2 Artificial Intelligence (sbh11)

A *Boolean satisfiability problem* has four variables, x_1, x_2, x_3 and x_4. A literal l can be a variable or its negation, denoted $\bar{l}$. The formula of interest, in conjunctive normal form (CNF), is

$$f = (x_2 \vee \overline{x_3}) \wedge (\overline{x_2} \vee x_3) \wedge (x_1 \vee x_2 \vee \overline{x_4}). \qquad (1)$$

The aim is to find assignments to the variables such that f is true under the usual rules for Boolean operations. This question addresses the use of more general *constraint satisfaction* to solve this problem.

(*a*) Give a general description of a *constraint satisfaction problem (CSP)*. [3 marks]

(*b*) Explain how a Boolean satisfiability problem in CNF form and with n variables can be converted to a CSP, also having n variables and having a suitable constraint for each clause. Illustrate your answer using the 4-variable formula f in (1). [3 marks]

(*c*) Explain, again using a constraint corresponding to a clause from (1), how general constraints can be converted to binary constraints. Provide a graph illustrating the problem from (1) after it has been converted to a CSP with only binary constraints. [4 marks]

(*d*) Explain, how *forward checking* works in the context of a general CSP. How does this benefit a CSP solver? [3 marks]

(*e*) Using the CSP equivalent you developed for (1), with only binary constraints, demonstrate how *forward checking* works using the sequence of assignments $x_1 = F$, $x_2 = F$, $x_4 = T$. [5 marks]

(*f*) How would you expect the solution obtained when applying forward checking to be affected if constraints were allowed to propagate more widely? [2 marks]