

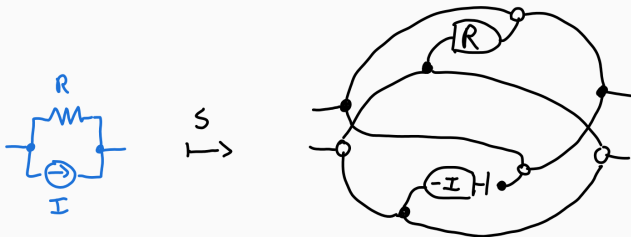
Practical Diagrammatic Electrical Circuit Theory

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Applied applied category theory

We already know we can analyze electrical circuits using diagrams.

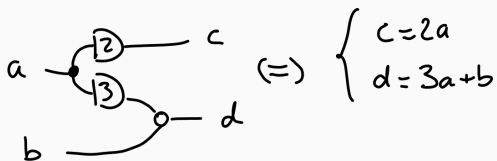


Q: is this practical? A: yes!

Graphical Linear Algebra

Background: Graphical Linear Algebra

Idea: we write affine equations using diagrams.



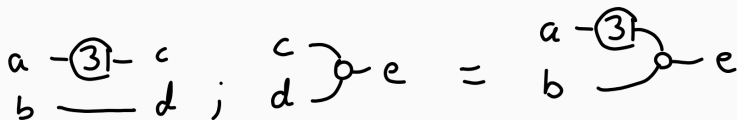
GLA: building blocks

$\begin{matrix} a \\ b \end{matrix} \rightarrow c$	$\bullet - a$	$\begin{matrix} a \\ b \end{matrix} \oplus c$	$\circ - a$	$\vdash a$	$a - \boxed{k} - b$
$a=b=c$	True	$c=a+b$	$a=0$	$a=1$	$b=ka$

also $\leftarrow, \rightarrow, \dots$

GLA: composing

Composition works by identifying connected variables.



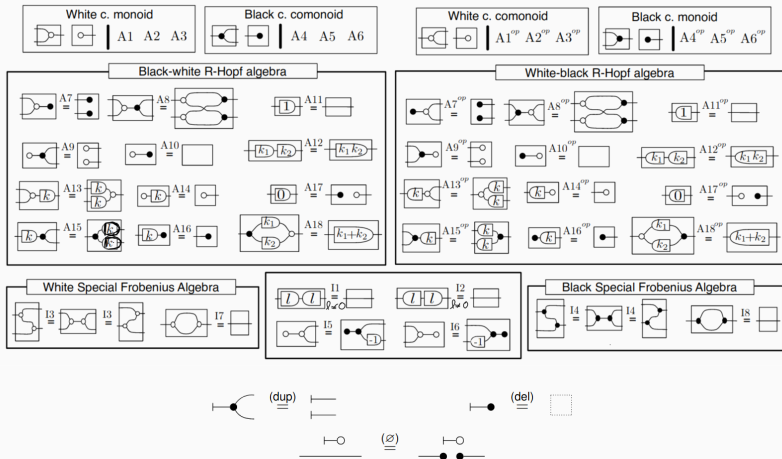
$$\begin{cases} a=3c \\ b=d \end{cases}$$

$$e=c+d$$

$$\exists c, d, \begin{cases} a=3c \\ b=d \\ e=c+d \end{cases}$$

$$\Leftrightarrow e = \frac{1}{3}a + b$$

GLA: reasoning



We can prove any equivalence.

