

13 Types (AMP)

- (a) Give an account of the Polymorphic Lambda Calculus (PLC). You should define the PLC types, expressions and typing environments, give the PLC typing rules, and define the relations of *beta-reduction* and *beta-conversion*. Explain why beta-conversion is a decidable relation for expressions that are typeable.

[10 marks]

- (b) Find, with justification, a PLC type τ for which the following typings are both provable:

$$\{ \} \vdash \Lambda \alpha, \beta (\lambda x : \alpha (\lambda y : \beta (\Lambda \gamma (\lambda z : \tau (z x y)))))) : \forall \alpha, \beta (\alpha \rightarrow (\beta \rightarrow \forall \gamma (\tau \rightarrow \gamma)))$$

$$\{ \} \vdash \Lambda \alpha, \beta (\lambda z : \forall \gamma (\tau \rightarrow \gamma) (z \alpha (\lambda x : \alpha (\lambda y : \beta (x))))) : \forall \alpha, \beta ((\forall \gamma (\tau \rightarrow \gamma)) \rightarrow \alpha)$$

[5 marks]

- (c) Give infinitely many different closed PLC expressions in beta-normal form of type $\forall \alpha ((\alpha \rightarrow \alpha) \rightarrow (\alpha \rightarrow \alpha))$.

[2 marks]

- (d) Use your answer to part (a) to show that there is no closed PLC expression Y of type $\forall \alpha ((\alpha \rightarrow \alpha) \rightarrow \alpha)$ for which the beta-reduction $Y \alpha f \rightarrow f(Y \alpha f)$ holds.

[3 marks]