10   Algorithms (TMS)

(a) Given any directed graph $G = (V, E)$ with non-negative edge weights, consider
the problem of *all-pairs shortest path* (APSP). Give the asymptotic runtimes of
the following four algorithms when applied (directly or iterated) to the APSP
problem as a function of $|V|$ and $|E|$, and provide a brief justification for your
answer: Bellman-Ford, Dijkstra, matrix multiplication and Johnson.   [8 marks]

(b) Consider the problem of converting currencies modelled by a directed graph
$G = (V, E)$ with $|V|$ vertices representing currencies and $|E|$ directed edges
(u, v) each of which has a strictly positive weight $w(u, v) > 0$ representing
the exchange rate. For instance, for any real number x, we have x USD $=$
$w(\text{dollars, pounds}) \cdot x$ GBP. Our goal is, given a pair of currencies $s, t \in V$, to
find the least expensive way of exchanging from s to t, possibly by using more
than one exchange.

(i)   How could you transform the graph by reweighting the edges so that the
problem could be solved with a shortest path algorithm? Indicate which
shortest path algorithm is used.   [8 marks]

(ii) How would you deal with negative-weight cycles if they occurred in the
transformed graph? Give the perspective of the currency trader as well as
that of a computer scientist.   [4 marks]