Dirichlet polynomials and entropy

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$$d(y) = a_n n^y + \ldots + a_2 2^y + a_1 1^y + a_0 0^y$$

$$n^y = \mathtt{FSet}(-, \underline{n})$$

$$0^{\underline{n}} = \begin{cases} 1 & \text{if } n = 0; \\ 0 & \text{if } n \geqslant 1 \end{cases}$$

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$$|d(0)| = \sum_{j=0}^{n} a_j$$

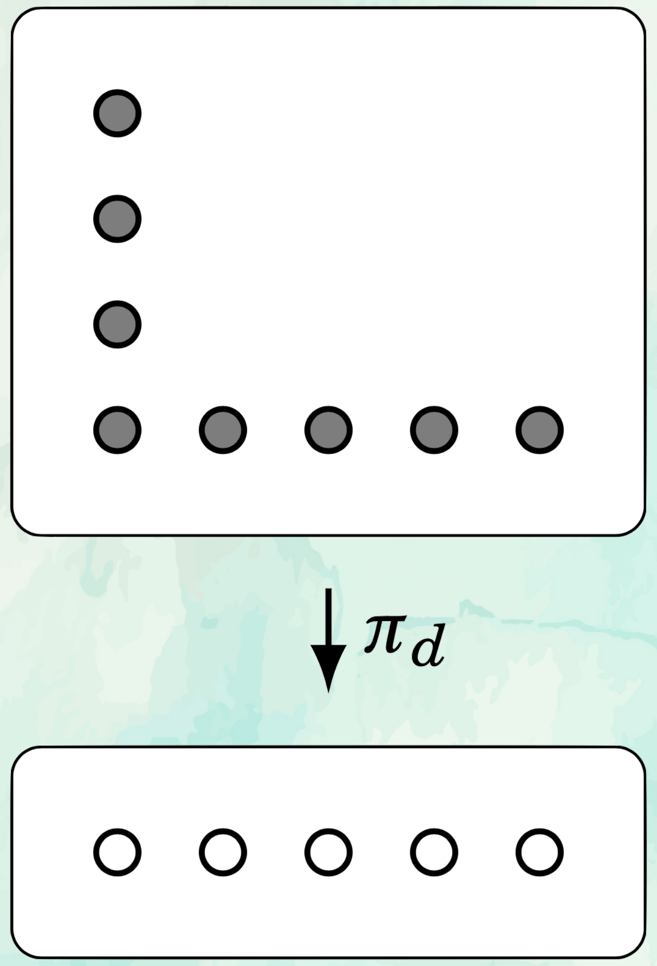
$$|d(1)| = \sum_{j=0}^{n} a_j j$$

$$d(y) = 4^y + 4 \cdot 1^y$$

$d(1) \cong \underline{8} \cong$

$\downarrow \pi_d$

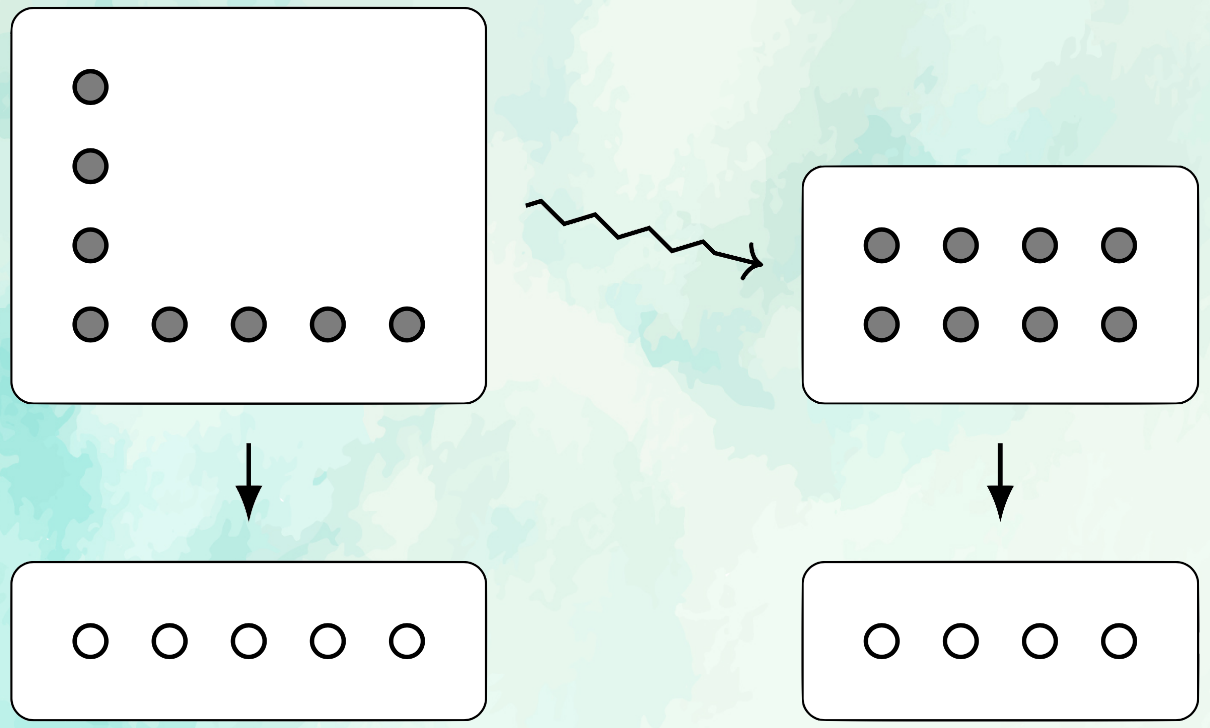
$d(0) \cong \underline{5} \cong$



2

$$(A_1, W_1) + (A_2, W_2) := \left(A_1 + A_2, (W_1^{A_1} W_2^{A_2})^{\frac{1}{A_1 + A_2}}\right)$$

$$(A_1, W_1) \cdot (A_2, W_2) := (A_1 A_2, W_1 W_2)$$



$$h : n^y \mapsto (n, n)$$

Corollary 4.5. *Let* $d \in \mathsf{Dir}$. *Then*

$$A(d) = |d(1)|$$

$$W(d)^{A(d)} = \left|\mathtt{Dir}_{/d(0)}(d, d)\right|.$$

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$$H(d) := -\sum_{i \in d(0)} \frac{|d[i]|}{|d(1)|} \log\left(\frac{|d[i]|}{|d(1)|}\right)$$

$$L(d) := 2^{H(d)}$$

Theorem 5.5. *For all* $d \in \mathsf{Dir}$,

$$A(d) = L(d) W(d).$$

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