5 Introduction to Probability (mj201)

(*a*) A travel agency is surveying their customer satisfaction by randomly polling 300 of their customers. From experience, 80% of their customers are typically happy with their service. Let H be the number of happy customers in the current poll.

(*i*) Randomly polling 300 different customers, specify a suitable distribution for H, including its parameters, expected value and variance.   [1 mark]

(*ii*) State a suitable approximation of H and specify its distribution including its parameters, and compute the expected value and variance.   [2 marks]

(*iii*) Using the approximation from Part (*a*)(*ii*), what is the probability that more than 220 and fewer that 260 customers are happy in the current poll?
[4 marks]

(*iv*) Now, let X be the proportion of customers that are happy in the current poll. Following your approximation from Part (*a*)(*ii*), give the distribution for X, including its parameters, expected value and variance.   [3 marks]

(*b*) Let X and Y have a joint density function

$$f(x, y) = \begin{cases} cx & \text{if } 0 < y < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

(*i*) Find the value of the constant $c > 0$.   [4 marks]

(*ii*) Find the marginal density functions of X and Y.   [4 marks]

(*iii*) Are X and Y independent? Justify your answer.   [2 marks]