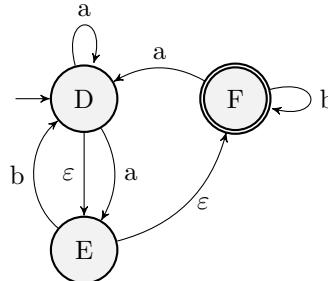


COMPUTER SCIENCE TRIPOS Part IA – 2021 – Paper 2

10 Discrete Mathematics (fms27)

(a) Consider the following NFA $^\epsilon$ , whose input alphabet is  $\{a, b, c\}$ .



- (i) For each of the two strings  $abc$  and  $bba$ , state whether the automaton accepts it, with justification. [2 marks]
  - (ii) Using the subset construction, produce the full unoptimized state transition table of an equivalent DFA, listing its states in lexicographic order (important!) and indicating the starting and accepting states. [6 marks]
  - (iii) Give a regular expression, no longer than six symbols (metacharacters included), that describes the strings accepted by the automaton, together with an intuitive explanation for it. [Hint: Part (a)(ii) helps.] [4 marks]
- (b) Consider language  $L_1$  of strings over alphabet  $\{0, 1\}$ , defined inductively as follows.

$$\overline{00} \quad (0) \qquad \frac{w}{1w} \quad (1) \qquad \frac{w}{w1} \quad (2)$$

- (i) Draw the diagram of a DFA that recognizes  $L_1$  in no more than four states. [4 marks]
- (ii) Considering the words in  $L_1$  as unsigned binary numerals, let language  $L_2$  of strings over  $\{0, 1\}$  be the set of all and only the binary numerals obtained by adding 1 to any numeral in  $L_1$  and removing any leading zeros. NB: “adding” here means arithmetic addition, not string concatenation. Produce a regular expression no longer than 11 symbols that recognizes  $L_2$ , with a clear and convincing explanation of how you derived it. [4 marks]