COMPUTER SCIENCE TRIPOS Part II – 2017 – Paper 7

1 Advanced Algorithms (TMS)

(a) Into what three cases can a linear program in standard form be classified? [3 marks]

(b) Consider the (unweighted) vertex cover problem for the graph $G = (V, E)$ with $V = \{1, 2, 3\}$ and $E = \{\{1, 2\}, \{2, 3\}, \{1, 3\}\}$.

(i) Write down the linear program relaxation for the vertex cover problem and solve the linear program. [6 marks]

(ii) Based on the solution of the linear program in (b)(i), derive an integer solution using the rounding approach described in the lecture. [2 marks]

(c) Consider the following randomised algorithm for the unweighted vertex cover problem:

```
Initialize S to be the empty set
For all edges e=(u,v) do
   If neither u nor v belongs to S
      Randomly choose u or v with probability 1/2
      and add the vertex to S
   End If
End For
Return S
```

Derive an upper bound, as tight as possible, on the approximation ratio of the algorithm.

Hint: Try to find an invariant that bounds from below the size of the intersection of the current solution $S = S(i)$ with the optimum solution, where $S(i)$ denotes the set S after the i-th iteration of the FOR loop. [9 marks]