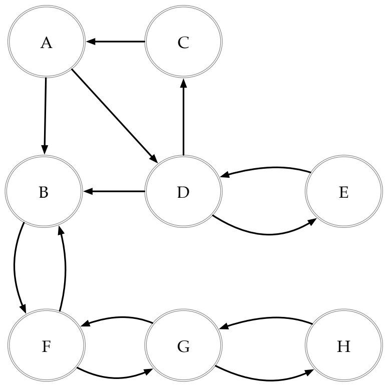
7 Machine Learning and Real-world Data (sht25)

Consider the directed graph shown in the figure below, which expresses cooperation amongst individuals (A, B, . . . , H) in a fishing village. The meaning of an edge from X to Y is that X has asked Y for advice or help during fishing at least once.



(*a*) Consider the betweenness centrality of each individual in this network, which is listed in the following table.

A	B	C	D	E	F	G	H
6	12	2	9	0	12	8	0

(*i*) Give a definition of the betweenness centrality of a node. [1 mark]

(*ii*) Explain intuitively why B and F have the highest betweenness centralities and why E and H have betweenness centralities of 0. [2 marks]

(*b*) We now look at what happens if the network is converted into an undirected network.

(*i*) What is the diameter of this network and why? Your question should include a definition of diameter. [2 marks]

(*ii*) Do the betweenness centralities of nodes A and C change, and why? Explain in terms of affected paths. [3 marks]

(*iii*) Consider the general case of two near-identical graphs S and T, where S is a directed graph and T is the undirected version of S, i.e., every edge (u, v) in S is replaced by an undirected edge (u, v) in T. Which of the following statements are true about the betweenness centrality of any pair of nodes X_S and X_T, which are in identical relative position in the graphs? Justify your answer or provide a counter example.

[continued . . .]

(A) The betweenness centrality of X_S is always at least that of X_T. [2 marks]

(B) The betweenness centrality of X_S is always equal to that of X_T. [1 mark]

(C) The betweenness centrality of X_S is always at most that of X_T. [2 marks]

(*c*) In directed graphs, the in-degree of a node v is defined as the number of incoming edges (u, v), whereas the node's out-degree is defined as the number of outgoing edges (v, u).

(*i*) What does high in-degree and out-degree mean in the context of the fishing collaboration? [2 marks]

(*ii*) Directed graphs are called "strongly connected" if there exists a path from every node to every other node. Is the graph in Figure 1 strongly connected? Justify your answer. [2 marks]

(*iii*) What is the relation between strong connectedness of a directed graph and its nodes' in- and out-degrees? [3 marks]