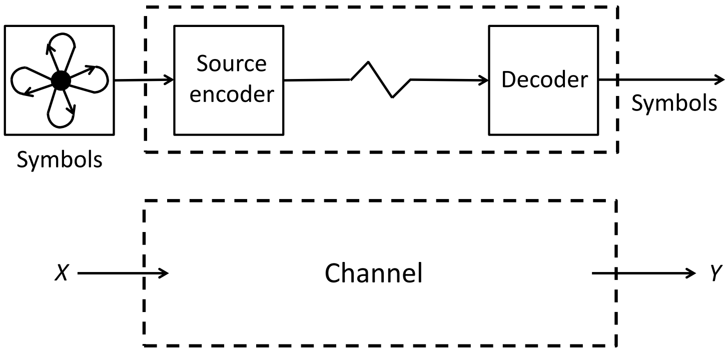
10 Information Theory (JGD)

(*a*) Consider a discrete memoryless channel whose input symbol source is a random variable $X \in \{x_1, \ldots, x_J\}$ having probability distribution $p(x_j)$, and whose output symbol (possibly corrupted) is a random variable $Y \in \{y_1, \ldots, y_K\}$ (see figure below).

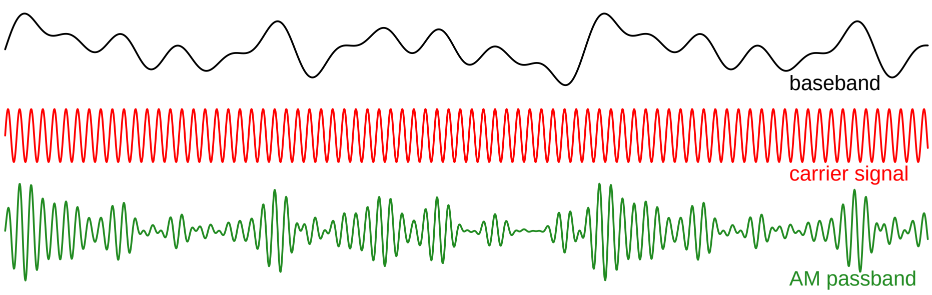


(*i*) Provide its channel matrix. [3 marks]

(*ii*) Give the average probability of correct reception, meaning the probability that the same symbol is emitted as was injected into the channel, averaged over all the cases. [3 marks]

(*b*) Show that convolution of any continuous signal with a Dirac delta function reproduces the signal. [4 marks]

(*c*) A frequency-shifting modulation of signals into different channels of a shared medium multiplies the baseband signal $f(t)$ by a complex exponential carrier wave e^{ict} of some (channel-specific) frequency c to produce a passband $f(t)\,e^{ict}$ (see figure below). Upon reception of such a passband, what process of *demodulation* would recover the original baseband?



[5 marks]

(*d*) Explain the "information diagram" of Gabor, and why the Uncertainty Principle gives it a quantal structure with an irreducible representation of the data. [5 marks]