5 Numerical Methods (DJG)

A programmer is asked to write a function that raises a single-precision number to the power of 2.4. His manager advises him to take the following approach involving squaring and fifth roots, and to use Newton Raphson for the fifth root step.

$$f(x) = x^{2.4} = x^2 \times \sqrt[5]{x^2}$$

(*a*) State why the above identity holds. [1 mark]

(*b*) Sketch an implementation of this approach coded in Java or ML. Use the four standard arithmetic operators but do not use any other built-in maths operators or libraries. You must include an iteration termination strategy. [8 marks]

(*c*) Assuming values of x of around unity, and stating any other assumptions you make, give the worst-case error expected in your result assuming the input x is perfectly-representable. [*Note:* macheps for single precision $\epsilon = 1.19 \times 10^{-7}$ but you may give your answer in terms of ϵ.] [3 marks]

(*d*) What worst-case error arises when the input value was approximated by the nearest representable value and so is already out by as much as $\pm\epsilon/2$? [2 marks]

(*e*) A colleague suggests it would be better to simply use the logarithm and exponential functions in the standard library. Another says you should just use a suitable polynomial evaluation (i.e. a power series) for the complete problem. Discuss whether either of these might be better or worse than the original suggestion in terms of performance and accuracy. [6 marks]