2 Artificial Intelligence I (SBH)

This question relates to binary *constraint satisfaction problems (CSPs).* A CSP has a set $X = \{x_1, \ldots, x_n\}$ of variables, each having a domain $D_i = \{v_1, \ldots, v_{n_i}\}$ of values. In addition, a CSP has a set $C = \{C_1, \ldots, C_m\}$ of constraints, each relating to a subset of X and specifying the allowable combinations of assignments to the variables in that subset.

(*a*) Give a general definition of a *solution* to a CSP. [1 mark]

(*b*) Given a binary CSP, define what it means for a directed arc $x_i \rightarrow x_j$ between variables x_i and x_j to be *arc consistent.* [2 marks]

(*c*) Give an example of how a directed arc $x_i \rightarrow x_j$ can fail to be arc consistent. Explain how this can be fixed. [2 marks]

(*d*) Describe the *AC-3* algorithm for enforcing arc consistency. [5 marks]

(*e*) Prove that the time complexity of the *AC-3* algorithm is $O(n^2d^3)$ where d is the size of the largest domain. [3 marks]

(*f*) Suggest a way in which the concept of arc consistency, also known as *2-consistency* can be extended to sets of three, rather than two variables. In the remainder of the question we will refer to this as *3-consistency.* [1 mark]

(*g*) Give an example of how a set of three variables might fail to be 3-consistent, and show how 3-consistency might then be imposed. [2 marks]

(*h*) Suggest a modified version of the *AC-3* algorithm that can be used to enforce 3-consistency. [4 marks]