

7 Denotational Semantics (amp12)

- (a) Suppose that  $(D, \sqsubseteq)$  is a poset which is chain-complete but does not have a least element, and that  $f : D \rightarrow D$  is a continuous function.
- (i) Give an example of such  $(D, \sqsubseteq)$  and  $f$  for which  $f$  has no fixed point. [1 mark]
- (ii) If  $d \in D$  satisfies  $d \sqsubseteq f(d)$ , prove that there is a least element  $e \in D$  satisfying  $d \sqsubseteq e = f(e)$ . [Hint: consider the method used to prove Tarski's fixed point theorem.] [7 marks]
- (b) (i) Define the notion of *contextual equivalence* for the language PCF. (You need not describe the syntax and semantics of PCF.) [2 marks]
- (ii) State the *compositionality*, *soundness* and *adequacy* properties of the denotational semantics of PCF. Explain why they imply that any two closed PCF terms of the same type with equal denotations are contextually equivalent. [8 marks]
- (iii) Give, without proof, an example of two contextually equivalent PCF terms that have unequal denotation. [2 marks]