COMPUTER SCIENCE TRIPOS Part II – 2017 – Paper 9

1 Advanced Algorithms (TMS)

(*a*) Give two examples of greedy algorithms and state their approximation ratios. [4 marks]

(*b*) Consider the Centre Selection Problem, defined as follows. The input consists of n points $p_1, p_2, \ldots, p_n$ in a metric space, and an integer $k > 0$. The goal is to find k centres $C = \{c_1, c_2, \ldots, c_k\}$ (not necessarily from among the n points) so that $r(C) = \max_{1 \le i \le n} \mathrm{dist}(p_i, C)$, where $\mathrm{dist}(p_i, C) = \min_{1 \le j \le k} \mathrm{dist}(p_i, c_j)$, is minimised.

(*i*) Consider the standard greedy approach: solve the problem optimally for $k = 1$ and then extend the solution to larger values of k by adding the optimal point to the current solution. Why is this likely to give a poor result? [4 marks]

(*ii*) Consider the following algorithm to solve the Centre Selection Problem:

```
Let C be the empty set
Repeat k times
   Select a point p_i with maximum distance dist(p_i,C)
   Add point p_i to the set C
Return C
```

Derive a lower bound for this algorithm on the minimum pairwise distance among the chosen centres C. [4 marks]

(*iii*) Give an upper bound, as tight as possible, on the approximation ratio of the algorithm in part (*b*)(*ii*). [2 marks]

(*iv*) Give a detailed analysis in order to justify your answer for part (*b*)(*iii*). *Hint*: Exploit the lower bound derived in part (*b*)(*ii*) in order to construct disjoint balls around the centre points. [6 marks]