

COMPUTER SCIENCE TRIPOS Part IB – 2013 – Paper 6

3 Computation Theory (AMP)

(a) What does it mean for a register machine to be *universal*? [4 marks]

(b) Define what it means for a partial function $f \in \mathbb{N}^n \rightarrow \mathbb{N}$ to be *register machine computable*. [3 marks]

(c) Show that the following functions f, g, h, k are register machine computable.

(i) The partial function $f \in \mathbb{N} \rightarrow \mathbb{N}$ that is everywhere undefined. [1 mark]

(ii) $g(x_1, x_2) = \begin{cases} x_1 - x_2 & \text{if } x_1 \geq x_2 \\ 0 & \text{if } x_1 < x_2 \end{cases}$ [4 marks]

(iii) $h(x_1) = \begin{cases} 2^{x_1-1} & \text{if } x_1 > 0 \\ \text{undefined} & \text{if } x_1 = 0 \end{cases}$ [4 marks]

(iv) $k(x_1, x_2) = 1$ if the register machine program with index x_1 , when started with 0 in all registers, halts in at most x_2 steps; and $k(x_1, x_2) = 0$ otherwise. [4 marks]