1 Advanced Algorithms (TMS)

(*a*) Given an algorithm for the SET-COVER problem as a black box, how could you use this to solve the unweighted VERTEX-COVER problem? [4 marks]

(*b*) Following the approach in Part (*a*), which approximation ratio for the VERTEX-COVER problem do you achieve by applying the greedy algorithm for the SET-COVER problem? What happens if every vertex in the graph has at most 4 neighbours? [6 marks]

(*c*) Consider the following greedy algorithm for the unweighted VERTEX-COVER problem:

Compute a directed Depth-First-Search tree (DFS-tree) from every connected component in the graph, and output all nodes which are not leaves in the DFS-tree (a vertex is a leaf if it has no outgoing edges in the DFS-tree).

(*i*) What is the running time of this algorithm? [2 marks]

(*ii*) Why is the returned solution a valid vertex cover? [4 marks]

(*iii*) Derive a bound, as good as possible, on the approximation ratio of this algorithm.
Hint: You may use the fact that in any undirected graph $G = (V, E)$, $\sum_{u \in V} \deg(u) = 2|E|$, where $\deg(u)$ denotes the number of neighbours of u. [4 marks]