6 Introduction to Probability (mj201+tms41)

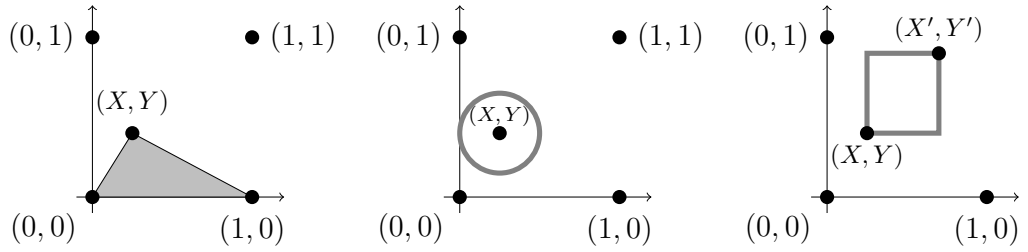
(*a*) Let $X \sim \mathsf{Uni}(0, 1/2)$ be a uniform continuous random variable. What are $\mathbf{E}[X]$ and $\mathbf{V}[X]$? [3 marks]

(*b*) Let $X \sim \mathsf{Uni}(0, 1/2)$ and $Y \sim \mathsf{Uni}(0, 1/2)$ be two independent uniform continuous random variables, and define $Z = \min(X, Y)$.

(*i*) What is the cumulative distribution function of Z? [2 marks]

(*ii*) What is $\mathbf{E}[Z]$? [3 marks]

(*c*) Let $X \sim \mathsf{Uni}(0, 1)$ and $Y \sim \mathsf{Uni}(0, 1)$ be two independent uniform continuous random variables.



(*i*) (*ii*) (*iv*)

(*i*) Consider a random triangle between the three points $(0, 0)$, $(1, 0)$ and (X, Y), as illustrated in the figure above. What is the expectation of the area? [2 marks]

(*ii*) Now consider a random circle with center (X, Y) such that the circumference is as large as possible but remains within the unit-square $[0, 1]^2$ (see figure). What is the expectation of the circumference? [4 marks]

(*iii*) Based on your answer from (*c*)(*ii*), what can you conclude about the expectation of the area of this circle? [2 marks]

(*iv*) Additionally, let $X' \sim \mathsf{Uni}(0, 1)$ and $Y' \sim \mathsf{Uni}(0, 1)$ be two uniform continuous random variables and assume X, Y, X', Y' are mutually independent. Consider a random rectangle with corner points (X, Y) and (X', Y'), which are diagonally opposite. What is the expectation of the circumference? [4 marks]