7   Logic and Proof (mj201)

(*a*) Exhibit a model for the following set of formulas, or prove that none exists. Briefly explain your work in each step.

$$P \qquad P \to (R \to Q) \qquad P \vee \neg Q \vee \neg P \qquad Q \to S \wedge \neg T \qquad S \to Q \vee T$$

[6 marks]

(*b*) For each of the following sets of formulas, either exhibit an interpretation in S4 modal logic that satisfies them simultaneously at a particular world, w, or show through a formal proof that they cannot be satisfied.

(*i*) $\Diamond \Box P, \quad Q, \quad \Box \Diamond \Box \neg Q, \quad \Box(P \to \Diamond R \wedge \Diamond \neg R), \quad \Box(\Box \neg Q \vee \neg \Diamond P)$

[8 marks]

(*ii*) $\Box(P \vee Q), \quad \Diamond \neg P, \quad \neg \Diamond Q$

[6 marks]