7 Probability (RJG)

Suppose that N is a random variable taking values $j = 0, 1, 2 \dots$ with

$$\mathbb{P}(N = j) = (1 - \rho)\rho^j$$

and where $0 < \rho < 1$.

(a) Show that $\mathbb{P}(N = j)$ is a *probability mass function*. [2 marks]

(b) For $k = 0, 1, 2, \dots$ derive an expression for $\mathbb{P}(N > k)$. [2 marks]

(c) Derive the *probability generating function*, $G_N(z)$, for the random variable N, stating carefully any conditions required for it to be well-defined and use it to determine

(i) $\mathbb{E}(N)$

(ii) $\mathrm{Var}(N)$

[8 marks]

(d) For a random variable X, which takes non-negative integer values, show that

$$\mathbb{E}(X) = \sum_{k=0}^{\infty} \mathbb{P}(X > k).$$

[6 marks]

(e) By evaluating $\sum_{k=0}^{\infty} \mathbb{P}(N > k)$ show that this expression equals the value derived in part (c)(i) for $\mathbb{E}(N)$.

[2 marks]