COMPUTER SCIENCE TRIPOS Part II – 2019 – Paper 9

1 Advanced Algorithms (tms41)

(a) Consider the definition of an approximation algorithm.

(i) Explain the meaning of approximation ratio in the case of a maximisation problem. [2 marks]

(ii) How is this definition adjusted to the case of a randomised approximation algorithm? [2 marks]

(b) State the definition of PTAS and FPTAS. [4 marks]

(c) Let $G = (V, E)$ be an undirected graph. For any $k \geq 1$, define $G^{(k)}$ to be the undirected graph $(V^{(k)}, E^{(k)})$, where $V^{(k)}$ is the set of all ordered k-tuples of vertices from V and $E^{(k)}$ is defined so that $(v_1, v_2, \ldots, v_k)$ is adjacent to $(w_1, w_2, \ldots, w_k)$ if and only if $\{v_1, v_2, \ldots, v_k, w_1, w_2, \ldots, w_k\}$ forms a clique.

(i) Argue that the graph $G^{(k)}$ can be constructed in time polynomial in n (for any fixed value of k). [3 marks]

(ii) Prove that the size of the maximum clique in $G^{(k)}$ is equal to the k-th power of the size of the maximum clique in G. [5 marks]

(iii) Argue that if there is a polynomial-time approximation algorithm that has a constant approximation ratio for finding a maximum clique, then there is a polynomial-time approximation scheme (PTAS) for the problem.
Hint: Your PTAS should be based on applying the given approximation algorithm with constant approximation ratio to $G^{(k)}$ for a proper choice of $k > 0$. Then use the equivalence in part *(ii)* to analyse its approximation ratio. [4 marks]