## 9 Logic and Proof (lp15)

(a) Prof Blunder was using a SAT solver to solve some propositional logic problems he found in a textbook, presented in clause form. Unfortunately, he typed in the problems incorrectly, making five types of error.

In each of the following cases, briefly indicate what can be deduced about the original problem from the SAT solver output for the modified problem. Consider both possible outputs for the SAT solver: reporting "unsatisfiable" and outputting a model.

(i) Mistyping some occurrences of a propositional symbol so that it becomes two different symbols.

(ii) Mistyping two different propositional symbols such that they become the same symbol.

(iii) Splitting a clause in two, e.g. replacing $\{P, \neg Q, R\}$ by $\{P, \neg Q\}$ and $\{R\}$.

(iv) Deleting a clause.

(v) Moving a literal from one clause to another.

[10 marks]

(b) This part is concerned with Binary Decision Diagrams. Use the variable ordering $P$, $Q$, $R$.

(i) Write down the BDDs for $P \wedge Q \wedge R$ and $(\neg R \wedge Q) \to P$. There is no need to show your work. [2 marks]

(ii) Use the results above to obtain the BDD of

$$[P \wedge Q \wedge R] \leftrightarrow [(\neg R \wedge Q) \to P] \leftrightarrow P,$$

showing your working. [8 marks]

*Hint*: In $A \leftrightarrow B \leftrightarrow C$, the order of the operands is insignificant.