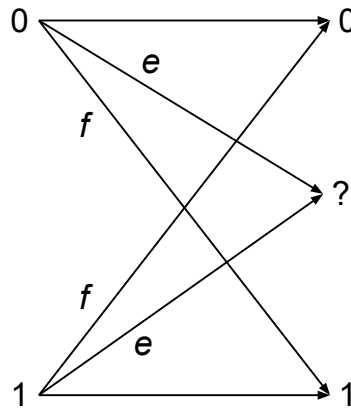


7 Information Theory (rkh23)

- (a) Describe the concepts of discrete entropy and mutual information and how they relate to each other. [4 marks]
- (b) How does your answer to part (a) change when the system is continuous rather than discrete? [2 marks]
- (c) How do entropy and mutual information relate to the capacity of a noisy channel? [3 marks]
- (d) Consider a noisy binary channel with input  $X$  and output  $Y$ . Under what circumstances is  $H(Y|X)$  independent of the distribution of  $X$ ? [3 marks]
- (e) A noisy binary channel is modeled as shown in the diagram below:



The probability of a bit being flipped is  $f$ . The probability of a bit being erased is  $e$ . Derive the capacity of this channel and the probability distribution of the input bits that achieves it. [8 marks]

You may use the following equality without proof:

$$\begin{aligned}
 H(a, 1 - a - b) &= -a \log_2(a) - (1 - a - b) \log_2(1 - a - b) \\
 &= (1 - b) H\left(\frac{a}{1 - b}, 1 - \frac{a}{1 - b}\right) - (1 - b) \log_2(1 - b)
 \end{aligned}$$