7 Further Graphics (aco41)

(*a*) Recall that the (local) rendering equation is given as $L_o(\mathbf{x}, \vec{\omega}_o) = L_e(\mathbf{x}, \vec{\omega}_o) + \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_o) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i d\vec{\omega}_i$. Simplify this expression progressively as much as possible after each of the following assumptions. [5 marks]

(*i*) There is no light emitted from this surface point $\mathbf{x}$.

(*ii*) Diffuse reflection.

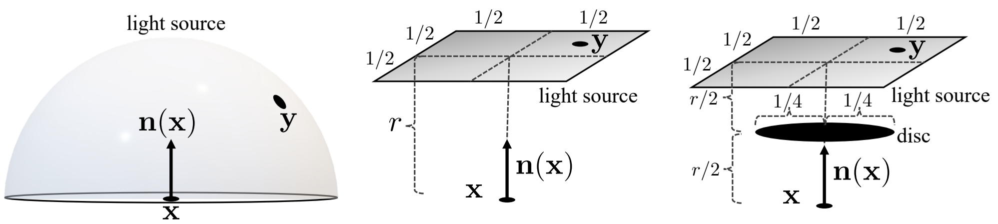
(*iii*) Reflection is constant at all surface points.

(*iv*) The incoming light is the same for all incident angles but there can be occluders.

(*v*) There is a single object and we ignore self-occlusions.

(*b*) Under certain assumptions, we can simplify the rendering equation to the following: $L_o(\mathbf{x}, \vec{\omega}_o) = f_r \int_{H^2} L_i(\vec{\omega}_i) \cos\theta_i d\vec{\omega}_i$. Assume we have a single known geometry for which we can compute surface normals $\mathbf{n}(\mathbf{x})$. Given three measurements from the surface $c_k = L_o(\mathbf{x}_k, \vec{\omega}_o)$ at known points $\mathbf{x}_k$, compute the reflected radiance $L_o(\mathbf{x}, \vec{\omega}_o)$ at an arbitrary point $\mathbf{x}$ on the surface. [*Hint:* If you encounter a linear system $\mathbf{A}\mathbf{x} = \mathbf{b}$, assume $\mathbf{A}^{-1}$ is known.] [4 marks]

(*c*) Recall that for direct illumination, we have the following form for the reflection equation: $L_r(\mathbf{x}, \mathbf{z}) = \int_A f_r(\mathbf{x}, \mathbf{y}, \mathbf{z}) L_i(\mathbf{x}, \mathbf{y}) G(\mathbf{x}, \mathbf{y}) dA(\mathbf{y})$. Starting from this form, provide answers to the following in terms of the provided parameters.



(*i*) Assuming a surface point $\mathbf{x}$ lit by a hemispherical light source with constant emitted light radiance L_i (left in the figure), derive the equation for the reflected light at $\mathbf{x}$ if the BRDF is a constant c. [*Hint:* $\int \cos\theta \sin\theta d\theta \equiv -0.5\cos^2\theta$] [4 marks]

(*ii*) Assuming a surface point $\mathbf{x}$ with a constant BRDF c lit by a planar 1×1 square light source distance r away (middle of the figure) where the emitted light radiance decays as $L_i(\mathbf{x}, \mathbf{y}) = 1/\cos^4\theta$ with θ the angle between the emitted light direction and plane normal, derive the equation for the reflected light at $\mathbf{x}$ (the surface normal $\mathbf{n}(\mathbf{x})$ is orthogonal to the planar light source). [4 marks]

(*iii*) Now assume we put an occluding opaque disc of radius 0.25 parallel to the light plane and centred at $\mathbf{x}$ between the light plane and $\mathbf{x}$ at a distance $r/2$ from $\mathbf{x}$ (right in the figure). Derive the equation for the reflected light at $\mathbf{x}$. [3 marks]