COMPUTER SCIENCE TRIPOS Part IA – 2014 – Paper 2

10 Discrete Mathematics (AMP)

(*a*) For each symbol x in the alphabet $\Sigma = \{a, b, c\}$, let O_x be the language over Σ consisting of all strings that contain an odd number of occurrences of the symbol x; and let E_x be the language of strings over Σ containing an even number of occurrences of the symbol x.

(*i*) Give a deterministic finite automaton whose language of accepted strings is O_a. [2 marks]

(*ii*) Give a regular expression whose language of matching strings is O_a. [2 marks]

(*iii*) Give a deterministic finite automaton whose language of accepted strings is $O_a \cap E_b$. [4 marks]

(*b*) $M = (Q, \Sigma, \delta, s, F)$ is a deterministic finite automaton whose set of states Q has ℓ elements. Suppose that M accepts a string $w \in \Sigma^*$ whose length $|w|$ satisfies $|w| \geq \ell$.

(*i*) Show that $w = u_1 v u_2$ for some strings $u_1, v, u_2 \in \Sigma^*$ such that $|u_1| < \ell$, $1 \leq |v| \leq \ell$ and M accepts $u_1 v^n u_2$ for all $n \in \mathbb{N} = \{0, 1, 2, \ldots\}$. [7 marks]

(*ii*) Hence show that if infinitely many strings are accepted by M, then it must accept some string w' with $\ell \leq |w'| < 2\ell$. [5 marks]