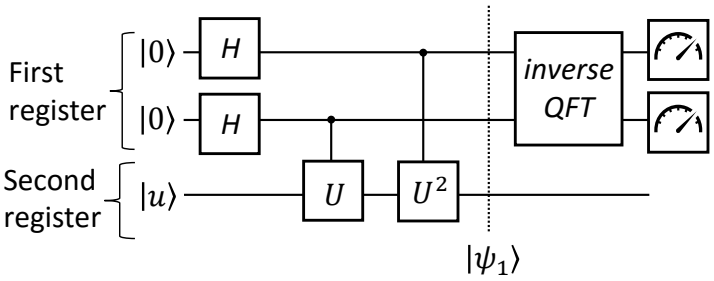
COMPUTER SCIENCE TRIPOS Part II – 2020 – Paper 9

14 Quantum Computing (sjh227)



(*a*) Show that $U = \frac{1}{\sqrt{5}}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ is unitary. [1 mark]

(*b*) Show that $\begin{bmatrix} 1+\sqrt{5} \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1-\sqrt{5} \\ 2 \end{bmatrix}$ are (un-normalised) eigenvectors of U, and give the eigenvalues. [2 marks]

(*c*) The figure shows the quantum circuit for quantum phase estimation to two bits of precision for a single-qubit unitary. Quantum phase estimation is performed to two bits of precision with $U = \frac{1}{\sqrt{5}}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ and $|u\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, find the quantum state, $|\psi_1\rangle$, before the inverse quantum Fourier transform is executed. [7 marks]

(*d*) What possible measurement outcomes can occur (i.e., after the inverse quantum Fourier transform, with measurement in the computational basis)? Give probabilities for each possible outcome. [6 marks]

(*e*) Give two applications of quantum phase estimation, and for each give the state, $|u\rangle$, in which the second register should be prepared, and briefly outline how these can be prepared in practise. [4 marks]