

7 Information Theory (rkh23)

- (a) Explain the notions of Entropy and Mutual Information. Explain how they relate to channel capacity. For a noisy channel, how does an optimal coding affect the distribution of the input? [6 marks]
- (b) Consider random variables X and Y and let $Z = X + Y$.
- (i) Can $H(X)$ be greater than $H(Z)$? Either prove it cannot or provide a counterexample. [3 marks]
- (ii) If X and Y are independent find an expression for $I(X; Z) - I(Y; Z)$ in terms of $H(X)$ and $H(Y)$ only. [5 marks]
- (c) Consider a random variable, X , and a second random variable $Y = f(X)$, where f is a function. Show that $H(Y) \leq H(X)$ and explain what conditions are necessary for equality. [6 marks]