

8 Further Graphics (aco41)

- (a) We are given a scene with a point light source at a point \mathbf{p} that emits light of radiance L in all directions, and a plane with equation $\mathbf{n}^T(\mathbf{x} - \mathbf{c}) = 0$ and BRDF f_r . We send a ray from the camera sensor with origin \mathbf{o} and direction \mathbf{d} such that each point on the ray is $\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$ for $t > 0$.
- (i) What is the intersection point of the ray and the plane? [2 marks]
 - (ii) What is the incoming light direction? [1 mark]
 - (iii) What is the direction of the outgoing reflected light towards the sensor origin \mathbf{o} ? [1 mark]
 - (iv) What is the radiance of the light reflected towards the sensor origin \mathbf{o} ? [2 marks]
- (b) A surface represented by an implicit function $f(\mathbf{x})$ is transformed by linear blend skinning such that each point \mathbf{x} is mapped to \mathbf{x}' . There are two bones with transformation matrices \mathbf{T}_1 and \mathbf{T}_2 and corresponding blending weights $w_1(\mathbf{x})$ and $w_2(\mathbf{x}) = 1 - w_1(\mathbf{x})$. In addition, we are given $s_1(\mathbf{x}') = w_1(\mathbf{x})$ and $s_2(\mathbf{x}') = w_2(\mathbf{x})$.
- (i) Write the expression for \mathbf{x}' . [1 mark]
 - (ii) Simplify the expression for \mathbf{x}' assuming no rotations are stored on the bones. Modify the expression further for the case where the weights are not summing up to 1. [2 marks]
 - (iii) What is the implicit function of the transformed surface assuming all blended rotations are invertible? Simplify as much as possible. [3 marks]
 - (iv) What is the implicit function of the transformed surface if there are no rotations stored on the bones? Simplify as much as possible. [2 marks]
- (c) In this question, we will think about how quaternions represent spatial rotations.
- (i) Write the form of a general quaternion and the form of a quaternion representing rotations. [2 marks]
 - (ii) Prove that a quaternion \mathbf{q} and $-\mathbf{q}$ represents the same rotation. [2 marks]
 - (iii) Given a quaternion \mathbf{q} with no real part, what is $\mathbf{q}\mathbf{q}$? [2 marks]