

COMPUTER SCIENCE TRIPOS Part IB – 2023 – Paper 6

1 Complexity Theory (ad260)

Recall that a (*simple, undirected*) graph G is a set of vertices V along with a set of edges E , where each edge $e \in E$ is a two-element subset of V . For the purpose of this question, all graphs are simple, undirected graphs.

For a graph $G = (V, E)$ and an edge $e \in E$, we write $G - e$ to denote the graph obtained from G by *removing* the edge e . That is $G - e$ has exactly the same vertices as G and all edges in E except for e .

- (a) What is a *Hamiltonian cycle* in a graph $G = (V, E)$? [2 marks]
- (b) What is known about the complexity of deciding whether a given graph G has a Hamiltonian cycle? [2 marks]
- (c) Show that G has a Hamiltonian cycle that does not include the edge e if, and only if, $G - e$ has a Hamiltonian cycle. [4 marks]
- (d) Assume that $P=NP$. Using this assumption, show that there is a polynomial-time algorithm A such that if A is given a graph $G = (V, E)$, it will return “no” if G does not contain a Hamiltonian cycle and return a Hamiltonian cycle of G otherwise. [12 marks]