Electrical circuits

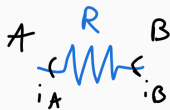
Electrical circuits

Ohm's law:



$$U = R \cdot I$$

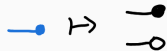
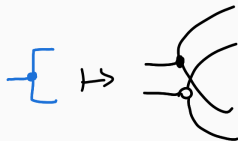
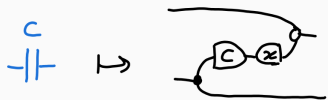
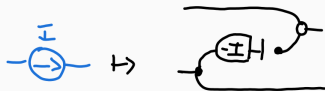
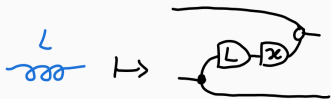
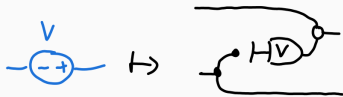
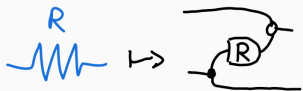
Diagrammatically:



$$\begin{cases} V_B - V_A = R i_A \\ i_A = i_B \end{cases} \quad (\equiv)$$

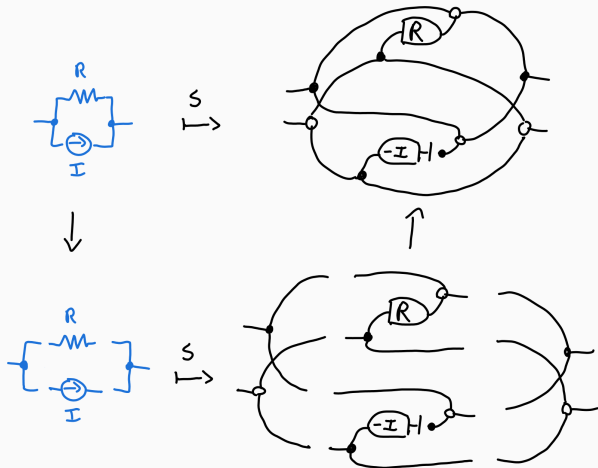


Electrical circuits: building blocks

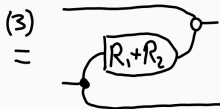
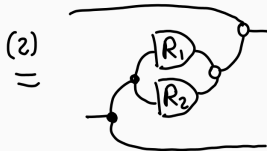
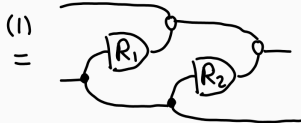
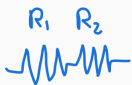


Electrical circuits: composition

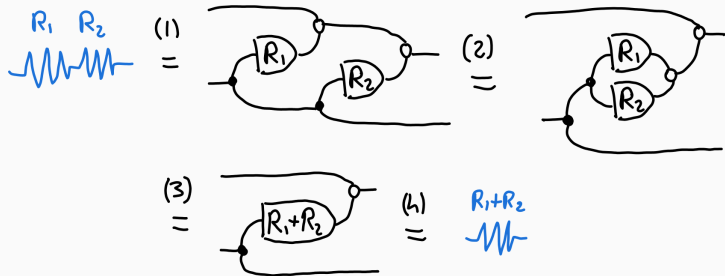
The behavior of a circuit is made by plugging together the behavior of its elements.









Impedance boxes



Impedance boxes



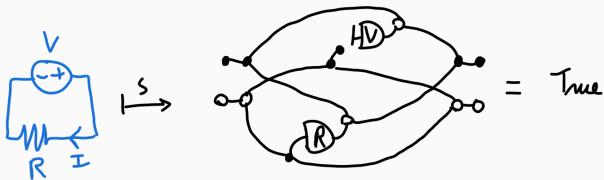
Notice a pattern: write  for the circuit .

Then   = , and  = .

This generalizes complex impedance.

