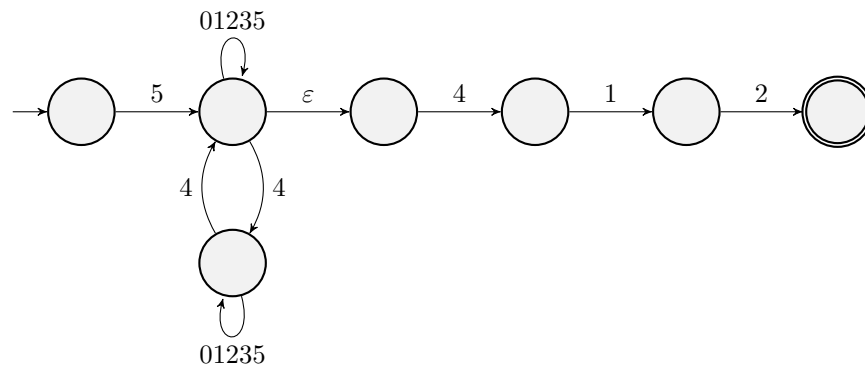


COMPUTER SCIENCE TRIPOS Part IA – 2022 – Paper 2

10 Discrete Mathematics (fms27)

All the formal languages associated with finite automata in this question are defined over the alphabet  $\Sigma = \{0, 1, 2, 3, 4, 5\}$ . [Note: Ensure that any answer DFA you provide is actually a DFA.]

- (a) Build a 4-state DFA  $A_0$  to recognise the set of strings that start with 5 and have an even number of 0s. [2 marks]
- (b) Build a 6-state DFA  $A_1$  to recognise the same language as  $A_0$ . Every state in  $A_1$  must be reachable by some string in  $\Sigma^*$ . [5 marks]
- (c) Here is a 7-state NFA <sup>$\epsilon$</sup>   $A_2$ .



- (i) Find strings  $x, y, z \in \Sigma^*$  such that the following statement is true: “ $A_2$  accepts all and only the strings that start with  $x$ , contain an odd number of  $y$  and end with  $z$ ”. [2 marks]
- (ii) Build a DFA  $A_3$  with no more than seven states that recognises the same language as  $A_2$ . [Hint: Check whether the property in part (c)(i) still holds for your  $A_3$ .] [7 marks]
- (d) For each of the following four strings, state which of  $A_0$  and  $A_2$  recognise it. [Note: Spaces have been inserted for legibility but have no other significance.]
  - (i) 5234 5543 2100 1412
  - (ii) 5555 5500 5031 0041 2
  - (iii) 5430 4041 2
  - (iv) 5421

[4 marks]