Computer Science Admissions Test

— Example only (not actual test) —

Instructions:

· The test duration is 100 minutes. Section A has 8 questions. Section B has 12 questions.

· Questions in Section B are more difficult and worth more marks.

· All questions attempted are marked. We will consider your best 5 questions from each section.

· Write your solutions on the paper provided and clearly mark the question number you are solving. Solutions should be complete and clear, but they need not be thorough (e.g. skip induction proofs).

· Calculators, phones, watches, other electronic devices or other paper are not permitted.

· All paper must be handed in at the end of the test.

· Do not discuss any test questions with others (e.g. candidates at the same or another College, the Internet, or elsewhere), especially before February. You would disadvantage yourself.

It is recommended that you:
- take 5 minutes first to read through all questions,
- start with Section A,
- spend up to around 5 minutes on a question in Section A and
- aim for 5 questions in each section; only attempt more if you finish early.

Good luck!
Section A — aim for 5 questions out of 8, of your choosing

1. Find all integers \( a, b, c \) that satisfy \( a\sqrt{2} - b = c\sqrt{3} \).

2. You decide to take the stairs and to climb either 1 or 2 stairs at a time, at any given time. In how many different ways can you get up a flight of 11 stairs? Prove your answer.

3. What is the units digit of the number \( \sum_{n=1}^{1337} (n!)^4 \) ?

4. 

5. 

6. 

8. Let the function \( f \) be as defined below. What is the value of \( f(1337) \)?

```python
function f(n):
    if n < 1:
        return 1
    else if n is even:
        return 1 + f(n-1)
    else:
        return 1 + f(n-3)
```

Section B — aim for 5 questions out of 12, of your choosing

9. You travel on Earth’s surface south \( n \) miles, then east \( n \) miles, then north \( n \) miles and find yourself back where you started, without visiting any point more than twice. What is the closest you could have been to the south pole when you started? Assume Earth is a sphere with radius \( R > n \).

10. Let \( n < 10 \) be a non-negative integer. How many integers from 0 to 999 inclusive have the sum of their digits equal to \( n \)? Give your answer in terms of \( n \). **Hint:** Try first for integers from 0 to 99.

11. You must slice a square of side length \( b \) into 6 pieces with equal areas, using 3 lines that intersect each other inside the square, one of which is a diagonal. Where on the sides of the square, with respect to the nearest corner, should the other two lines cut? Give your answer in terms of \( b \).

12. 

13. 

14. 

20. Does 30 divide \( n^5 - n \) for all positive integers \( n \)?