9 Discrete Mathematics (mpf23)

(*a*) Prove that $4^n + 6n - 1 \equiv 0 \pmod 9$ for all natural numbers n. [5 marks]

(*b*) (*i*) State one of the standard characterizations of the reflexive-transitive closure $R^\star \subseteq A \times A$ of a binary relation R on a set A. [1 mark]

(*ii*) For the given characterization for a binary relation R on a set A, prove that $R^\star = \bigcup_{n \in \mathbb{N}} R_n$ where $R_0 = \emptyset$ and, for $n \in \mathbb{N}$, $R_{n+1} = \mathrm{id}_A \cup (R \circ R_n)$. You may use standard results provided that you state them clearly. [8 marks]

(*c*) Let $\mathcal{F} \subseteq \mathcal{P}(\mathbb{N})$ be a family of pairwise-disjoint subsets of natural numbers; that is, such that $\forall\, S, T \in \mathcal{F}.\ S \neq T \implies S \cap T = \emptyset$.

State whether or not the set $\mathcal{F}$ may be uncountable and prove your claim. [6 marks]