## 9 Semantics of Programming Languages (nk480)

(*a*) Suppose that $l : \mathsf{intref} \in \Gamma$. Now, consider the following program equivalence for L1:

$$(\mathsf{if}\ !l \leq 0\ \mathsf{then}\ e_1\ \mathsf{else}\ e_2); e_3 \simeq^{\Gamma}_{\mathsf{unit}} (\mathsf{if}\ !l \leq 0\ \mathsf{then}\ e_1; e_3\ \mathsf{else}\ e_2; e_3)$$

(*i*) Explain informally but carefully why this equivalence holds. [3 marks]

(*ii*) Using the definition of semantic equivalence, prove that this equivalence holds. [7 marks]

(*b*) Now, consider the following *non*-equivalence:

$$e_3; (\mathsf{if}\ !l \leq 0\ \mathsf{then}\ e_1\ \mathsf{else}\ e_2) \not\simeq^{\Gamma}_{\mathsf{unit}} (\mathsf{if}\ !l \leq 0\ \mathsf{then}\ e_3; e_1\ \mathsf{else}\ e_3; e_2)$$

(*i*) Give a well-typed example exhibiting a counterexample of this equivalence. [5 marks]

(*ii*) Give a statically decidable condition under which the transformation is valid. [5 marks]