in S where n_i is the number of occurrences the i-th transition rule in ρ.

Example:



$L((\mathscr{A}, S)) = \text{MIX} = \{w \in \{a, b, c\}^* \mid |w|_a = |w|_b = |w|_c\}$
where $S = \{(n, n, n) \mid n \in \mathbb{N}\}$.

Digression: constrained automata

- The class of **unambiguous** constrained automata is a very well-behaved class:
  - Many counting-type languages (including MIX, O_3, O_4, M and $\overline{\mathrm{M}}_2$) are in UnCA (UnCA = the class of unambiguous constrained automata recognisable languages).
  - Every UnCA language has a holonomic generating function (cf. [Bostan et al. 2020]).
  - UnCA is closed under Boolean operations and quotients [Cadilhac et al. 2012].
  - The regularity for UnCA is decidable [Cadilhac et al. 2012].
  - The context-freeness for some subclass of UnCA is decidable [S3].

Open problems

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2. Can we give an alternative characterisation of REG-measurable (context-free) languages?

3. Can we find a language class that "separates" Q and CFLs? i.e., is there a language class $\mathscr{C}$ such that
   - Q has full $\mathscr{C}$-gap but no co-null context-free language has full $\mathscr{C}$-gap, or
   - Q is $\mathscr{C}$-immeasurable but every co-null context-free language is $\mathscr{C}$-measurable?

(Akita-Inu)

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The full versions are all available at http://www.math.akita-u.ac.jp/~ryoma