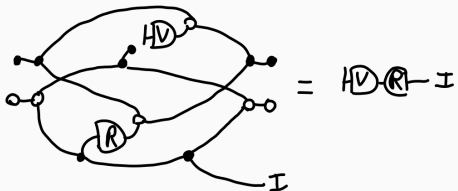
Closed circuits

Problem: a closed circuit has a trivial equation



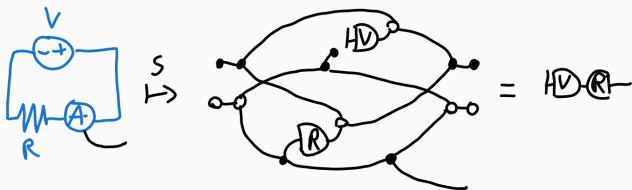
Closed circuits

If only we could tap a wire



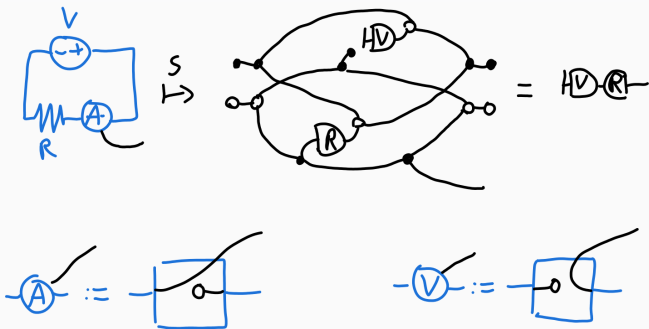
Closed circuits

Solution: add a measuring element



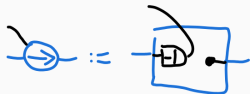
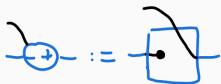
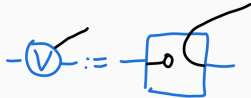
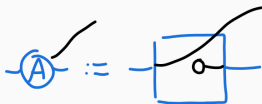
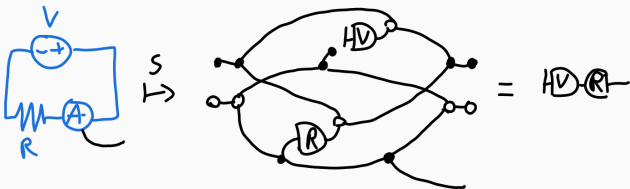
Closed circuits

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Closed circuits

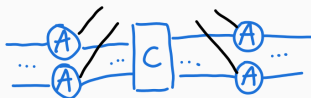
Solution: add a measuring element



Let's prove some theorems!

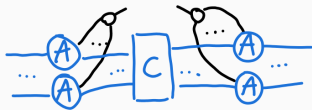
Conservation of currents

Currents entering a circuit C must sum to zero.



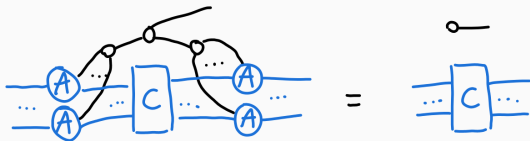
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Conservation of currents

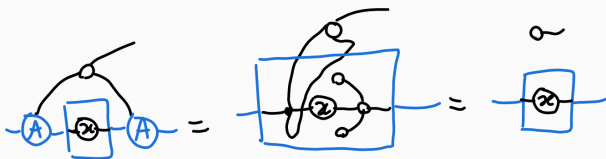
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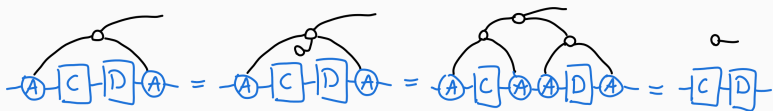
Conservation of currents: proof

Proof: by induction.

Base case:

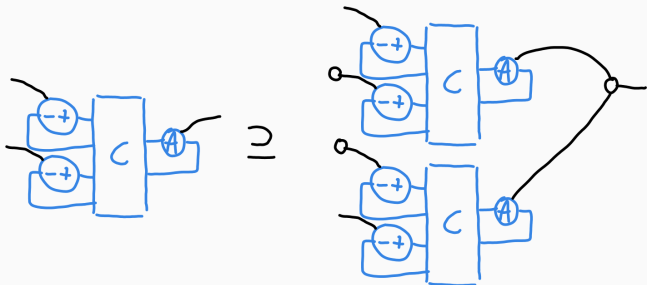


Inductive case:



Superposition theorem

The behavior of a circuit is linear in its sources.



Conclusion

Conclusion

- This is a credible alternative to standard linear algebra.
- Diagrams are good at inductive & topological properties.
- Diagrams are extensible.
- Diagrams are truer to the model because relational.
- This is fun, I want to see more of that.

Questions?
