

6 Computation Theory (amp12)

- (a) Define the Church numerals for zero ($\underline{0}$), one ($\underline{1}$) and for an arbitrary natural number (\underline{n}). [2 marks]
- (b) Define encodings of Booleans as λ -terms (\mathbf{True} , \mathbf{False} and \mathbf{If}). [1 mark]
- (c) Explain what it means for a λ term to *represent* a number-valued partial function of n numerical arguments; do the same for one returning Boolean instead of numerical results. [3 marks]
- (d) Give λ -terms that represent the following functions:
- (i) successor (\mathbf{Succ}) [1 mark]
- (ii) test for zero (\mathbf{Eq}_0) [1 mark]
- (e) Define encodings of pairing and projections (\mathbf{Pair} , \mathbf{Fst} and \mathbf{Snd}). [2 marks]
- (f) What function $\mathbb{N} \rightarrow \mathbb{N}$ is represented by the following λ -term? Carefully justify your answer.

$$\lambda x. \mathbf{Snd}(x(\lambda y. \mathbf{Pair}(\mathbf{Succ}(\mathbf{Fst} y))(\mathbf{Fst} y))(\mathbf{Pair} \underline{0} \underline{0}))$$

[6 marks]

- (g) Give with justification a λ -term that represents the function mapping each pair of numbers (m, n) to \mathbf{True} if $m \leq n$ and to \mathbf{False} otherwise. [Hint: use the λ -terms from parts (d)(ii) and (f).] [4 marks]